User Guide for CHOLMOD: a sparse Cholesky factorization and modification package

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VERSION 5.2.0, Jan 20, 2024

Abstract

CHOLMOD[1] is a set of routines for factorizing sparse symmetric positive definite matrices of the form $\mathbf{A}$ or $\mathbf{AA}^{\mathsf{T}}$, updating/downdating a sparse Cholesky factorization, solving linear systems, updating/downdating the solution to the triangular system $\mathbf{Lx} = \mathbf{b}$, and many other sparse matrix functions for both symmetric and unsymmetric matrices. Its supernodal Cholesky factorization relies on LAPACK and the Level-3 BLAS, and obtains a substantial fraction of the peak performance of the BLAS. Both real and complex matrices are supported. It also includes a non-supernodal $\mathbf{LDL}^T$ factorization method that can factorize symmetric indefinite matrices if all of their leading submatrices are well-conditioned ($\mathbf{D}$ is diagonal). CHOLMOD is written in C11, with both C and MATLAB interfaces. The package works on Linux, Mac, and Windows.

[1] CHOLMOD is short for CHOLesky MODification, since a key feature of the package is its ability to update/downdate a sparse Cholesky factorization

Contents

1 Overview

CHOLMOD is a set of ANSI C routines for solving systems of linear equations, $\mathbf{Ax} = \mathbf{b}$, when $\mathbf{A}$ is sparse and symmetric positive definite, and $\mathbf{x}$ and $\mathbf{b}$ can be either sparse or dense.[2] Complex matrices are supported, in two different formats. CHOLMOD includes high-performance left-looking supernodal factorization and solve methods [21], based on LAPACK [3] and the BLAS [12]. After a matrix is factorized, its factors can be updated or downdated using the techniques described by Davis and Hager in [8, 9, 10]. Many additional sparse matrix operations are provided, for both symmetric and unsymmetric matrices (square or rectangular), including sparse matrix multiply, add, transpose, permutation, scaling, norm, concatenation, sub-matrix access, and converting to alternate data structures. My recent GraphBLAS package will typically be faster for these kinds of operations, however. Interfaces to many ordering methods are provided, including minimum degree (AMD [1, 2], COLAMD [6, 7]), constrained minimum degree (CSYMAMD, CCOLAMD, CAMD), and graph-partitioning-based nested dissection (METIS [18]). Most of its operations are available within MATLAB via mexFunction interfaces.

CHOLMOD also includes a non-supernodal $\mathbf{LDL}^T$ factorization method that can factorize symmetric indefinite matrices if all of their leading submatrices are well-conditioned ($\mathbf{D}$ is diagonal).

A pair of articles on CHOLMOD appears in the ACM Transactions on Mathematical Software: [4, 11].

CHOLMOD appears as `chol` (the sparse case), `symbfact`, and `etree` in MATLAB 7.2 (R2006a) and later, and is used for `x=A\b` when `A` is symmetric positive definite [14].

The C-callable CHOLMOD library consists of many user-callable routines and one include file. Each routine comes in two versions, one for `int32_t` integers and another for `int64_t`. Many of the routines can support either real or complex matrices, simply by passing a matrix of the appropriate type. All of the routines support both `double` and single (`float`) precision matrices (this is new in CHOLMOD 5.1).

Nick Gould, Yifan Hu, and Jennifer Scott have independently tested CHOLMOD's performance, comparing it with nearly a dozen or so other solvers [17, 16]. Its performance was quite competitive.

2 Single-precision sparse matrix support

CHOLMOD v5.1.0: introduces full of support for single precision sparse matrices, with the introduction of the new CHOLMOD:Utility Module. The CHOLMOD:Utility Module replaces the CHOLMOD:Core Module that appeared in prior versions of CHOLMOD.

[2] Some support is provided for symmetric indefinite matrices.

3 Primary routines and data structures

Five primary CHOLMOD routines are required to factorize $\mathbf{A}$ or $\mathbf{AA}^\mathsf{T}$ and solve the related system $\mathbf{Ax} = \mathbf{b}$ or $\mathbf{AA}^\mathsf{T}\mathbf{x} = \mathbf{b}$, for either the real or complex cases:

1. `cholmod_start`: This must be the first call to CHOLMOD.
2. `cholmod_analyze`: Finds a fill-reducing ordering, and performs the symbolic factorization, either simplicial (non-supernodal) or supernodal.
3. `cholmod_factorize`: Numerical factorization, either simplicial or supernodal, $\mathbf{LL}^\mathsf{T}$ or $\mathbf{LDL}^\mathsf{T}$ using either the symbolic factorization from `cholmod_analyze` or the numerical factorization from a prior call to `cholmod_factorize`.
4. `cholmod_solve`: Solves $\mathbf{Ax} = \mathbf{b}$, or many other related systems, where $\mathbf{x}$ and $\mathbf{b}$ are dense matrices. The `cholmod_spsolve` routine handles the sparse case. Any mixture of real and complex $\mathbf{A}$ and $\mathbf{b}$ are allowed.
5. `cholmod_finish`: This must be the last call to CHOLMOD.

Additional routines are also required to create and destroy the matrices $\mathbf{A}$, $\mathbf{x}$, $\mathbf{b}$, and the $\mathbf{LL}^\mathsf{T}$ or $\mathbf{LDL}^\mathsf{T}$ factorization. CHOLMOD has five kinds of data structures, referred to as objects and implemented as pointers to `struct`'s:

1. `cholmod_common`: parameter settings, statistics, and workspace used internally by CHOLMOD. See Section 13 for details.
2. `cholmod_sparse`: a sparse matrix in compressed-column form, either pattern-only, real, complex, or "zomplex." In its basic form, the matrix `A` contains:
   - `A->p`, an integer array of size `A->ncol+1`.
   - `A->i`, an integer array of size `A->nzmax`.
   - `A->x`, a `double` or `float` array of size `A->nzmax` or twice that for the complex case. This is compatible with the Fortran and ANSI C99 complex data type.
   - `A->z`, a `double` or `float` array of size `A->nzmax` if `A` is zomplex. A zomplex matrix has a `z` array, thus the name. This is compatible with the MATLAB representation of complex matrices.
   - `A->nz`, an integer array of size `A->ncol`, if `A` is in the *unpacked* format. In this format, the columns of `A` can appear out of order, and gaps of unused space can appear between columns.

   For all four types of matrices, the row indices of entries of column `j` are located in `A->i [A->p [j] ... A->p [j+1]-1]`, or in `A->i [A->p [j] ... A->p [j] + A->nz [j] -1]`, if in the unpacked format. For a real matrix, the corresponding numerical values are in `A->x` at the same location. For a complex matrix, the entry whose row index is `A->i [p]` is contained in `A->x [2*p]` (the real part) and `A->x [2*p+1]` (the imaginary part). For a zomplex matrix, the real part is in `A->x [p]` and imaginary part is in `A->z [p]`. See Section 14 for more details.

3. `cholmod_factor`: A symbolic or numeric factorization, either real, complex, or zomplex. It can be either an $\mathbf{LL}^{\mathsf{T}}$ or $\mathbf{LDL}^{\mathsf{T}}$ factorization, and either simplicial or supernodal. You will normally not need to examine its contents. See Section 15 for more details.

4. `cholmod_dense`: A dense matrix, either real, complex or zomplex, in column-major order. This differs from the row-major convention used in C. A dense matrix `X` contains

   - `X->x`, a `double` or `float` array of size `X->nzmax` or twice that for the complex case.
   - `X->z`, a `double` or `float` array of size `X->nzmax` if `X` is zomplex.

   For a real dense matrix x_{ij} is `X->x [i+j*d]` where `d = X->d` is the leading dimension of `X`. For a complex dense matrix, the real part of x_{ij} is `X->x [2*(i+j*d)]` and the imaginary part is `X->x [2*(i+j*d)+1]`. For a zomplex dense matrix, the real part of x_{ij} is `X->x [i+j*d]` and the imaginary part is `X->z [i+j*d]`. Real and complex dense matrices can be passed to LAPACK and the BLAS. See Section 16 for more details.

5. `cholmod_triplet`: CHOLMOD's sparse matrix (`cholmod_sparse`) is the primary input for nearly all CHOLMOD routines, but it can be difficult for the user to construct. A simpler method of creating a sparse matrix is to first create a `cholmod_triplet` matrix, and then convert it to a `cholmod_sparse` matrix via the `cholmod_triplet_to_sparse` routine. In its basic form, the triplet matrix `T` contains

   - `T->i` and `T->j`, integer arrays of size `T->nzmax`.
   - `T->x`, a `double` or `float` array of size `T->nzmax` or twice that for the complex case.
   - `T->z`, a `double` or `float` array of size `T->nzmax` if `T` is zomplex.

   The `k`th entry in the data structure has row index `T->i [k]` and column index `T->j [k]`. For a real triplet matrix, its numerical value is `T->x [k]`. For a complex triplet matrix, its real part is `T->x [2*k]` and its imaginary part is `T->x [2*k+1]`. For a zomplex matrix, the real part is `T->x [k]` and imaginary part is `T->z [k]`. The entries can be in any order, and duplicates are permitted. See Section 17 for more details.

Each of the five objects has a routine in CHOLMOD to create and destroy it. CHOLMOD provides many other operations on these objects as well. A few of the most important ones are illustrated in the sample program in the next section.

4 Simple example program

```
#include "cholmod.h"
int main (void)
{
    cholmod_sparse *A ;
    cholmod_dense *x, *b, *r ;
    cholmod_factor *L ;
    double one [2] = {1,0}, m1 [2] = {-1,0} ;       // basic scalars
    cholmod_common c ;
    cholmod_start (&c) ;                            // start CHOLMOD
    int dtype = CHOLMOD_DOUBLE ;                    // use double precision
    A = cholmod_read_sparse2 (stdin, dtype, &c) ;   // read in a matrix
    c.precise = true ;
    c.print = (A->nrow > 5) ? 3 : 5 ;
    cholmod_print_sparse (A, "A", &c) ;             // print the matrix
    if (A == NULL || A->stype == 0)                 // A must be symmetric
    {
        cholmod_free_sparse (&A, &c) ;
        cholmod_finish (&c) ;
        return (0) ;
    }
    b = cholmod_ones (A->nrow, 1, A->xtype + dtype, &c) ;   // b = ones(n,1)

    double t1 = SUITESPARSE_TIME ;
    L = cholmod_analyze (A, &c) ;                   // analyze
    t1 = SUITESPARSE_TIME - t1 ;
    double t2 = SUITESPARSE_TIME ;
    cholmod_factorize (A, L, &c) ;                  // factorize
    t2 = SUITESPARSE_TIME - t2 ;
    double t3 = SUITESPARSE_TIME ;
    x = cholmod_solve (CHOLMOD_A, L, b, &c) ;       // solve Ax=b
    t3 = SUITESPARSE_TIME - t3 ;
    printf ("analyze   time: %10.3f sec\n", t1) ;
    printf ("factorize time: %10.3f sec\n", t2) ;
    printf ("solve     time: %10.3f sec\n", t3) ;
    printf ("total     time: %10.3f sec\n", t1 + t2 + t3) ;

    cholmod_print_factor (L, "L", &c) ;             // print the factorization
    cholmod_print_dense (x, "x", &c) ;              // print the solution
    r = cholmod_copy_dense (b, &c) ;                // r = b
#ifndef NMATRIXOPS
    cholmod_sdmult (A, 0, m1, one, x, r, &c) ;      // r = r-Ax
    double rnorm = cholmod_norm_dense (r, 0, &c) ;  // compute inf-norm of r
    double anorm = cholmod_norm_sparse (A, 0, &c) ; // compute inf-norm of A
    printf ("\n%s precision results:\n", dtype ? "single" : "double") ;
    printf ("norm(b-Ax) %8.1e\n", rnorm) ;
    printf ("norm(A)    %8.1e\n", anorm) ;
    double relresid = rnorm / anorm ;
    printf ("resid: norm(b-Ax)/norm(A) %8.1e\n", relresid) ;
    fprintf (stderr, "resid: norm(b-Ax)/norm(A) %8.1e\n", relresid) ;
#else
    printf ("residual norm not computed (requires CHOLMOD/MatrixOps)\n") ;
#endif
    cholmod_free_factor (&L, &c) ;                  // free matrices
```

```
    cholmod_free_sparse (&A, &c) ;
    cholmod_free_dense (&r, &c) ;
    cholmod_free_dense (&x, &c) ;
    cholmod_free_dense (&b, &c) ;
    cholmod_print_common ("common", &c) ;
    cholmod_gpu_stats (&c) ;
    cholmod_finish (&c) ;                          // finish CHOLMOD
    return (0) ;
}
```

Purpose: The `Demo/cholmod_*_simple.c` programs illustrate the basic usage of CHOLMOD. Each of them reads a triplet matrix from a file (in Matrix Market format), convert it into a sparse matrix, creates a linear system, solves it, and prints the norm of the residual. See the `CHOLMOD/Demo/cholmod_*_demo.c` program for a more elaborate example. These two sets of programs come in four variants:

- di: `double` values, `int32_t` integers.
- dl: `double` values, `int64_t` integers.
- si: `float` values, `int32_t` integers.
- sl: `float` values, `int64_t` integers.

5 Installation of the C-callable library

CHOLMOD requires a suite of external packages, many of which are distributed along with CHOLMOD, but three of which are not. Those included with CHOLMOD are:

- AMD: an approximate minimum degree ordering algorithm, by Tim Davis, Patrick Amestoy, and Iain Duff [1, 2].
- COLAMD: an approximate column minimum degree ordering algorithm, by Tim Davis, Stefan Larimore, John Gilbert, and Esmond Ng [6, 7].
- CCOLAMD: a constrained approximate column minimum degree ordering algorithm, by Tim Davis and Siva Rajamanickam, based directly on COLAMD. This package is not required if CHOLMOD is compiled with the `-DNCAMD` flag.
- CAMD: a constrained approximate minimum degree ordering algorithm, by Tim Davis and Yanqing Chen, based directly on AMD. This package is not required if CHOLMOD is compiled with the `-DNCAMD` flag.
- `SuiteSparse_config`: a single place where all sparse matrix packages authored or co-authored by Davis are configured.
- `METIS 5.1.0`: a graph partitioning package by George Karypis, Univ. of Minnesota. A slightly revised copy appears in `CHOLMOD/SuiteSparse_metis`, but with a revised namespace so that no METIS functions in CHOLMOD conflict with the unmodified METIS. SuiteSparse cannot use any other METIS library except for `CHOLMOD/SuiteSparse_metis`. There is no conflict with the unmodified METIS, so it can be linked into the user's application alongside `CHOLMOD/SuiteSparse_metis`. This folder containing the modified METIS is not needed if `-DNPARTITION` is used. See http://www-users.cs.umn.edu/~karypis/metis for the original version of METIS.

Three other packages are required for optimal performance:

- BLAS: the Basic Linear Algebra Subprograms. Not needed if `-DNSUPERNODAL` is used. See http://www.netlib.org for the reference BLAS (not meant for production use). I recommend the Intel MKL BLAS.
- LAPACK: the Basic Linear Algebra Subprograms. Not needed if `-DNSUPERNODAL` is used. See http://www.netlib.org.
- CUDA BLAS: CHOLMOD can exploit an NVIDIA GPU by using the CUDA BLAS for large supernodes.

You must first obtain and install LAPACK, and the BLAS.
CHOLMOD's specific settings are revised by the following compile-time flags for the compiler:

- `-DNCHECK`: do not include the Check module.
- `-DNCHOLESKY`: do not include the Cholesky module.

- `-DNPARTITION`: do not include the interface to METIS in the Partition module.
- `-DNCAMD`: do not include the interfaces to CAMD, CCOLAMD, and CSYMAMD in the Partition module.
- `-DNMATRIXOPS`: do not include the MatrixOps module. Note that the Demo requires the MatrixOps module.
- `-DNMODIFY`: do not include the Modify module.
- `-DNSUPERNODAL`: do not include the Supernodal module.
- `-DNPRINT`: do not print anything.

These settings are controlled in cmake by the following variables, which are all `ON` by default:

- `CHOLMOD_CHECK`: if `OFF`: do not use the Check module.
- `CHOLMOD_CHOLESKY`: if `OFF`: do not use the Cholesky module.
- `CHOLMOD_PARTITION`: if `OFF`: do not use the interface to METIS in the Partition module.
- `CHOLMOD_CAMD`: if `OFF`: do not use the interfaces to CAMD, CCOLAMD, and CSYMAMD in the Partition module.
- `CHOLMOD_MATRIXOPS`: if `OFF`: do not use the MatrixOps module.
- `CHOLMOD_MODIFY`: if `OFF`: do not use the Modify module.
- `CHOLMOD_SUPERNODAL`: if `OFF`: do not use the Supernodal module.
- `CHOLMOD_GPL`: if `OFF`: do not use any GPL-licensed module (MatrixOps, Modify, Supernodal, and GPU-module).

SuiteSparse now has a complete `cmake`-based build system. Each package (SuiteSparse_congig, AMD, CAMD, CCOLAMD, CAMD, and CHOLMOD) has its own `CMakeLists.txt`. Use `cmake` to build each package in that order. Alternatively, you can use a single top-level `CMakeLists.txt` file to build all packages in SuiteSparse.

An optional `Makefile` is provided at the top-level of `SuiteSparse` Type `make` in that directory. The AMD, COLAMD, CAMD, CCOLAMD, and `CHOLMOD` libraries will be compiled. To compile and run demo programs for each package, type `make demos`. For CHOLMOD, the residuals should all be small.

CHOLMOD is now ready for use in your own applications. You must link your programs with the `libcholmod.*`, `libamd.*`, `libcolamd.*`, LAPACK, and BLAS libraries. Unless `-DNCAMD` is present at compile time, you must link with `CAMD/libcamd.*`, and `CCOLAMD/libccolamd.*`. Each library has its own `*Config.cmake` script to use in the `cmake find_package` command.

To install CHOLMOD in default locations use `make install`. To remove CHOLMOD, do `make uninstall`.

6 Using CHOLMOD in MATLAB

CHOLMOD includes a set of m-files and mexFunctions in the CHOLMOD/MATLAB directory. The following functions are provided:

`analyze`	order and analyze a matrix
`bisect`	find a node separator
`chol2`	same as `chol`
`cholmod2`	same as `x=A\b` `A` is symmetric positive definite
`cholmod_demo`	a short demo program
`cholmod_make`	compiles CHOLMOD for use in MATLAB
`etree2`	same as `etree`
`graph_demo`	graph partitioning demo
`lchol`	`L*L'` factorization
`ldlchol`	`L*D*L'` factorization
`ldl_normest`	estimate `norm(A-L*D*L')`
`ldlsolve`	`x = L'\ (D \(L \ b))`
`ldlsplit`	split the output of `ldlchol` into `L` and `D`
`ldlupdate`	update/downdate an `L*D*L'` factorization
`ldlrowmod`	add/delete a row from an `L*D*L'` factorization
`metis`	interface to `METIS_NodeND` ordering
`mread`	read a sparse or dense Matrix Market file
`mwrite`	write a sparse or dense Matrix Market file
`nesdis`	CHOLMOD's nested dissection ordering
`resymbol`	recomputes the symbolic factorization
`sdmult`	`S*F` where `S` is sparse and `F` is dense
`spsym`	determine symmetry
`symbfact2`	same as `symbfact`

Each function is described in the next sections.

6.1 `analyze`: order and analyze

```
ANALYZE order and analyze a matrix using CHOLMOD's best-effort ordering.

   Example:
   [p count] = analyze (A)         orders A, using just tril(A)
   [p count] = analyze (A,'sym')   orders A, using just tril(A)
   [p count] = analyze (A,'row')   orders A*A'
   [p count] = analyze (A,'col')   orders A'*A

   an optional 3rd parameter modifies the ordering strategy:

   [p count] = analyze (A,'sym',k) orders A, using just tril(A)
   [p count] = analyze (A,'row',k) orders A*A'
   [p count] = analyze (A,'col',k) orders A'*A

 Returns a permutation and the count of the number of nonzeros in each
 column of L for the permuted matrix A.  That is, count is returned as:
```

```
    count = symbfact2 (A (p,p))       if ordering A
    count = symbfact2 (A (p,:),'row') if ordering A*A'
    count = symbfact2 (A (:,p),'col') if ordering A'*A

CHOLMOD uses the following ordering strategy:

     k = 0:  Try AMD.  If that ordering gives a flop count >= 500 *
         nnz(L) and a fill-in of nnz(L) >= 5*nnz(C), then try metis
         (where C=A, A*A', or A'*A is the matrix being ordered.
         Selects the best ordering tried.  This is the default.

     if k > 0, then multiple orderings are attempted.

     k = 1 or 2: just try AMD
     k = 3: also try METIS_NodeND
     k = 4: also try NESDIS, CHOLMOD's nested dissection (NESDIS), with
          default parameters.  Uses METIS's node bisector and CCOLAMD.
     k = 5: also try the natural ordering (p = 1:n)
     k = 6: also try NESDIS with large leaves of the separator tree
     k = 7: also try NESDIS with tiny leaves and no CCOLAMD ordering
     k = 8: also try NESDIS with no dense-node removal
     k = 9: also try COLAMD if ordering A'*A or A*A', AMD if ordering A
     k > 9 is treated as k = 9

     k = -1: just use AMD
     k = -2: just use METIS
     k = -3: just use NESDIS

     The method returning the smallest nnz(L) is used for p and count.
     k = 4 takes much longer than (say) k = 0, but it can reduce
     nnz(L) by a typical 5% to 10%.  k = 5 to 9 is getting extreme,
     but if you have lots of time and want to find the best ordering
     possible, set k = 9.

If METIS is not installed for use in CHOLMOD, then the strategy is
different:

     k = 1 to 4: just try AMD
     k = 5 to 8: also try the natural ordering (p = 1:n)
     k = 9: also try COLAMD if ordering A'*A or A*A', AMD if ordering A
     k > 9 is treated as k = 9

See also metis, nesdis, bisect, symbfact, amd.
```

6.2 `bisect`: find a node separator

```
BISECT computes a node separator based on METIS_ComputeVertexSeparator.

  Example:
  s = bisect(A)       bisects A. Uses tril(A) and assumes A is symmetric
  s = bisect(A,'sym') the same as p=bisect(A)
  s = bisect(A,'col') bisects A'*A
```

```
  s = bisect(A,'row') bisects A*A'

A must be square for p=bisect(A) and bisect(A,'sym').

s is a vector of length equal to the dimension of A, A'*A, or A*A',
depending on the matrix bisected.  s(i)=0 if node i is in the left
subgraph, s(i)=1 if it is in the right subgraph, and s(i)=2 if node i
is in the node separator.

Requires METIS, authored by George Karypis, Univ. of Minnesota.  This
MATLAB interface, via CHOLMOD, is by Tim Davis.

See also metis, nesdis.
```

6.3 chol2: same as chol

```
CHOL2 sparse Cholesky factorization, A=R'R.
 Note that A=L*L' (LCHOL) and A=L*D*L' (LDLCHOL) factorizations are
 faster than R'*R (CHOL2 and CHOL) and use less memory.  The LL' and
 LDL' factorization methods use tril(A).  This method uses triu(A), just
 like the built-in CHOL.

   Example:
   R = chol2 (A)                 same as R = chol (A), just faster
   [R,p] = chol2 (A)             same as [R,p] = chol(A), just faster
   [R,p,q] = chol2 (A)           factorizes A(q,q) into R'*R, where q is
                                 a fill-reducing ordering

 A must be sparse.

 See also lchol, ldlchol, chol, ldlupdate.
```

6.4 cholmod2: supernodal backslash

```
CHOLMOD2 supernodal sparse Cholesky backslash, x = A\b

   Example:
   x = cholmod2 (A,b)

 Computes the LL' factorization of A(p,p), where p is a fill-reducing
 ordering, then solves a sparse linear system Ax=b. A must be sparse,
 symmetric, and positive definite).  Uses only the upper triangular part
 of A.  A second output, [x,stats]=cholmod2(A,b), returns statistics:

   stats(1)    estimate of the reciprocal of the condition number
   stats(2)    ordering used:
                   0: natural, 1: given, 2:amd, 3:metis, 4:nesdis,
                   5:colamd, 6: natural but postordered.
   stats(3)    nnz(L)
   stats(4)    flop count in Cholesky factorization.  Excludes solution
```

```
              of upper/lower triangular systems, which can be easily
              computed from stats(3) (roughly 4*nnz(L)*size(b,2)).
  stats(5)    memory usage in MB.

The 3rd argument select the ordering method to use.  If not present or
-1, the default ordering strategy is used (AMD, and then try METIS if
AMD finds an ordering with high fill-in, and use the best method
tried).

  Other options for the ordering parameter:

      0   natural (no etree postordering)
      -1  use CHOLMOD's default ordering strategy (AMD, then try METIS)
      -2  AMD, and then try NESDIS (not METIS) if AMD has high fill-in
      -3  use AMD only
      -4  use METIS only
      -5  use NESDIS only
      -6  natural, but with etree postordering
      p   user permutation (vector of size n, with a permutation of 1:n)

See also chol, mldivide.
```

6.5 cholmod_demo: a short demo program

```
CHOLMOD_DEMO a demo for CHOLMOD

Tests CHOLMOD with the sparse matrix problem used in the MATLAB bench
program, with various sizes.  Note that MATLAB uses CHOLMOD itself for
x=A\b, chol, etc. so the timings should be comparable.

See CHOLMOD/MATLAB/Test/cholmod_test.m for a lengthy test using
matrices from the SuiteSparse Matrix Collection.

Example:
  cholmod_demo

See also bench.
```

6.6 cholmod_make: compile CHOLMOD in MATLAB

```
CHOLMOD_MAKE compiles the CHOLMOD mexFunctions

Example:
  cholmod_make

CHOLMOD relies on AMD and COLAMD, and optionally CCOLAMD, CAMD, and
METIS.  You must type the cholmod_make command while in the
CHOLMOD/MATLAB directory.

See also analyze, bisect, chol2, cholmod2, etree2, lchol, ldlchol,
```

```
   ldlsolve, ldlupdate, metis, spsym, nesdis, septree, resymbol, sdmult,
   symbfact2, mread, mwrite, ldlrowmod.
```

6.7 etree2: same as etree

```
ETREE2  sparse elimination tree.
 Finds the elimination tree of A, A'*A, or A*A', and optionaly
 postorders the tree.  parent(j) is the parent of node j in the tree, or
 0 if j is a root.  The symmetric case uses only the upper or lower
 triangular part of A (etree2(A) uses the upper part, and etree2(A,'lo')
 uses the lower part).

 Example:
   parent = etree2 (A)         finds the etree of A, using triu(A)
   parent = etree2 (A,'sym')   same as etree2(A)
   parent = etree2 (A,'col')   finds the etree of A'*A
   parent = etree2 (A,'row')   finds the etree of A*A'
   parent = etree2 (A,'lo')    finds the etree of A, using tril(A)

 [parent,post] = etree2 (...) also returns a post-ordering of the tree.

 If you have a fill-reducing permutation p, you can combine it with an
 elimination tree post-ordering using the following code.  Post-ordering
 has no effect on fill-in (except for lu), but it does improve the
 performance of the subsequent factorization.

 For the symmetric case, suitable for chol(A(p,p)):

       [parent post] = etree2 (A (p,p)) ;
       p = p (post) ;

 For the column case, suitable for qr(A(:,p)) or lu(A(:,p)):

       [parent post] = etree2 (A (:,p), 'col') ;
       p = p (post) ;

 For the row case, suitable for qr(A(p,:)') or chol(A(p,:)*A(p,:)'):

       [parent post] = etree2 (A (p,:), 'row') ;
       p = p (post) ;

 See also treelayout, treeplot, etreeplot, etree.
```

6.8 graph_demo: graph partitioning demo

```
GRAPH_DEMO graph partitioning demo
 graph_demo(n) constructs an set of n-by-n 2D grids, partitions them, and
 plots them in one-second intervals.  n is optional; it defaults to 60.

 Example:
```

```
      graph_demo

 See also delsq, numgrid, gplot, treeplot.
```

6.9 `lchol`: LL^T factorization

```
LCHOL sparse A=L*L' factorization.
 Note that L*L' (LCHOL) and L*D*L' (LDLCHOL) factorizations are faster
 than R'*R (CHOL2 and CHOL) and use less memory.  The LL' and LDL'
 factorization methods use tril(A).  A must be sparse.

 Example:
   L = lchol (A)         same as L = chol (A')', just faster
   [L,p] = lchol (A)     same as [R,p] = chol(A') ; L=R', just faster
   [L,p,q] = lchol (A)   factorizes A(q,q) into L*L', where q is a
                         fill-reducing ordering

 See also chol2, ldlchol, chol.
```

6.10 `ldlchol`: LDL^T factorization

```
LDLCHOL sparse A=LDL' factorization
 Note that L*L' (LCHOL) and L*D*L' (LDLCHOL) factorizations are faster
 than R'*R (CHOL2 and CHOL) and use less memory.  The LL' and LDL'
 factorization methods use tril(A).  A must be sparse.

 Example:
   LD = ldlchol (A)            return the LDL' factorization of A
   [LD,p] = ldlchol (A)        similar [R,p] = chol(A), but for L*D*L'
   [LD,p,q] = ldlchol (A)      factorizes A(q,q) into L*D*L', where q is
                               a fill-reducing ordering
   LD = ldlchol (A,beta)       LDL' factorization of A*A'+beta*I
   [LD,p] = ldlchol (A,beta)   like [R,p] = chol(A*A'+beta+I)
   [LD,p,q] = ldlchol (A,beta) factorizes A(q,:)*A(q,:)'+beta*I = L*D*L'

 The output matrix LD contains both L and D.  D is on the diagonal of
 LD, and L is contained in the strictly lower triangular part of LD.
 The unit- diagonal of L is not stored.  You can obtain the L and D
 matrices with [L,D] = ldlsplit (LD).  LD is in the form needed by
 ldlupdate.

 Explicit zeros may appear in the LD matrix.  The pattern of LD matches
 the pattern of L as computed by symbfact2, even if some entries in LD
 are explicitly zero.  This is to ensure that ldlupdate and ldlsolve
 work properly.  You must NOT modify LD in MATLAB itself and then use
 ldlupdate or ldlsolve if LD contains explicit zero entries; ldlupdate
 and ldlsolve will fail catastrophically in this case.

 You MAY modify LD in MATLAB if you do not pass it back to ldlupdate or
 ldlsolve.  Just be aware that LD contains explicit zero entries,
```

```
contrary to the standard practice in MATLAB of removing those entries
from all sparse matrices.  LD = sparse (LD) will remove any zero
entries in LD.

See also ldlupdate, ldlsolve, ldlsplit, chol2, lchol, chol.
```

6.11 `ldlsolve`: solve using an $\mathbf{LDL}^{\mathsf{T}}$ factorization

```
LDLSOLVE solve LDL'x=b using a sparse LDL' factorization

 Example:
   x = ldlsolve (LD,b)

 solves the system L*D*L'*x=b for x.  This is equivalent to

   [L,D] = ldlsplit (LD) ;
   x = L' \ (D \ (L \ b)) ;

 LD is from ldlchol, or as updated by ldlupdate or ldlrowmod.  You must
 not modify LD as obtained from ldlchol, ldlupdate, or ldlrowmod prior
 to passing it to this function.  See ldlupdate for more details.

 See also ldlchol, ldlupdate, ldlsplit, ldlrowmod.
```

6.12 `ldlsplit`: split an $\mathbf{LDL}^{\mathsf{T}}$ factorization

```
LDLSPLIT split an LDL' factorization into L and D

 Example:
   [L,D] = ldlsplit (LD)

 LD contains an LDL' factorization, computed with LD = ldlchol(A), for
 example.  The diagonal of LD contains D, and the entries below the
 diagonal contain L (which has a unit diagonal).  This function splits
 LD into its two components L and D so that L*D*L' = A.

 See also ldlchol, ldlsolve, ldlupdate.
```

6.13 `ldlupdate`: update/downdate an $\mathbf{LDL}^{\mathsf{T}}$ factorization

```
LDLUPDATE multiple-rank update or downdate of a sparse LDL' factorization

 On input, LD contains the LDL' factorization of A (L*D*L'=A or A(q,q)).
 The unit-diagonal of L is not stored.  In its place is the diagonal
 matrix D.  LD can be computed using the CHOLMOD mexFunctions:

   LD = ldlchol (A) ;
```

```
or
  [LD,p,q] = ldlchol (A) ;

With this LD, either of the following MATLAB statements:

Example:
      LD = ldlupdate (LD,C)
      LD = ldlupdate (LD,C,'+')

return the LDL' factorization of A+C*C' or A(q,q)-C*C' if LD holds the
LDL' factorization of A(q,q) on input.  For a downdate:

      LD = ldlupdate (LD,C,'-')

returns the LDL' factorization of A-C*C' or A(q,q)-C*C'.

LD and C must be sparse and real.  LD must be square, and C must have
the same number of rows as LD.  You must not modify LD in MATLAB (see
the WARNING below).

Note that if C is sparse with few columns, most of the time spent in
this routine is taken by copying the input LD to the output LD.  If
MATLAB allowed mexFunctions to safely modify its inputs, this
mexFunction would be much faster, since not all of LD changes.

See also ldlchol, ldlsplit, ldlsolve, cholupdate.

========================================================================
============================ WARNING ===================================
========================================================================

MATLAB drops zero entries from its sparse matrices.  LD can contain
numerically zero entries that are symbolically present in the sparse
matrix data structure.  These are essential for ldlupdate and ldlsolve
to work properly, since they exploit the graph-theoretic structure of a
sparse Cholesky factorization. If you modify LD in MATLAB, those zero
entries may get dropped and the required graph property will be
destroyed.  In this case, ldlupdate and ldlsolve will fail
catastrophically (possibly with a segmentation fault, terminating
MATLAB).  It takes much more time to ensure this property holds than
the time it takes to do the update/downdate or the solve, so ldlupdate
and ldlsolve simply assume the propertly holds.
```

6.14 `ldlrowmod`: add/delete a row from an LDL^T factorization

```
LDLROWMOD add/delete a row from a sparse LDL' factorization.

 On input, LD contains the LDL' factorization of A (L*D*L'=A or A(q,q)).
 The unit-diagonal of L is not stored.  In its place is the diagonal
 matrix D.  LD can be computed using the CHOLMOD mexFunctions:

   LD = ldlchol (A) ;
 or
```

```
  [LD,p,q] = ldlchol (A) ;

With this LD, either of the following MATLAB statements,

Example:
  LD = ldlrowmod (LD,k,C)     add row k to an LDL' factorization

  returns the LDL' factorization of S, where S = A except for S(:,k) =
  C and S (k,:) = C.  The kth row of A is assumed to initially be equal
  to the kth row of identity.  To delete a row:

  LD = ldlrowmod (LD,k)       delete row k from an LDL' factorization

  returns the LDL' factorization of S, where S = A except that S(:,k)
  and S (k,:) become the kth column/row of speye(n), repespectively.

  LD and C must be sparse and real.  LD must be square, and C must have
  the same number of rows as LD.  You must not modify LD in MATLAB (see
  the WARNING below).

Note that if C is sparse with few columns, most of the time spent in
this routine is taken by copying the input LD to the output LD.  If
MATLAB allowed mexFunctions to safely modify its inputs, this
mexFunction would be much faster, since not all of LD changes.

See also ldlchol, ldlsplit, ldlsolve, cholupdate, ldlupdate.

=========================================================================
============================ WARNING ====================================
=========================================================================

MATLAB drops zero entries from its sparse matrices.  LD can contain
numerically zero entries that are symbolically present in the sparse
matrix data structure.  These are essential for ldlrowmod and ldlsolve
to work properly, since they exploit the graph-theoretic structure of a
sparse Cholesky factorization. If you modify LD in MATLAB, those zero
entries may get dropped and the required graph property will be
destroyed.  In this case, ldlrowmod and ldlsolve will fail
catastrophically (possibly with a segmentation fault, terminating
MATLAB).  It takes much more time to ensure this property holds than
the time it takes to do the row add/delete or the solve, so ldlrowmod
and ldlsolve simply assume the propertly holds.
```

6.15 mread: read a sparse or dense matrix from a Matrix Market file

```
MREAD read a sparse matrix from a file in Matrix Market format

 Example:
   A = mread (filename)
   [A Z] = mread (filename, prefer_binary)

 Unlike MMREAD, only the matrix is returned; the file format is not
 returned.  Explicit zero entries can be present in the file; these are
```

```
not included in A.  They appear as the nonzero pattern of the binary
matrix Z.

If prefer_binary is not present, or zero, a symmetric pattern-only
matrix is returned with A(i,i) = 1+length(find(A(:,i))) if it is
present in the pattern, and A(i,j) = -1 for off-diagonal entries.  If
you want the original Matrix Market matrix in this case, simply use
A=mread(filename,1).

Compare with mmread.m at http://math.nist.gov/MatrixMarket

See also load.
```

6.16 mwrite: write a sparse or dense matrix to a Matrix Market file

```
MWRITE write a matrix to a file in Matrix Market form.

 Example:
   mtype = mwrite (filename, A, Z, comments_filename)

 A can be sparse or full.

 If present and non-empty, A and Z must have the same dimension.  Z
 contains the explicit zero entries in the matrix (which MATLAB drops).
 The entries of Z appear as explicit zeros in the output file.  Z is
 optional.  If it is an empty matrix it is ignored.  Z must be sparse or
 empty, if present.  It is ignored if A is full.

 filename is the name of the output file.  comments_filename is the file
 whose contents are include after the Matrix Market header and before
 the first data line.  Ignored if an empty string or not present.

 See also mread.
```

6.17 metis: order with METIS

```
METIS nested dissection ordering via METIS_NodeND

 Example:
   p = metis(A)        returns p such chol(A(p,p)) is typically sparser
                       than chol(A).  Uses tril(A) and assumes A is
                       symmetric.
   p = metis(A,'sym')  the same as p=metis(A).
   p = metis(A,'col')  returns p so that chol(A(:,p)'*A(:,p)) is
                       typically sparser than chol(A'*A).
   p = metis(A,'row')  returns p so that chol(A(p,:)*A(p,:)') is
                       typically sparser than chol(A'*A).

 A must be square for p=metis(A) or metis(A,'sym')
```

```
Requires METIS, authored by George Karypis, Univ. of Minnesota.  This
MATLAB interface, via CHOLMOD, is by Tim Davis.

See also nesdis, bisect.
```

6.18 `nesdis`: order with CHOLMOD nested dissection

```
NESDIS nested dissection ordering via CHOLMOD's nested dissection.

  Example:
  p = nesdis(A)         returns p such chol(A(p,p)) is typically
                        sparser than chol(A).  Uses tril(A) and assumes
                        A is symmetric.
  p = nesdis(A,'sym')   the same as p=nesdis(A).
  p = nesdis(A,'col')   returns p so that chol(A(:,p)'*A(:,p)) is
                        typically sparser than chol(A'*A).
  p = nesdis(A,'row')   returns p so that chol(A(p,:)*A(p,:)') is
                        typically sparser than chol(A'*A).

A must be square for p=nesdis(A) or nesdis(A,'sym').

With three output arguments, [p cp cmember] = nesdis(...), the
separator tree and node-to-component mapping is returned.  cmember(i)=c
means that node i is in component c, where c is in the range of 1 to
the number of components.  length(cp) is the number of components
found.  cp is the separator tree; cp(c) is the parent of component c,
or 0 if c is a root.  There can be anywhere from 1 to n components,
where n is dimension of A, A*A', or A'*A.  cmember is a vector of
length n.

An optional 3rd input argument, nesdis (A,mode,opts), modifies the
default parameters.  opts(1) specifies the smallest subgraph that
should not be partitioned (default is 200).  opts(2) is 0 by default;
if nonzero, connected components (formed after the node separator is
removed) are partitioned independently.  The default value tends to
lead to a more balanced separator tree, cp.  opts(3) defines when a
separator is kept; it is kept if the separator size is < opts(3) times
the number of nodes in the graph being cut (valid range is 0 to 1,
default is 1).

opts(4) specifies graph is to be ordered after it is dissected.  For
the 'sym' case: 0: natural ordering, 1: CAMD, 2: CSYMAMD.  For other
cases: 0: natural ordering, nonzero: CCOLAMD.  The default is 1, to use
CAMD for the symmetric case and CCOLAMD for the other cases.

If opts is shorter than length 4, defaults are used for entries that
are not present.

NESDIS uses METIS' node separator algorithm to recursively partition
the graph.  This gives a set of constraints (cmember) that is then
passed to CCOLAMD, CSYMAMD, or CAMD, constrained minimum degree
ordering algorithms.  NESDIS typically takes slightly more time than
METIS (METIS_NodeND), but tends to produce better orderings.
```

```
Requires METIS, authored by George Karypis, Univ. of Minnesota.  This
MATLAB interface, via CHOLMOD, is by Tim Davis.

See also metis, bisect, amd.
```

6.19 resymbol: re-do symbolic factorization

```
RESYMBOL recomputes the symbolic Cholesky factorization of the matrix A

 Example:
   L = resymbol (L, A)

 Recompute the symbolic Cholesky factorization of the matrix A.  A must
 be symmetric.  Only tril(A) is used.  Entries in L that are not in the
 Cholesky factorization of A are removed from L.  L can be from an LL'
 or LDL' factorization (lchol or ldlchol).  resymbol is useful after a
 series of downdates via ldlupdate or ldlrowmod, since downdates do not
 remove any entries in L.  The numerical values of A are ignored; only
 its nonzero pattern is used.

 See also lchol, ldlupdate, ldlrowmod.
```

6.20 sdmult: sparse matrix times dense matrix

```
SDMULT sparse matrix times dense matrix
 Compute C = S*F or S'*F where S is sparse and F is full (C is also
 sparse).  S and F must both be real or both be complex.

   Example:
       C = sdmult (S,F) ;       C = S*F
       C = sdmult (S,F,0) ;     C = S*F
       C = sdmult (S,F,1) ;     C = S'*F

 See also mtimes.
```

6.21 spsym: determine symmetry

```
SPSYM check if a matrix is symmetric, Hermitian, or skew-symmetric.
 If so, also determine if its diagonal has all positive real entries.
 A must be sparse.

 Example:
   result = spsym (A) ;
   result = spsym (A,quick) ;

 If quick = 0, or is not present, then this routine returns:
```

```
      1: if A is rectangular
      2: if A is unsymmetric
      3: if A is symmetric, but with one or more A(j,j) <= 0
      4: if A is Hermitian, but with one or more A(j,j) <= 0 or with
          nonzero imaginary part
      5: if A is skew symmetric (and thus the diagonal is all zero)
      6: if A is symmetric with real positive diagonal
      7: if A is Hermitian with real positive diagonal

If quick is nonzero, then the function can return more quickly, as soon
as it finds a diagonal entry that is <= 0 or with a nonzero imaginary
part.  In this case, it returns 2 for a square matrix, even if the
matrix might otherwise be symmetric or Hermitian.  Regardless of the
value of "quick", this function returns 6 or 7 if A is a candidate for
sparse Cholesky.  spsym does not compute the transpose of A, nor does
it need to examine the entire matrix if it is unsymmetric.

Examples:
  load west0479
  A = west0479 ;
  spsym (A)
  spsym (A+A')
  spsym (A-A')
  spsym (A+A'+3*speye(size(A,1)))

With additional outputs, spsym computes the following for square
matrices (in this case "quick" is ignored, and treated as zero):

[result xmatched pmatched nzoffdiag nnzdiag] = spsym(A)

  xmatched is the number of nonzero entries for which A(i,j) =
  conj(A(j,i)).  pmatched is the number of entries (i,j) for which
  A(i,j) and A(j,i) are both in the pattern of A (the value doesn't
  matter).  nzoffdiag is the total number of off-diagonal entries in
  the pattern.  nzdiag is the number of diagonal entries in the
  pattern.  If the matrix is rectangular, xmatched, pmatched,
  nzoffdiag, and nzdiag are not computed (all of them are returned as
  zero).  Note that a matched pair, A(i,j) and A(j,i) for i != j, is
  counted twice (once per entry).

See also mldivide.
```

6.22 symbfact2: same as symbfact

```
SYMBFACT2  symbolic factorization

Analyzes the Cholesky factorization of A, A'*A, or A*A'.

  Example:
  count = symbfact2 (A)           returns row counts of R=chol(A)
  count = symbfact2 (A,'col')     returns row counts of R=chol(A'*A)
  count = symbfact2 (A,'sym')     same as symbfact2(A)
```

```
  count = symbfact2 (A,'lo')       same as symbfact2(A'), uses tril(A)
  count = symbfact2 (A,'row')      returns row counts of R=chol(A*A')

The flop count for a subsequent LL' factorization is sum(count.^2)

[count, h, parent, post, R] = symbfact2 (...) returns:

      h: height of the elimination tree
      parent: the elimination tree itself
      post: postordering of the elimination tree
      R: a 0-1 matrix whose structure is that of chol(A) for the
          symmetric case, chol(A'*A) for the 'col' case, or chol(A*A')
          for the 'row' case.

symbfact2(A) and symbfact2(A,'sym') uses the upper triangular part of A
(triu(A)) and assumes the lower triangular part is the transpose of the
upper triangular part.  symbfact2(A,'lo') uses tril(A) instead.

With one to four output arguments, symbfact2 takes time almost
proportional to nnz(A)+n where n is the dimension of R, and memory
proportional to nnz(A).  Computing the 5th argument takes more time and
memory, both O(nnz(L)).  Internally, the pattern of L is computed and
R=L' is returned.

The following forms return L = R' instead of R.  They are faster and
take less memory than the forms above.  They return the same count, h,
parent, and post outputs.

  [count, h, parent, post, L] = symbfact2 (A,'col','L')
  [count, h, parent, post, L] = symbfact2 (A,'sym','L')
  [count, h, parent, post, L] = symbfact2 (A,'lo', 'L')
  [count, h, parent, post, L] = symbfact2 (A,'row','L')

See also chol, etree, treelayout, symbfact.
```

---

7 Installation for use in MATLAB

7.1 `cholmod_make`: compiling CHOLMOD in MATLAB

Start MATLAB, `cd` to the `CHOLMOD/MATLAB` directory, and type `cholmod_make` in the MATLAB command window. This will compile the MATLAB interfaces for METIS and CHOLMOD.

8 Using CHOLMOD with OpenMP acceleration

CHOLMOD includes OpenMP acceleration for some operations. In CHOLMOD versions prior to v6.0.0, the number of threads to use was controlled by a compile time parameter. This is now replaced with run-time controls.

`Common->nthreads_max` defaults to `omp_get_max_threads()`, or 1 if OpenMP is not in use. This value controls the maximum number of threads that CHOLMOD will use. If zero or less, the default is used. The `Common->chunk` parameter controls how many threads are used when the work to do is low. If w is a count of operations to perform, c = `Common->chunk`, and m = `Common->nthreads_max`, then a parallel region will use $\max(1, \min(\lfloor w/c \rfloor, m))$ threads. These parameters can be revised by the user application at run time.

9 Using CHOLMOD with GPU acceleration

Starting with CHOLMOD v2.0.0, it is possible to accelerate the numerical factorization phase of CHOLMOD using NVIDIA GPUs. Due to the large computational capability of the GPUs, enabling this capability can result in significant performance improvements. Similar to CPU processing, the GPU is better able to accelerate the dense math associated with larger supernodes. Hence the GPU will provide more significant performance improvements for larger matrices that have more, larger supernodes.

In CHOLMOD v2.3.0 this GPU capability has been improved to provide a significant increase in performance and the interface has been expanded to make the use of GPUs more flexible. CHOLMOD can take advantage of a single NVIDIA GPU that supports CUDA and has at least 64MB of memory. (But substantially more memory, typically about 3 GB, is recommended for best performance.)

Only the (`double`, `int64_t`) version of CHOLMOD can leverage GPU acceleration (both real and complex).

9.1 Compiling CHOLMOD with GPU support

In order to support GPU processing, CHOLMOD must be compiled with the preprocessor macro `CHOLMOD_HAS_CUDA` defined. It is enabled by default in `cmake` if you have a GPU, but this can be disabled by setting this to false when using cmake, via the cmake variables `CHOLMOD_USE_CUDA` or `SUITESPARSE_USE_CUDA`.

9.2 Enabling GPU acceleration in CHOLMOD

Even if compiled with GPU support, in CHOLMOD v.2.3.0, GPU processing is not enabled by default and must be specifically requested. There are two ways to do this, either in the code calling

CHOLMOD or using environment variables.

The code author can specify the use of GPU processing with the `Common->useGPU` variable. If this is set to `1`, CHOLMOD will attempt to use the GPU. If this is set to `0` the use of the GPU will be prohibited. If this is set to `-1`, which is the default case, then the environment variables (following paragraph) will be queried to determine if the GPU is to be used. Note that the default value of `-1` is set when `cholmod_start(Common)` is called, so the code author must set `Common->useGPU` after calling `cholmod_start`.

Alternatively, or if it is not possible to modify the code calling CHOLMOD, GPU processing can invoked using the `CHOLMOD_USE_GPU` environment variable. This makes it possible for any CHOLMOD user to invoke GPU processing even if the author of the calling program did not consider this. The interpretation of the environment variable `CHOLMOD_USE_GPU` is that if the string evaluates to an integer other than zero, GPU processing will be enabled. Note that the setting of `Common->useGPU` takes precedence and the environment variable `CHOLMOD_USE_GPU` will only be queried if `Common->useGPU = -1`.

Note that in either case, if GPU processing is requested, but there is no GPU present, CHOLMOD will continue using the CPU only. Consequently it is always safe to request GPU processing.

9.3 Adjustable parameters

There are a number of parameters that have been added to CHOLMOD to control GPU processing. All of these have appropriate defaults such that GPU processing can be used without any modification. However, for any particular combination of CPU/GPU, better performance might be obtained by adjusting these parameters.

From `t_cholmod_gpu.c`

`CHOLMOD_ND_ROW_LIMIT` : Minimum number of rows required in a descendant supernode to be eligible for GPU processing during supernode assembly

`CHOLMOD_ND_COL_LIMIT` : Minimum number of columns in a descendant supernode to be eligible for GPU processing during supernode assembly

`CHOLMOD_POTRF_LIMIT` : Minimum number of columns in a supernode to be eligible for `POTRF` and `TRSM` processing on the GPU

`CHOLMOD_GPU_SKIP` : Number of small descendant supernodes to assembled on the CPU before querying if the GPU is needs more descendant supernodes queued.

From `cholmod_core.h`

`CHOLMOD_HOST_SUPERNODE_BUFFERS` : Number of buffers in which to queue descendant supernodes for GPU processing

Programatically

`Common->maxGpuMemBytes` : Specifies the maximum amount of memory, in bytes, that CHOLMOD can allocate on the GPU. If this parameter is not set, CHOLMOD will allocate as much GPU memory as possible. Hence, the purpose of this parameter is to

restrict CHOLMOD's GPU memory use so that CHOLMOD can be used simultaneously with other codes that also use GPU acceleration and require some amount of GPU memory. If the specified amount of GPU memory is not allocatable, CHOLMOD will allocate the available memory and continue.

`Common->maxGpuMemFraction` : Entirely similar to `Common->maxGpuMemBytes` but with the memory specified as a fraction of total GPU memory. Note that if both `maxGpuMemBytes` and `maxGpuMemFraction` are specified, whichever results in the minimum amount of memory will be used.

Environment variables

`CHOLMOD_GPU_MEM_BYTES` : Environment variable with a meaning equivalent to `Common->maxGpuMemBytes`. This will only be queried if `Common->useGPU = -1`.

`CHOLMOD_GPU_MEM_FRACTION` : Environment variable with a meaning equivalent to `Common->maxGpuMemFraction`. This will only be queried if `Common->useGPU = -1`.

10 Integer and floating-point types, and notation used

CHOLMOD supports both `int32_t` and `int64_t` integers. CHOLMOD routines with the prefix `cholmod_` use `int32_t` integers, `cholmod_l_` routines use `int64_t`. Floating-point values are `double` or `float`, depending on the `A->dtype` field for a sparse matrix, triplet matrix, dense matrix, or factorization object.

Two kinds of complex matrices are supported: complex and zomplex. A complex matrix is held in a manner that is compatible with the Fortran and ANSI C99 complex data type. A complex array of size `n` is a `double` or `float` array `x` of size `2*n`, with the real and imaginary parts interleaved, with the real part comes first, as a `double` or `float` followed the imaginary part, also as a `double` or `float`. Thus, the real part of the `k`th entry is `x[2*k]` and the imaginary part is `x[2*k+1]`.

A zomplex matrix of size `n` stores its real part in one `double` or `float` array of size `n` called `x` and its imaginary part in another `double` or `float` array of size `n` called `z` (thus the name "zomplex"). This also how MATLAB stored its complex matrices in R2017b and earlier. The real part of the `k`th entry is `x[k]` and the imaginary part is `z[k]`. In MATLAB R2018a, complex matrices are stored in the standard interleaved format. The CHOLMOD MATLAB interface uses this, and thus requires MATLAB R2018a or later.

Unlike `UMFPACK`, the same routine name in CHOLMOD is used for pattern-only, real, complex, and zomplex matrices, and also for both `double` and `float` values. For example, the statement

```
C = cholmod_copy_sparse (A, &Common) ;
```

creates a copy of a pattern, real, complex, or zomplex sparse matrix `A`. The xtype (pattern, real, complex, or zomplex) of the resulting sparse matrix `C` is the same as `A` (a pattern-only sparse matrix contains no floating-point values). In the above case, `C` and `A` use `int32_t` integers. For `int64_t` integers, the statement would become:

```
C = cholmod_l_copy_sparse (A, &Common) ;
```

The last parameter of all CHOLMOD routines is always `&Common`, a pointer to the `cholmod_common` object, which contains parameters, statistics, and workspace used throughout CHOLMOD.

The `xtype` of a CHOLMOD object (sparse matrix, triplet matrix, dense matrix, or factorization) determines whether it is pattern-only, real, complex, or zomplex. The `dtype` of a CHOLMOD object (sparse matrix, triplet matrix, dense matrix, or factorization) determines whether it is `double` or `float`. These two terms are often added together when passing parameters to CHOLMOD, as `xtype + dtype`. This API design was chosen for backward compatibility with CHOLMOD 4.x and earlier.

The names of the `int32_t` versions are primarily used in this document. To obtain the name of the `int64_t` version of the same routine, simply replace `cholmod_` with `cholmod_l_`.

MATLAB matrix notation is used throughout this document and in the comments in the CHOLMOD code itself. If you are not familiar with MATLAB, here is a short introduction to the notation, and a few minor variations used in CHOLMOD:

- `C=A+B` and `C=A*B`, respectively are a matrix add and multiply if both `A` and `B` are matrices of appropriate size. If `A` is a scalar, then it is added to or multiplied with every entry in `B`.
- `a:b` where `a` and `b` are integers refers to the sequence `a`, `a+1`, ... `b`.

- `[A B]` and `[A,B]` are the horizontal concatenation of `A` and `B`.
- `[A;B]` is the vertical concatenation of `A` and `B`.
- `A(i,j)` can refer either to a scalar or a submatrix. For example:

`A(1,1)`	a scalar.
`A(:,j)`	column `j` of `A`.
`A(i,:)`	row `i` of `A`.
`A([1 2], [1 2])`	a 2-by-2 matrix containing the 2-by-2 leading minor of `A`.

  If `p` is a permutation of `1:n`, and `A` is `n`-by-`n`, then `A(p,p)` corresponds to the permuted matrix $\mathbf{PAP}^{\mathsf{T}}$.
- `tril(A)` is the lower triangular part of `A`, including the diagonal.
- `tril(A,k)` is the lower triangular part of `A`, including entries on and below the kth diagonal.
- `triu(A)` is the upper triangular part of `A`, including the diagonal.
- `triu(A,k)` is the upper triangular part of `A`, including entries on and above the kth diagonal.
- `size(A)` returns the dimensions of `A`.
- `find(x)` if `x` is a vector returns a list of indices `i` for which `x(i)` is nonzero.
- `A'` is the transpose of `A` if `A` is real, or the complex conjugate transpose if `A` is complex.
- `A.'` is the array transpose of `A`.
- `diag(A)` is the diagonal of `A` if `A` is a matrix.
- `C=diag(s)` is a diagonal matrix if `s` is a vector, with the values of `s` on the diagonal of `C`.
- `S=spones(A)` returns a binary matrix `S` with the same nonzero pattern of `A`.
- `nnz(A)` is the number of nonzero entries in `A`.

Variations to MATLAB notation used in this document:

- CHOLMOD uses 0-based notation (the first entry in the matrix is `A(0,0)`). MATLAB is 1-based. The context is usually clear.
- `I` is the identity matrix.
- `A(:,f)`, where `f` is a set of columns, is interpreted differently in CHOLMOD, but just for the set named `f`. See `cholmod_transpose_unsym` for details.

11 The CHOLMOD Modules, objects, and functions

CHOLMOD contains over 150 `int32_t`-based routines and the same number of `int64_t` routines with the same name except for `_l_` added. These are divided into a set of inter-related Modules. Each Module contains a set of related functions. The functions are divided into two types: Primary and Secondary, to reflect how a user will typically use CHOLMOD. Most users will find the Primary routines to be sufficient to use CHOLMOD in their programs. Each Module exists as a sub-directory (a folder for Windows users) within the CHOLMOD directory (or folder).

There are seven Modules that provide user-callable routines for CHOLMOD.

1. `Utility`: basic data structures and definitions
2. `Check`: prints/checks each of CHOLMOD's objects
3. `Cholesky`: sparse Cholesky factorization
4. `Modify`: sparse Cholesky update/downdate and row-add/row-delete
5. `MatrixOps`: sparse matrix operators (add, multiply, norm, scale)
6. `Supernodal`: supernodal sparse Cholesky factorization
7. `Partition`: graph-partitioning-based orderings, which uses a slightly modified copy of METIS 5.1.0 in the `SuiteSparse_metis` folder.

Additional directories provide support functions and documentation:

1. `Include`: include files for CHOLMOD and programs that use CHOLMOD
2. `Demo`: simple programs that illustrate the use of CHOLMOD
3. `Doc`: documentation (including this document)
4. `MATLAB`: CHOLMOD's interface to MATLAB
5. `Tcov`: an exhaustive test coverage (requires Linux or Solaris)
6. `cmake_modules`: how other packages can find CHOLMOD when using cmake.
7. `Config`: a folder containing the input files to create the `cholmod.h` include file, via cmake.

11.1 CHOLMOD objects

A CHOLMOD sparse, dense, or triplet matrix A, or a sparse factorization L can hold numeric values of 8 different types, according to its `A->xtype` and `A->dtype` parameters (or `L->xtype` and `L->dtype` for a sparse factor object). These values are held in the `A->x` array, and also `A->z` for zomplex matrices.

xtype: the matrix is real, complex, "zomplex", or pattern-only.

- (0): `CHOLMOD_PATTERN`: `A->x` and `A->z` are `NULL`. The matrix has no numerical values. Only the pattern is stored.

- (1): CHOLMOD_REAL: The matrix is real, and the values are held in A->x, whose size (in terms of double or float values) is given by A->nzmax. The kth value in the matrix is held in A->x[k].
- (2): CHOLMOD_COMPLEX: The matrix is complex, with interleaved real and imaginary parts. The kth value in the matrix is held in A->x[2*k] and A->x[2*k+1], where A->x can hold up to 2*A->nzmax values.
- (3): CHOLMOD_ZOMPLEX: The matrix is complex, with separate array for the real and imaginary parts. The kth value in the matrix is held in A->x[k], and A->z[k], where A->x and A->z can hold up to A-¿nzmax values each.

dtype: this parameter determines the type of values in A-¿x (and A-¿z if zomplex).

- (0) CHOLMOD_DOUBLE: A->x and A->z (for zomplex matrices) are double. If A is real, A->x has a size of A->nzmax * sizeof (double) bytes. If A is complex, A->x has size A->nzmax * 2 * sizeof (doub If zomplex, both A->x and A->z have size A->nzmax * sizeof (double).
- (4) CHOLMOD_SINGLE: A->x and A->z (for zomplex matrices) are float. If A is real, A->x has a size of A->nzmax * sizeof (float). If A is complex, A->x has size A->nzmax * 2 * sizeof (float). If zomplex, both A->x and A->z have size A->nzmax * sizeof (float). This feature is new to CHOLMOD v5.

Unless stated otherwise, the xtype and dtypes of all inputs to a method must be the same.

Many methods accept an xdtype parameter, which is simply xtype + dtype, combining the two parameters into a single number handling all 8 cases:

- (0) CHOLMOD_DOUBLE + CHOLMOD_PATTERN: a pattern-only matrix
- (1) CHOLMOD_DOUBLE + CHOLMOD_REAL: a double real matrix
- (2) CHOLMOD_DOUBLE + CHOLMOD_COMPLEX: a double complex matrix
- (3) CHOLMOD_DOUBLE + CHOLMOD_ZOMPLEX: a double zomplex matrix
- (4) CHOLMOD_SINGLE + CHOLMOD_PATTERN: a pattern-only matrix
- (5) CHOLMOD_SINGLE + CHOLMOD_REAL: a float real matrix
- (6) CHOLMOD_SINGLE + CHOLMOD_COMPLEX: a float complex matrix
- (7) CHOLMOD_SINGLE + CHOLMOD_ZOMPLEX: a float zomplex matrix

This approach was selected for backward compatibility with CHOLMOD v4 and earlier, where only the first four values were supported, and where the parameter was called xtype instead of xdtype. Several function names reflect the older parameter name (cholmod_*_xtype), but they have not been renamed _xdtype, for backward compatibility.

A CHOLMOD sparse or triplet matrix A can held in three symmetry formats according to its A->stype parameter. Dense matrices do not have this parameter and are always treated as unsymmetric. A sparse factor object L is always held in lower triangular form, with no entries ever held in the strictly upper triangular part.

- 0: the matrix is unsymmetric with both lower and upper parts stored.
- < 0: the matrix is symmetric, with just the lower triangular part and diagonal stored. Any entries in the upper part are ignored.
- > 0: the matrix is symmetric, with just the upper triangular part stored and diagonal. Any entries in the upper part are ignored.

If a sparse or triplet matrix A is complex or zomplex, most methods treat the matrix as Hermitian, where A(i,j) is the complex conjugate of A(j,i), when i is not equal to j. Some methods can also interpret the matrix as complex symmetric, where A(i,j) == A(j,i) when i != j. This is not determined by the matrix itself, but by a "mode" parameter of the function. This mode parameter also determines if the values of any matrix are to be ignored entirely, in which case only the pattern is operated on. Any output matrix will have an xtype of `CHOLMOD_PATTERN`.

The valid mode values are given below, except that many methods do not handle the negative cases. Values below the range accepted by the method are treated as its lowest accepted value, and values above the range accepted by the method are treated as its highest accepted value.

- mode = 2: the numerical values of a real, complex, or zomplex matrix are handled. If the matrix is complex or zomplex, an entry A(i,j) that not stored (or in the ignored part) is treated as the complex conjugate of A (j,i). Use this mode to treat a complex or zomplex matrix as Hermitian.
- mode = 1: the numerical values of a real, complex, or zomplex matrix are handled. If the matrix is complex or zomplex, an entry A(i,j) that not stored (or in the ignored part) is treated as equal A (j,i). Use this mode to treat a complex or zomplex matrix as complex symmetric.
- mode = 0: the numerical values are ignored. Any output matrix will have an xtype of `CHOLMOD_PATTERN`. This mode allows inputs to have different dtypes.
- mode = -1: the same as mode = 0, except that the diagonal entries are ignored, and do not appear in any output matrix.
- mode = -2: the same as mode = -1, except that the output matrix is given an additional slack space so that it can hold about 50entries. This mode is documented here but it is primarily meant for internal use, for CHOLMOD's interface to the AMD, CAMD, COLAMD, and CCOLAMD ordering methods.

The integer arrays in all objects are either int32 or int64, as determined by `A->type`. This integer type must be identical for all inputs, and must also match both the function name (`cholmod_method` for int32, or `cholmod_l_method` for int64) and the `Common->itype` as defined when CHOLMOD was initialized (via `cholmod_start` for int32, or `cholmod_l_start` for int64).

11.2 `Utility` Module: basic data structures and definitions

CHOLMOD includes five basic objects, defined in the `Utility` Module. The `Utility Module` provides basic operations for these objects and is required by all six other CHOLMOD library Modules:

11.2.1 cholmod_common: parameters, statistics, and workspace

You must call cholmod_start before calling any other CHOLMOD routine, and you must call cholmod_finish as your last call to CHOLMOD (with the exception of cholmod_print_common and cholmod_check_common in the Check Module). Once the cholmod_common object is initialized, the user may modify CHOLMOD's parameters held in this object, and obtain statistics on CHOLMOD's activity.

When using 64-bit integers, use cholmod_l_start and cholmod_l_finish instead. Matrices and other objects from different integer sizes cannot be mixed

Primary routines for the cholmod_common object:

- cholmod_start: the first call to CHOLMOD.
- cholmod_finish: the last call to CHOLMOD (frees workspace in the cholmod_common object).

Secondary routines for the cholmod_common object:

- cholmod_defaults: restores default parameters
- cholmod_maxrank: determine maximum rank for update/downdate.
- cholmod_allocate_work: allocate workspace (double only).
- cholmod_alloc_work: allocate workspace, double or float.
- cholmod_free_work: free workspace.
- cholmod_clear_flag: clear Flag array.
- cholmod_error: called when CHOLMOD encounters and error.
- cholmod_dbound: bounds the diagonal of **L** or **D** (double case).
- cholmod_sbound: bounds the diagonal of **L** or **D**. (float case).
- cholmod_hypot: compute sqrt(x*x+y*y) accurately.
- cholmod_divcomplex: complex divide.

11.2.2 cholmod_sparse: a sparse matrix in compressed column form

A sparse matrix A is held in compressed column form. In the basic type ("packed," which corresponds to how MATLAB stores its sparse matrices), and nrow-by-ncol matrix with nzmax entries is held in three arrays: p of size ncol+1, i of size nzmax, and x of size nzmax. Row indices of nonzero entries in column j are held in i [p[j] ... p[j+1]-1], and their corresponding numerical values are held in x [p[j] ... p[j+1]-1]. The first column starts at location zero (p[0]=0). There may be no duplicate entries. Row indices in each column may be sorted or unsorted (the A->sorted flag must be false if the columns are unsorted). The A->stype determines the storage: 0 if the matrix is unsymmetric, 1 if the matrix is symmetric with just the upper triangular part stored, and -1 if the matrix is symmetric with just the lower triangular part stored.

In "unpacked" form, an additional array `nz` of size `ncol` is used. The end of column `j` in `i` and `x` is given by `p[j]+nz[j]`. Columns not need be in any particular order (`p[0]` need not be zero), and there may be gaps between the columns.

Primary routines for the `cholmod_sparse` object:

- `cholmod_allocate_sparse`: allocate a sparse matrix
- `cholmod_free_sparse`: free a sparse matrix

Secondary routines for the `cholmod_sparse` object:

- `cholmod_reallocate_sparse`: change the size (number of entries) of a sparse matrix.
- `cholmod_nnz`: number of nonzeros in a sparse matrix.
- `cholmod_speye`: sparse identity matrix.
- `cholmod_spzeros`: sparse zero matrix.
- `cholmod_transpose`: transpose a sparse matrix.
- `cholmod_transpose_unsym`: transpose/permute an unsymmetric sparse matrix.
- `cholmod_transpose_sym`: transpose/permute a symmetric sparse matrix.
- `cholmod_ptranspose`: transpose/permute a sparse matrix.
- `cholmod_sort`: sort row indices in each column of a sparse matrix.
- `cholmod_band_nnz`: number of entries in a band of a sparse matrix.
- `cholmod_band`: extract a band of a sparse matrix.
- `cholmod_band_inplace`: remove entries not with a band.
- `cholmod_aat`: `C = A*A'`.
- `cholmod_copy_sparse`: `C = A`, create an exact copy of a sparse matrix.
- `cholmod_copy`: `C = A`, with possible change of `stype`.
- `cholmod_add`: `C = alpha*A + beta*B`.
- `cholmod_sparse_xtype`: change the `xtype` and/or `dtype` of a sparse matrix.

11.2.3 `cholmod_factor`: a symbolic or numeric factorization

A factor can be in $\mathbf{LL}^\mathsf{T}$ or $\mathbf{LDL}^\mathsf{T}$ form, and either supernodal or simplicial form. In simplicial form, this is very much like a packed or unpacked `cholmod_sparse` matrix. In supernodal form, adjacent columns with similar nonzero pattern are stored as a single block (a supernode).

Primary routine for the `cholmod_factor` object:

- `cholmod_free_factor`: free a factor

Secondary routines for the `cholmod_factor` object:

- `cholmod_allocate_factor`: allocate a factor. (`double` or `float`). You will normally use `cholmod_analyze` to create a factor.
- `cholmod_alloc_factor`: allocate a factor (`double` or `float`).
- `cholmod_reallocate_factor`: change the number of entries in a factor.
- `cholmod_change_factor`: change the type of a factor ($\mathbf{LDL}^\mathsf{T}$ to $\mathbf{LL}^\mathsf{T}$, supernodal to simplicial, etc.).
- `cholmod_pack_factor`: pack the columns of a factor.
- `cholmod_reallocate_column`: resize a single column of a factor.
- `cholmod_factor_to_sparse`: create a sparse matrix copy of a factor.
- `cholmod_copy_factor`: create a copy of a factor.
- `cholmod_factor_xtype`: change the `xtype` and/or `dtype` of a factor.

11.2.4 `cholmod_dense`: a dense matrix

This consists of a dense array of numerical values and its dimensions.

Primary routines for the `cholmod_dense` object:

- `cholmod_allocate_dense`: allocate a dense matrix.
- `cholmod_free_dense`: free a dense matrix.

Secondary routines for the `cholmod_dense` object:

- `cholmod_zeros`: allocate a dense matrix of all zeros.
- `cholmod_ones`: allocate a dense matrix of all ones.
- `cholmod_eye`: allocate a dense identity matrix .
- `cholmod_ensure_dense`: ensure a dense matrix has a given size and type.
- `cholmod_sparse_to_dense`: create a dense matrix copy of a sparse matrix.

- cholmod_dense_nnz: number of nonzeros in a dense matrix.
- cholmod_dense_to_sparse: create a sparse matrix copy of a dense matrix.
- cholmod_copy_dense: create a copy of a dense matrix.
- cholmod_copy_dense2: copy a dense matrix (pre-allocated).
- cholmod_dense_xtype: change the xtype and/or dtype of a dense matrix.

11.2.5 cholmod_triplet: a sparse matrix in "triplet" form

The cholmod_sparse matrix is the basic sparse matrix used in CHOLMOD, but it can be difficult for the user to construct. It also does not easily support the inclusion of new entries in the matrix. The cholmod_triplet matrix is provided to address these issues. A sparse matrix in triplet form consists of three arrays of size nzmax: i, j, and x, and a z array for the zomplex case.

Primary routines for the cholmod_triplet object:

- cholmod_allocate_triplet: allocate a triplet matrix.
- cholmod_free_triplet: free a triplet matrix.
- cholmod_triplet_to_sparse: create a sparse matrix copy of a triplet matrix.

Secondary routines for the cholmod_triplet object:

- cholmod_reallocate_triplet: change the number of entries in a triplet matrix.
- cholmod_sparse_to_triplet: create a triplet matrix copy of a sparse matrix.
- cholmod_copy_triplet: create a copy of a triplet matrix.
- cholmod_triplet_xtype: change the xtype and/or dtype of a triplet matrix.

11.2.6 Memory management routines

By default, CHOLMOD uses the ANSI C malloc, free, calloc, and realloc routines. You may use different routines by modifying function pointers in the SuiteSparse_config package. Refer to the user guide for that package for more details.

Primary routines:

- cholmod_malloc: malloc wrapper.
- cholmod_free: free wrapper.

Secondary routines:

- cholmod_calloc: calloc wrapper.
- cholmod_realloc: realloc wrapper.
- cholmod_realloc_multiple: realloc wrapper for multiple objects.

11.2.7 `cholmod_version`: Version control

The `cholmod_version` function returns the current version of CHOLMOD.

11.3 `Check` Module: print/check the CHOLMOD objects

The `Check` Module contains routines that check and print the five basic objects in CHOLMOD, and three kinds of integer vectors (a set, a permutation, and a tree). It also provides a routine to read a sparse matrix from a file in Matrix Market format (http://www.nist.gov/MatrixMarket). Requires the `Utility` Module.

Primary routines:

- `cholmod_print_common`: print the `cholmod_common` object, including statistics on CHOLMOD's behavior (fill-in, flop count, ordering methods used, and so on).
- `cholmod_write_sparse`: write a sparse matrix to a file in Matrix Market format.
- `cholmod_write_dense`: write a sparse matrix to a file in Matrix Market format.
- `cholmod_read_matrix`: read a sparse or dense matrix from a file in Matrix Market format.
- `cholmod_read_matrix2`: read a sparse or dense matrix from a file in Matrix Market format (`double` or `float`).

Secondary routines:

- `cholmod_check_common`: check the `cholmod_common` object
- `cholmod_check_sparse`: check a sparse matrix
- `cholmod_print_sparse`: print a sparse matrix
- `cholmod_check_dense`: check a dense matrix
- `cholmod_print_dense`: print a dense matrix
- `cholmod_check_factor`: check a Cholesky factorization
- `cholmod_print_factor`: print a Cholesky factorization
- `cholmod_check_triplet`: check a triplet matrix
- `cholmod_print_triplet`: print a triplet matrix
- `cholmod_check_subset`: check a subset (integer vector in given range)
- `cholmod_print_subset`: print a subset (integer vector in given range)
- `cholmod_check_perm`: check a permutation (an integer vector)
- `cholmod_print_perm`: print a permutation (an integer vector)

- `cholmod_check_parent`: check an elimination tree (an integer vector)
- `cholmod_print_parent`: print an elimination tree (an integer vector)
- `cholmod_read_triplet`: read a triplet matrix from a file
- `cholmod_read_triplet2`: read a triplet matrix from a file (`double` or `float`)
- `cholmod_read_sparse`: read a sparse matrix from a file
- `cholmod_read_sparse2`: read a sparse matrix from a file (`double` or `float`)
- `cholmod_read_dense`: read a dense matrix from a file
- `cholmod_read_dense2`: read a dense matrix from a file (`double` or `float`)
- `cholmod_gpu_stats`: print GPU timing statistics

11.4 `Cholesky` Module: sparse Cholesky factorization

The primary routines are all that a user requires to order, analyze, and factorize a sparse symmetric positive definite matrix $\mathbf{A}$ (or $\mathbf{A}\mathbf{A}^{\mathsf{T}}$), and to solve $\mathbf{A}\mathbf{x} = \mathbf{b}$ (or $\mathbf{A}\mathbf{A}^{\mathsf{T}}\mathbf{x} = \mathbf{b}$). The primary routines rely on the secondary routines, the `Utility` Module, and the AMD and COLAMD packages. They make optional use of the `Supernodal` and `Partition` Modules, the METIS package, the CAMD package, and the CCOLAMD package. The `Cholesky` Module is required by the `Partition` Module.

Primary routines:

- `cholmod_analyze`: order and analyze (simplicial or supernodal).
- `cholmod_factorize`: simplicial or supernodal Cholesky factorization.
- `cholmod_solve`: solve a linear system (simplicial or supernodal, dense $\mathbf{x}$ and $\mathbf{b}$).
- `cholmod_spsolve`: solve a linear system (simplicial or supernodal, sparse $\mathbf{x}$ and $\mathbf{b}$).

Secondary routines:

- `cholmod_analyze_p`: analyze, with user-provided permutation or $\mathbf{f}$ set.
- `cholmod_factorize_p`: factorize, with user-provided permutation or $\mathbf{f}$.
- `cholmod_analyze_ordering`: analyze a permutation
- `cholmod_solve2`: solve a linear system, reusing workspace.
- `cholmod_etree`: find the elimination tree.
- `cholmod_rowcolcounts`: compute the row/column counts of $\mathbf{L}$.
- `cholmod_amd`: order using AMD.
- `cholmod_colamd`: order using COLAMD.

- `cholmod_rowfac`: incremental simplicial factorization.
- `cholmod_row_subtree`: find the nonzero pattern of a row of **L**.
- `cholmod_lsolve_pattern`: find the nonzero pattern of $\mathbf{L}^{-1}b$.
- `cholmod_row_lsubtree`: find the nonzero pattern of a row of **L**.
- `cholmod_resymbol`: recompute the symbolic pattern of **L**.
- `cholmod_resymbol_noperm`: recompute the symbolic pattern of **L**, no permutation.
- `cholmod_rcond`: compute the reciprocal condition number
- `cholmod_postorder`: postorder a tree. estimate.

11.5 `Modify` Module: update/downdate a sparse Cholesky factorization

The `Modify` Module contains sparse Cholesky modification routines: update, downdate, row-add, and row-delete. It can also modify a corresponding solution to $\mathbf{Lx} = \mathbf{b}$ when L is modified. This module is most useful when applied on a Cholesky factorization computed by the `Cholesky` module, but it does not actually require the `Cholesky` module. The `Utility` module can create an identity Cholesky factorization ($\mathbf{LDL}^\mathsf{T}$ where $\mathbf{L} = \mathbf{D} = \mathbf{I}$) that can then be modified by these routines. Requires the `Utility` module. Not required by any other CHOLMOD Module.

Primary routine:

- `cholmod_updown`: multiple rank update/downdate

Secondary routines:

- `cholmod_updown_solve`: update/downdate, and modify solution to $\mathbf{Lx} = \mathbf{b}$
- `cholmod_rowadd`: add a row to an $\mathbf{LDL}^\mathsf{T}$ factorization
- `cholmod_rowadd_solve`: add a row, and update solution to $\mathbf{Lx} = \mathbf{b}$
- `cholmod_rowdel`: delete a row from an $\mathbf{LDL}^\mathsf{T}$ factorization
- `cholmod_rowdel_solve`: delete a row, and downdate $\mathbf{Lx} = \mathbf{b}$

11.6 `MatrixOps` Module: basic sparse matrix operations

The `MatrixOps` Module provides basic operations on sparse and dense matrices. Requires the `Utility` module. Not required by any other CHOLMOD module. In the descriptions below, `A`, `B`, and `C:` are sparse matrices (`cholmod_sparse`), `X` and `Y` are dense matrices (`cholmod_dense`), `s` is a scalar or vector, and `alpha beta` are scalars.

Many of these operations are also available in GraphBLAS, with better performance.

- `cholmod_drop`: drop entries from A with absolute value $\geq$ a given tolerance.

- `cholmod_norm_dense`: `s = norm (X)`, 1-norm, infinity-norm, or 2-norm
- `cholmod_norm_sparse`: `s = norm (A)`, 1-norm or infinity-norm
- `cholmod_scale`: `A = diag(s)*A`, `A*diag(s)`, `s*A` or `diag(s)*A*diag(s)`.
- `cholmod_sdmult`: `Y = alpha*(A*X) + beta*Y` or `alpha*(A'*X) + beta*Y`.
- `cholmod_ssmult`: `C = A*B`
- `cholmod_submatrix`: `C = A (i,j)`, where `i` and `j` are arbitrary integer vectors.
- `cholmod_horzcat`: `C = [A,B]`
- `cholmod_vertcat`: `C = [A ; B]`.
- `cholmod_symmetry`: determine symmetry of a matrix.

11.7 `Supernodal` Module: supernodal sparse Cholesky factorization

The `Supernodal` Module performs supernodal analysis, factorization, and solve. The simplest way to use these routines is via the `Cholesky` Module. This Module does not provide any fill-reducing orderings. It normally operates on matrices ordered by the `Cholesky` Module. It does not require the `Cholesky` Module itself, however. Requires the `Utility` Module, and two external packages: LAPACK and the BLAS. Optionally used by the `Cholesky` Module. All are secondary routines since these functions are more easily used via the `Cholesky` Module.

Secondary routines:

- `cholmod_super_symbolic`: supernodal symbolic analysis
- `cholmod_super_numeric`: supernodal numeric factorization
- `cholmod_super_lsolve`: supernodal $\mathbf{Lx} = \mathbf{b}$ solve
- `cholmod_super_ltsolve`: supernodal $\mathbf{L}^{\mathsf{T}}\mathbf{x} = \mathbf{b}$ solve

11.8 `Partition` Module: graph-partitioning-based orderings

The `Partition` Module provides graph partitioning and graph-partition-based orderings. It includes an interface to CAMD, CCOLAMD, and CSYMAMD, constrained minimum degree ordering methods which order a matrix following constraints determined via nested dissection. Requires the `Utility` and `Cholesky` Modules, and two packages: `METIS 5.1.0`, CAMD, and CCOLAMD. Optionally used by the `Cholesky` Module. All are secondary routines since these are more easily used by the `Cholesky` Module.

Secondary routines:

- `cholmod_ccolamd`: interface to CCOLAMD ordering
- `cholmod_csymamd`: interface to CSYMAMD ordering

- `cholmod_camd`: interface to CAMD ordering
- `cholmod_nested_dissection`: CHOLMOD nested dissection ordering
- `cholmod_metis`: METIS nested dissection ordering (`METIS_NodeND`)
- `cholmod_bisect`: graph partitioner (currently based on METIS)
- `cholmod_metis_bisector`: direct interface to `METIS_NodeComputeSeparator`.
- `cholmod_collapse_septree`: pruned a separator tree from `cholmod_nested_dissection`.

12 CHOLMOD naming convention, parameters, and return values

All routine names, data types, and CHOLMOD library files use the `cholmod_` prefix. All macros and other `#define` statements visible to the user program use the `CHOLMOD` prefix. The `cholmod.h` file must be included in user programs that use CHOLMOD:

```
#include "cholmod.h"
```

All CHOLMOD routines (in all modules) use the following protocol for return values:

- `int`: `TRUE` (1) if successful, or `FALSE` (0) otherwise. (exception: `cholmod_divcomplex`).
- `int32_t` or `int64_t`: a value ≥ 0 if successful, or -1 otherwise.
- `float` or `double`: a value ≥ 0 if successful, or -1 otherwise.
- `size_t`: a value > 0 if successful, or 0 otherwise.
- `void *`: a non-`NULL` pointer to newly allocated memory if successful, or `NULL` otherwise.
- `cholmod_sparse *`: a non-`NULL` pointer to a newly allocated sparse matrix if successful, or `NULL` otherwise.
- `cholmod_factor *`: a non-`NULL` pointer to a newly allocated factor if successful, or `NULL` otherwise.
- `cholmod_triplet *`: a non-`NULL` pointer to a newly allocated triplet matrix if successful, or `NULL` otherwise.
- `cholmod_dense *`: a non-`NULL` pointer to a newly allocated dense matrix if successful, or `NULL` otherwise.

`TRUE` and `FALSE` are not defined in `cholmod.h`, since they may conflict with the user program. A routine that described here returning `TRUE` or `FALSE` returns 1 or 0, respectively. Any `TRUE`/`FALSE` parameter is true if nonzero, false if zero.
Input, output, and input/output parameters:

- Input parameters appear first in the parameter lists of all CHOLMOD routines. They are not modified by CHOLMOD.
- Input/output parameters (except for `Common`) appear next. They must be defined on input, and are modified on output.
- Output parameters are listed next. If they are pointers, they must point to allocated space on input, but their contents are not defined on input.
- Workspace parameters appear next. They are used in only two routines in the Supernodal module.

- The `cholmod_common *Common` parameter always appears as the last parameter (with two exceptions: `cholmod_hypot` and `cholmod_divcomplex`). It is always an input/output parameter.

A floating-point scalar is passed to CHOLMOD as a pointer to a `double` array of size two. The first entry in this array is the real part of the scalar, and the second entry is the imaginary part. The imaginary part is only accessed if the other inputs are complex or zomplex. In some cases the imaginary part is always ignored (`cholmod_factor_p`, for example). This method for passing scalars is used when the computations are done both in `double` and single (`float`) precision.

13 Utility Module: cholmod_common object

13.1 cholmod_common: parameters, statistics, and workspace

```
typedef struct cholmod_common_struct
{

    //------------------------------------------------------------------------
    // primary parameters for factorization and update/downdate
    //------------------------------------------------------------------------

    double dbound ; // Bounds the diagonal entries of D for LDL'
        // factorization and update/downdate/rowadd.  Entries outside this
        // bound are replaced with dbound.  Default: 0.  dbound is used for
        // double precision factorization only.  See sbound for single
        // precision factorization.

    double grow0 ;      // default: 1.2
    double grow1 ;      // default: 1.2
    size_t grow2 ;      // default: 5
        // Initial space for simplicial factorization is max(grow0,1) times the
        // required space.  If space is exhausted, L is grown by max(grow0,1.2)
        // times the required space.  grow1 and grow2 control how each column
        // of L can grow in an update/downdate; if space runs out, then
        // grow1*(required space) + grow2 is allocated.

    size_t maxrank ;    // maximum rank for update/downdate.  Valid values are
        // 2, 4, and 8.  Default is 8.  If a larger update/downdate is done, it
        // is done in steps of maxrank.

    double supernodal_switch ;  // default: 40
    int supernodal ;            // default: CHOLMOD_AUTO.
        // Controls supernodal vs simplicial factorization.  If
        // Common->supernodal is CHOLMOD_SIMPLICIAL, a simplicial factorization
        // is always done; if CHOLMOD_SUPERNODAL, a supernodal factorization is
        // always done.  If CHOLMOD_AUTO, then a simplicial factorization is
        // down if flops/nnz(L) < Common->supernodal_switch.

        #define CHOLMOD_SIMPLICIAL 0 /* always use simplicial method         */
        #define CHOLMOD_AUTO       1 /* auto select simplicial vs supernodal */
        #define CHOLMOD_SUPERNODAL 2 /* always use supernoda method          */

    int final_asis ;    // if true, other final_* parameters are ignored,
        // except for final_pack and the factors are left as-is when done.
        // Default: true.

    int final_super ;   // if true, leave factor in supernodal form.
        // if false, convert to simplicial.  Default: true.

    int final_ll ;      // if true, simplicial factors are converted to LL',
        // otherwise left as LDL.  Default: false.

    int final_pack ;    // if true, the factorize are allocated with exactly
        // the space required.  Set this to false if you expect future
        // updates/downdates (giving a little extra space for future growth),
```

```
    // Default: true.

int final_monotonic ;   // if true, columns are sorted when done, by
    // ascending row index.  Default: true.

int final_resymbol ;    // if true, a supernodal factorization converted
    // to simplicial is reanalyzed, to remove zeros added for relaxed
    // amalgamation.  Default: false.

double zrelax [3] ; size_t nrelax [3] ;
    // The zrelax and nrelax parameters control relaxed supernodal
    // amalgamation,  If ns is the # of columns in two adjacent supernodes,
    // and z is the fraction of zeros in the two supernodes if merged, then
    // the two supernodes are merged if any of the 5 following condition
    // are true:
    //
    //      no new zero entries added if the two supernodes are merged
    //      (ns <= nrelax [0])
    //      (ns <= nrelax [1] && z < zrelax [0])
    //      (ns <= nrelax [2] && z < zrelax [1])
    //      (z < zrelax [2])
    //
    // With the defaults, the rules become:
    //
    //      no new zero entries added if the two supernodes are merged
    //      (ns <=  4)
    //      (ns <= 16 && z < 0.8)
    //      (ns <= 48 && z < 0.1)
    //      (z < 0.05)

int prefer_zomplex ;    // if true, and a complex system is solved,
    // X is returned as zomplex (with two arrays, one for the real part
    // and one for the imaginary part).  If false, then X is returned as
    // a single array with interleaved real and imaginary parts.
    // Default: false.

int prefer_upper ;  // if true, then a preference is given for holding
    // a symmetric matrix by just its upper triangular form.  This gives
    // the best performance by the CHOLMOD analysis and factorization
    // methods.  Only used by cholmod_read.  Default: true.

int quick_return_if_not_posdef ;    // if true, a supernodal factorization
    // returns immediately if it finds the matrix is not positive definite.
    // If false, the failed supernode is refactorized, up to but not
    // including the failed column (required by MATLAB).

int prefer_binary ; // if true, cholmod_read_triplet converts a symmetric
    // pattern-only matrix to a real matrix with all values set to 1.
    // if false, diagonal entries A(k,k) are set to one plus the # of
    // entries in row/column k, and off-diagonals are set to -1.
    // Default: false.

//------------------------------------------------------------------------
// printing and error handling options
//------------------------------------------------------------------------
```

```
int print ;     // print level.  Default is 3.
int precise ;   // if true, print 16 digits, otherwise 5. Default: false.

int try_catch ; // if true, ignore errors (CHOLMOD is assumed to be inside
    // a try/catch block.  No error messages are printed and the
    // error_handler function is not called.  Default: false.

void (*error_handler) (int status, const char *file, int line,
    const char *message) ;
    // User error handling routine; default is NULL.
    // This function is called if an error occurs, with parameters:
    // status: the Common->status result.
    // file: filename where the error occurred.
    // line: line number where the error occurred.
    // message: a string that describes the error.

//------------------------------------------------------------------------------
// ordering options
//------------------------------------------------------------------------------

// CHOLMOD can try many ordering options and then pick the best result it
// finds.  The default is to use one or two orderings: the user's
// permutation (if given), and AMD.

// Common->nmethods is the number of methods to try.  If the
// Common->method array is left unmodified, the methods are:

// (0) given (skipped if no user permutation)
// (1) amd
// (2) metis
// (3) nesdis with defaults (CHOLMOD's nested dissection, based on METIS)
// (4) natural
// (5) nesdis: stop at subgraphs of 20000 nodes
// (6) nesdis: stop at subgraphs of 4 nodes, do not use CAMD
// (7) nesdis: no pruning on of dense rows/cols
// (8) colamd

// To use all 9 of the above methods, set Common->nmethods to 9.  The
// analysis will take a long time, but that might be worth it if the
// ordering will be reused many many times.

// Common->nmethods and Common->methods can be revised to use a different
// set of orderings.  For example, to use just a single method
// (AMD with a weighted postordering):
//
//      Common->nmethods = 1 ;
//      Common->method [0].ordering = CHOLMOD_AMD ;
//      Common->postorder = TRUE ;
//
//

int nmethods ;  // Number of methods to try, default is 0.
    // The value of 0 is a special case, and tells CHOLMOD to use the user
    // permutation (if not NULL) and then AMD.  Next, if fl is lnz are the
```

```
    // flop counts and number of nonzeros in L as found by AMD, then the
    // this ordering is used if fl/lnz < 500 or lnz/anz < 5, where anz is
    // the number of entries in A.  If this condition fails, METIS is tried
    // as well.
    //
    // Otherwise, if Common->nmethods > 0, then the methods defined by
    // Common->method [0 ... Common->nmethods-1] are used.

int current ;   // The current method being tried in the analysis.
int selected ;  // The selected method: Common->method [Common->selected]

// The Common->method parameter is an array of structs that defines up
// to 9 methods:
struct cholmod_method_struct
{

    //--------------------------------------------------------------------
    // statistics from the ordering
    //--------------------------------------------------------------------

    double lnz ;    // number of nonzeros in L
    double fl ;     // Cholesky flop count for this ordering (each
        // multiply and each add counted once (doesn't count complex
        // flops).

    //--------------------------------------------------------------------
    // ordering parameters:
    //--------------------------------------------------------------------

    double prune_dense ;    // dense row/col control.  Default: 10.
        // Rows/cols with more than max (prune_dense*sqrt(n),16) are
        // removed prior to orderingm and placed last.  If negative,
        // only completely dense rows/cols are removed.  Removing these
        // rows/cols with many entries can speed up the ordering, but
        // removing too many can reduce the ordering quality.
        //
        // For AMD, SYMAMD, and CSYMAMD, this is the only dense row/col
        // parameter.  For COLAMD and CCOLAMD, this parameter controls
        // how dense columns are handled.

    double prune_dense2 ;   // dense row control for COLAMD and CCOLAMD.
        // Default -1.  When computing the Cholesky factorization of AA'
        // rows with more than max(prune_dense2*sqrt(n),16) entries
        // are removed prior to ordering.  If negative, only completely
        // dense rows are removed.

    double nd_oksep ;   // for CHOLMOD's nesdis method. Default 1.
        // A node separator with nsep nodes is discarded if
        // nsep >= nd_oksep*n.

    double other_1 [4] ;    // unused, for future expansion

    size_t nd_small ;   // for CHOLMOD's nesdis method. Default 200.
        // Subgraphs with fewer than nd_small nodes are not partitioned.
```

```
    double other_2 [4] ;    // unused, for future expansion

    int aggressive ;    // if true, AMD, COLAMD, SYMAMD, CCOLAMD, and
        // CSYMAMD perform aggresive absorption.  Default: true

    int order_for_lu ;  // Default: false.  If the CHOLMOD analysis/
        // ordering methods are used as an ordering method for an LU
        // factorization, then set this to true.  For use in a Cholesky
        // factorization by CHOLMOD itself, never set this to true.

    int nd_compress ;   // if true, then the graph and subgraphs are
        // compressed before partitioning them in CHOLMOD's nesdis
        // method.  Default: true.

    int nd_camd ;   // if 1, then CHOLMOD's nesdis is followed by
        // CAMD.  If 2: followed by CSYMAMD.  If nd_small is very small,
        // then use 0, which skips CAMD or CSYMAMD.  Default: 1.

    int nd_components ; // CHOLMOD's nesdis can partition a graph and then
        // find that the subgraphs are unconnected.  If true, each of these
        // components is partitioned separately.  If false, the whole
        // subgraph is partitioned.  Default: false.

    int ordering ;  // ordering method to use:

        #define CHOLMOD_NATURAL     0 /* no preordering                   */
        #define CHOLMOD_GIVEN       1 /* user-provided permutation        */
        #define CHOLMOD_AMD         2 /* AMD: approximate minimum degree  */
        #define CHOLMOD_METIS       3 /* METIS: mested dissection         */
        #define CHOLMOD_NESDIS      4 /* CHOLMOD's nested dissection      */
        #define CHOLMOD_COLAMD      5 /* AMD for A, COLAMD for AA' or A'A */
        #define CHOLMOD_POSTORDERED 6 /* natural then postordered         */

    size_t other_3 [4] ;    // unused, for future expansion

}
#define CHOLMOD_MAXMETHODS 9    /* max # of methods in Common->method */
method [CHOLMOD_MAXMETHODS + 1] ;

int postorder ; // if true, CHOLMOD performs a weighted postordering
    // after its fill-reducing ordering, which improves supernodal
    // amalgamation.  Has no effect on flop count or nnz(L).
    // Default: true.

int default_nesdis ;    //  If false, then the default ordering strategy
    // when Common->nmethods is zero is to try the user's permutation
    // if given, then AMD, and then METIS if the AMD ordering results in
    // a lot of fill-in.  If true, then nesdis is used instead of METIS.
    // Default: false.

//------------------------------------------------------------------------------
// METIS workarounds
//------------------------------------------------------------------------------

// These workarounds were put into place for METIS 4.0.1.  They are safe
```

```
// to use with METIS 5.1.0, but they might not longer be necessary.

double metis_memory ;   // default: 0.  If METIS terminates your
    // program when it runs out of memory, try 2, or higher.
double metis_dswitch ;  // default: 0.66
size_t metis_nswitch ;  // default: 3000
    // If a matrix has n > metis_nswitch and a density (nnz(A)/n^2) >
    // metis_dswitch, then METIS is not used.

//------------------------------------------------------------------------------
// workspace
//------------------------------------------------------------------------------

// This workspace is kept in the CHOLMOD Common object.  cholmod_start
// sets these arrays to NULL, and cholmod_finish frees them.

size_t nrow ;       // Flag has size nrow, Head has size nrow+1
int64_t mark ;      // Flag is cleared if Flag [0..nrow-1] < mark.
size_t iworksize ;  // size of Iwork, in Ints (int32 or int64).
                    // This is at most 6*nrow + ncol.
size_t xworkbytes ; // size of Xwork, in bytes.
    // NOTE: in CHOLMOD v4 and earlier, this variable was called xworksize,
    // and was in terms of # of doubles, not # of bytes.

void *Flag ;    // size nrow.  If this is "cleared" then
    // Flag [i] < mark for all i = 0:nrow-1.  Flag is kept cleared between
    // calls to CHOLMOD.

void *Head ;    // size nrow+1.  If Head [i] = EMPTY (-1) then that
    // entry is "cleared".  Head is kept cleared between calls to CHOLMOD.

void *Xwork ;   // a double or float array.  It has size nrow for most
    // routines, or 2*nrow if complex matrices are being handled.
    // It has size 2*nrow for cholmod_rowadd/rowdel, and maxrank*nrow for
    // cholmod_updown, where maxrank is 2, 4, or 8.  Xwork is kept all
    // zero between calls to CHOLMOD.

void *Iwork ;   // size iworksize integers (int32's or int64's).
    // Uninitialized integer workspace, of size at most 6*nrow+ncol.

int itype ;     // cholmod_start (for int32's) sets this to CHOLMOD_INT,
    // and cholmod_l_start sets this to CHOLMOD_LONG.  It defines the
    // integer sizes for th Flag, Head, and Iwork arrays, and also
    // defines the integers for all objects created by CHOLMOD.
    // The itype of the Common object must match the function name
    // and all objects passed to it.

int other_5 ;   // unused: for future expansion

int no_workspace_reallocate ;   // an internal flag, usually false.
    // This is set true to disable any reallocation of the workspace
    // in the Common object.

//------------------------------------------------------------------------------
// statistics
```

```
//------------------------------------------------------------------------------

int status ;    // status code (0: ok, negative: error, pos: warning)

    // Common->status for error handling: 0 is ok, negative is a fatal
    // error, and positive is a warning
    #define CHOLMOD_OK            (0)
    #define CHOLMOD_NOT_INSTALLED (-1) /* module not installed          */
    #define CHOLMOD_OUT_OF_MEMORY (-2) /* malloc/calloc/realloc failed  */
    #define CHOLMOD_TOO_LARGE     (-3) /* integer overflow              */
    #define CHOLMOD_INVALID       (-4) /* input invalid                 */
    #define CHOLMOD_GPU_PROBLEM   (-5) /* CUDA error                    */
    #define CHOLMOD_NOT_POSDEF    (1)  /* matrix not positive definite  */
    #define CHOLMOD_DSMALL        (2)  /* diagonal entry very small     */

double fl ;     // flop count from last analysis
double lnz ;    // nnz(L) from last analysis
double anz ;    // in last analysis: nnz(tril(A)) or nnz(triu(A)) if A
                // symmetric, or tril(A*A') if A is unsymmetric.
double modfl ;  // flop count from last update/downdate/rowadd/rowdel,
                // not included the flops to revise the solution to Lx=b,
                // if that was performed.

size_t malloc_count ;   // # of malloc'd objects not yet freed
size_t memory_usage ;   // peak memory usage in bytes
size_t memory_inuse ;   // current memory usage in bytes

double nrealloc_col ;   // # of column reallocations
double nrealloc_factor ;// # of factor reallocations due to col. reallocs
double ndbounds_hit ;   // # of times diagonal modified by dbound

double rowfacfl ;       // flop count of cholmod_rowfac
double aatfl ;          // flop count to compute A(:,f)*A(:,f)'

int called_nd ; // true if last analysis used nesdis or METIS.
int blas_ok ;   // true if no integer overflow has occured when trying to
    // call the BLAS.  The typical BLAS library uses 32-bit integers for
    // its input parameters, even on a 64-bit platform.  CHOLMOD uses int64
    // in its cholmod_l_* methods, and these must be typecast to the BLAS
    // integer.  If integer overflow occurs, this is set false.

//------------------------------------------------------------------------------
// SuiteSparseQR control parameters and statistics
//------------------------------------------------------------------------------

// SPQR uses the CHOLMOD Common object for its control and statistics.
// These parameters are not used by CHOLMOD itself.

// control parameters:
double SPQR_grain ;     // task size is >= max (total flops / grain)
double SPQR_small ;     // task size is >= small
int SPQR_shrink ;       // controls stack realloc method
int SPQR_nthreads ;     // number of TBB threads, 0 = auto

// statistics:
```

```
double SPQR_flopcount ;         // flop count for SPQR
double SPQR_analyze_time ;      // analysis time in seconds for SPQR
double SPQR_factorize_time ;    // factorize time in seconds for SPQR
double SPQR_solve_time ;        // backsolve time in seconds
double SPQR_flopcount_bound ;   // upper bound on flop count
double SPQR_tol_used ;          // tolerance used
double SPQR_norm_E_fro ;        // Frobenius norm of dropped entries

//------------------------------------------------------------------------------
// Revised for CHOLMOD v5.0
//------------------------------------------------------------------------------

// was size 10 in CHOLMOD v4.2; reduced to 8 in CHOLMOD v5:
int64_t SPQR_istat [8] ;        // other statistics

//------------------------------------------------------------------------------
// Added for CHOLMOD v5.0
//------------------------------------------------------------------------------

// These terms have been added to the CHOLMOD Common struct for v5.0, and
// on most systems they will total 16 bytes.  The preceding term,
// SPQR_istat, was reduced by 16 bytes, since those last 2 entries were
// unused in CHOLMOD v4.2.  As a result, the Common struct in v5.0 has the
// same size as v4.0, and all entries would normally be in the same offset,
// as well.  This mitigates any changes between v4.0 and v5.0, and may make
// it easier to upgrade from v4 to v5.

double nsbounds_hit ;   // # of times diagonal modified by sbound.
                // This ought to be int64_t, but ndbounds_hit was double in
                // v4 (see above), so nsbounds_hit is made the same type
                // for consistency.
float sbound ;  // Same as dbound,
                // but for single precision factorization.
float other_6 ; // for future expansion

//------------------------------------------------------------------------------
// GPU configuration and statistics
//------------------------------------------------------------------------------

int useGPU ; // 1 if GPU is requested for CHOLMOD
             // 0 if GPU is not requested for CHOLMOD
             // -1 if the use of the GPU is in CHOLMOD controled by the
             // CHOLMOD_USE_GPU environment variable.

size_t maxGpuMemBytes ;     // GPU control for CHOLMOD
double maxGpuMemFraction ;  // GPU control for CHOLMOD

// for SPQR:
size_t gpuMemorySize ;      // Amount of memory in bytes on the GPU
double gpuKernelTime ;      // Time taken by GPU kernels
int64_t gpuFlops ;          // Number of flops performed by the GPU
int gpuNumKernelLaunches ;  // Number of GPU kernel launches

#ifdef CHOLMOD_HAS_CUDA
    // these three types are pointers defined by CUDA:
```

```
    #define CHOLMOD_CUBLAS_HANDLE cublasHandle_t
    #define CHOLMOD_CUDASTREAM    cudaStream_t
    #define CHOLMOD_CUDAEVENT     cudaEvent_t
#else
    // they are (void *) if CUDA is not in use:
    #define CHOLMOD_CUBLAS_HANDLE void *
    #define CHOLMOD_CUDASTREAM    void *
    #define CHOLMOD_CUDAEVENT     void *
#endif

CHOLMOD_CUBLAS_HANDLE cublasHandle ;

// a set of streams for general use
CHOLMOD_CUDASTREAM  gpuStream [CHOLMOD_HOST_SUPERNODE_BUFFERS] ;

CHOLMOD_CUDAEVENT   cublasEventPotrf [3] ;
CHOLMOD_CUDAEVENT   updateCKernelsComplete ;
CHOLMOD_CUDAEVENT   updateCBuffersFree [CHOLMOD_HOST_SUPERNODE_BUFFERS] ;

void *dev_mempool ; // pointer to single allocation of device memory
size_t dev_mempool_size ;

void *host_pinned_mempool ; // pointer to single alloc of pinned mem
size_t host_pinned_mempool_size ;

size_t devBuffSize ;
int    ibuffer ;
double syrkStart ;          // time syrk started

// run times of the different parts of CHOLMOD (GPU and CPU):
double cholmod_cpu_gemm_time ;
double cholmod_cpu_syrk_time ;
double cholmod_cpu_trsm_time ;
double cholmod_cpu_potrf_time ;
double cholmod_gpu_gemm_time ;
double cholmod_gpu_syrk_time ;
double cholmod_gpu_trsm_time ;
double cholmod_gpu_potrf_time ;
double cholmod_assemble_time ;
double cholmod_assemble_time2 ;

// number of times the BLAS are called on the CPU and the GPU:
size_t cholmod_cpu_gemm_calls ;
size_t cholmod_cpu_syrk_calls ;
size_t cholmod_cpu_trsm_calls ;
size_t cholmod_cpu_potrf_calls ;
size_t cholmod_gpu_gemm_calls ;
size_t cholmod_gpu_syrk_calls ;
size_t cholmod_gpu_trsm_calls ;
size_t cholmod_gpu_potrf_calls ;

double chunk ;      // chunksize for computing # of OpenMP threads to use.
    // Given nwork work to do, # of threads is
    // max (1, min (floor (work / chunk), nthreads_max))
```

```
    int nthreads_max ; // max # of OpenMP threads to use in CHOLMOD.
        // Defaults to SUITESPARSE_OPENMP_MAX_THREADS.

    #ifdef BLAS_DUMP
    FILE *blas_dump ;  // only used if CHOLMOD is compiled with -DBLAS_DUMP
    #endif

} cholmod_common ;
```

Purpose: The `cholmod_common Common` object contains parameters, statistics, and workspace used within CHOLMOD. The first call to CHOLMOD must be `cholmod_start`, which initializes this object.

13.2 `cholmod_start`: start CHOLMOD

```
int cholmod_start (cholmod_common *Common) ;
int cholmod_l_start (cholmod_common *) ;
```

Purpose: Sets the default parameters, clears the statistics, and initializes all workspace pointers to `NULL`. The `int32/int64_t` type is set in `Common->itype`.

13.3 `cholmod_finish`: finish CHOLMOD

```
int cholmod_finish (cholmod_common *Common) ;
int cholmod_l_finish (cholmod_common *) ;
```

Purpose: This must be the last call to CHOLMOD.

13.4 `cholmod_defaults`: set default parameters

```
int cholmod_defaults (cholmod_common *Common) ;
int cholmod_l_defaults (cholmod_common *) ;
```

Purpose: Sets the default parameters.

13.5 `cholmod_maxrank`: maximum update/downdate rank

```
size_t cholmod_maxrank      // return validated Common->maxrank
(
    // input:
    size_t n,               // # of rows of L and A
    cholmod_common *Common
) ;
size_t cholmod_l_maxrank (size_t, cholmod_common *) ;
```

Purpose: Returns the maximum rank for an update/downdate.

13.6 cholmod_allocate_work: allocate workspace

```
int cholmod_allocate_work
(
    // input:
    size_t nrow,        // size of Common->Flag (nrow int32's)
                        // and Common->Head (nrow+1 int32's)
    size_t iworksize,   // size of Common->Iwork (# of int32's)
    size_t xworksize,   // size of Common->Xwork (# of double's)
    cholmod_common *Common
) ;
int cholmod_l_allocate_work (size_t, size_t, size_t, cholmod_common *) ;
```

Purpose: Allocates workspace in `Common`. The workspace consists of the integer `Head`, `Flag`, and `Iwork` arrays, of size `nrow+1`, `nrow`, and `iworksize`, respectively, and a `double` array `Xwork` of size `xworksize` entries. The `Head` array is normally equal to -1 when it is cleared. If the `Flag` array is cleared, all entries are less than `Common->mark`. The `Iwork` array is not kept in any particular state. The integer type is `int32_t` or `int64_t`, depending on whether the `cholmod_` or `cholmod_l_` routines are used.

13.7 cholmod_alloc_work: allocate workspace

```
int cholmod_alloc_work
(
    // input:
    size_t nrow,        // size of Common->Flag (nrow int32's)
                        // and Common->Head (nrow+1 int32's)
    size_t iworksize,   // size of Common->Iwork (# of int32's)
    size_t xworksize,   // size of Common->Xwork (# of entries)
    int dtype,          // CHOLMOD_DOUBLE or CHOLMOD_SINGLE
    cholmod_common *Common
) ;
int cholmod_l_alloc_work (size_t, size_t, size_t, int, cholmod_common *) ;
```

Purpose: This is the same as `cholmod_allocate_work`, except that the `Xwork` array can be `float` or `double`, as determined by the `dtype` input parameter.

13.8 cholmod_free_work: free workspace

```
int cholmod_free_work (cholmod_common *Common) ;
int cholmod_l_free_work (cholmod_common *) ;
```

Purpose: Frees the workspace in `Common`.

13.9 cholmod_clear_flag: clear Flag array

```
int64_t cholmod_clear_flag (cholmod_common *Common) ;
int64_t cholmod_l_clear_flag (cholmod_common *) ;
```

Purpose: Increments `Common->mark` so that the `Flag` array is now cleared.

13.10 cholmod_error: report error

```
int cholmod_error
(
    // input:
    int status,             // Common->status
    const char *file,       // source file where error occurred
    int line,               // line number where error occurred
    const char *message,    // error message to print
    cholmod_common *Common
) ;
int cholmod_l_error (int, const char *, int, const char *, cholmod_common *) ;
```

Purpose: This routine is called when CHOLMOD encounters an error. It prints a message (if printing is enabled), sets `Common->status`. It then calls the user error handler routine `Common->error_handler`, if it is not `NULL`.

13.11 cholmod_dbound: bound diagonal of L

```
double cholmod_dbound   (double, cholmod_common *) ;
double cholmod_l_dbound (double, cholmod_common *) ;
```

Purpose: Ensures that entries on the diagonal of $\mathbf{L}$ for an $\mathbf{LL}^\mathsf{T}$ factorization are greater than or equal to `Common->dbound`, when computing in double precision (`double`). For an $\mathbf{LDL}^\mathsf{T}$ factorization, it ensures that the magnitude of the entries of $\mathbf{D}$ are greater than or equal to `Common->dbound`.

13.12 cholmod_sbound: bound diagonal of L

```
float cholmod_sbound   (float,  cholmod_common *) ;
float cholmod_l_sbound (float,  cholmod_common *) ;
```

Purpose: Ensures that entries on the diagonal of $\mathbf{L}$ for an $\mathbf{LL}^\mathsf{T}$ factorization are greater than or equal to `Common->sbound`, when computing in single precision (`float`). For an $\mathbf{LDL}^\mathsf{T}$ factorization, it ensures that the magnitude of the entries of $\mathbf{D}$ are greater than or equal to `Common->sbound`.

13.13 cholmod_hypot: sqrt(x*x+y*y)

```
double cholmod_hypot (double x, double y) ;
double cholmod_l_hypot (double, double) ;
```

Purpose: Computes the magnitude of a complex number. This routine calls `SuiteSparse_config_hypot`. Refer to the `SuiteSparse_config` package for details.

13.14 cholmod_divcomplex: complex divide

```
int cholmod_divcomplex          // return 1 if divide-by-zero, 0 if OK
(
    // input:
    double ar, double ai,       // a (real, imaginary)
    double br, double bi,       // b (real, imaginary)
    double *cr, double *ci      // c (real, imaginary)
) ;
int cholmod_l_divcomplex (double, double, double, double, double *, double *) ;
```

Purpose: Divides two complex numbers. This routine calls SuiteSparse_config_divcomplex. Refer to the SuiteSparse_config package for details.

14 Utility Module: cholmod_sparse object

14.1 cholmod_sparse: compressed-column sparse matrix

```
typedef struct cholmod_sparse_struct
{
    size_t nrow ;   // # of rows of the matrix
    size_t ncol ;   // # of colums of the matrix
    size_t nzmax ;  // max # of entries that can be held in the matrix

    // int32_t or int64_t arrays:
    void *p ;   // A->p [0..ncol], column "pointers" of the CSC matrix
    void *i ;   // A->i [0..nzmax-1], the row indices

    // for unpacked matrices only:
    void *nz ;  // A->nz [0..ncol-1], is the # of nonzeros in each col.
        // This is NULL for a "packed" matrix (conventional CSC).
        // For a packed matrix, the jth column is held in A->i and A->x in
        // postions A->p [j] to A->p [j+1]-1, with no gaps between columns.
        // For an "unpacked" matrix, there can be gaps between columns, so
        // the jth columns appears in positions A-p [j] to
        // A->p [j] + A->nz [j] - 1.

    // double or float arrays:
    void *x ;   // size nzmax or 2*nzmax, or NULL
    void *z ;   // size nzmax, or NULL

    int stype ; // A->stype defines what parts of the matrix is held:
        // 0:  the matrix is unsymmetric with both lower and upper parts stored.
        // >0: the matrix is square and symmetric, with just the upper
        //     triangular part stored.
        // <0: the matrix is square and symmetric, with just the lower
        //     triangular part stored.

    int itype ; // A->itype defines the integers used for A->p, A->i, and A->nz.
        // if CHOLMOD_INT, these arrays are all of type int32_t.
        // if CHOLMOD_LONG, these arrays are all of type int64_t.
    int xtype ;     // pattern, real, complex, or zomplex
    int dtype ;     // x and z are double or single
    int sorted ;    // true if columns are sorted, false otherwise
    int packed ;    // true if packed (A->nz ignored), false if unpacked

} cholmod_sparse ;
```

Purpose: Stores a sparse matrix in compressed-column form.

14.2 cholmod_allocate_sparse: allocate sparse matrix

```
cholmod_sparse *cholmod_allocate_sparse
(
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
```

```
    size_t nzmax,   // max # of entries the matrix can hold
    int sorted,     // true if columns are sorted
    int packed,     // true if A is be packed (A->nz NULL), false if unpacked
    int stype,      // the stype of the matrix (unsym, tril, or triu)
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (CHOLMOD_PATTERN, _REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_allocate_sparse (size_t, size_t, size_t, int, int,
    int, int, cholmod_common *) ;
```

Purpose: Allocates a sparse matrix. Indices and values (`A->i`, `A->x`, and `A->z`) are allocated but not initialized. The matrix returned is valid, has no entries, but contains space enough for `nzmax` entries.

14.3 `cholmod_free_sparse`: free sparse matrix

```
int cholmod_free_sparse
(
    // input/output:
    cholmod_sparse **A,         // handle of sparse matrix to free
    cholmod_common *Common
) ;
int cholmod_l_free_sparse (cholmod_sparse **, cholmod_common *) ;
```

Purpose: Frees a sparse matrix.

14.4 `cholmod_reallocate_sparse`: reallocate sparse matrix

```
int cholmod_reallocate_sparse
(
    // input:
    size_t nznew,       // new max # of nonzeros the sparse matrix can hold
    // input/output:
    cholmod_sparse *A,  // sparse matrix to reallocate
    cholmod_common *Common
) ;
int cholmod_l_reallocate_sparse (size_t, cholmod_sparse *, cholmod_common *) ;
```

Purpose: Reallocates a sparse matrix, so that it can contain `nznew` entries.

14.5 `cholmod_nnz`: number of entries in sparse matrix

```
int64_t cholmod_nnz             // return # of entries in the sparse matrix
(
    // input:
    cholmod_sparse *A,          // sparse matrix to query
    cholmod_common *Common
) ;
int64_t cholmod_l_nnz (cholmod_sparse *, cholmod_common *) ;
```

Purpose: Returns the number of entries in a sparse matrix.

14.6 cholmod_speye: sparse identity matrix

```
cholmod_sparse *cholmod_speye
(
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (CHOLMOD_PATTERN, _REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_speye (size_t, size_t, int, cholmod_common *) ;
```

Purpose: Returns the sparse identity matrix.

14.7 cholmod_spzeros: sparse zero matrix

```
cholmod_sparse *cholmod_spzeros     // return a sparse matrix with no entries
(
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
    size_t nzmax,   // max # of entries the matrix can hold
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (CHOLMOD_PATTERN, _REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_spzeros (size_t, size_t, size_t, int,
    cholmod_common *) ;
```

Purpose: Returns the sparse zero matrix. This is another name for `cholmod_allocate_sparse`, but with fewer parameters (the matrix is packed, sorted, and unsymmetric).

14.8 cholmod_transpose: transpose sparse matrix

```
cholmod_sparse *cholmod_transpose       // return new sparse matrix C
(
    // input:
    cholmod_sparse *A,  // input matrix
    int mode,           // 2: numerical (conj)
                        // 1: numerical (non-conj.)
                        // 0: pattern (with diag)
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_transpose (cholmod_sparse *, int, cholmod_common *) ;
```

Purpose: Returns the transpose or complex conjugate transpose of a sparse matrix. Three kinds of transposes are available, depending on the `mode` parameter:

- 0: do not compute the numerical values; create a `CHOLMOD_PATTERN` matrix
- 1: array transpose
- 2: complex conjugate transpose (same as 2 if input is real or pattern)

14.9 `cholmod_transpose_unsym`: transpose/permute unsymmetric sparse matrix

```
int cholmod_transpose_unsym
(
    // input:
    cholmod_sparse *A,  // input matrix
    int mode,           // 2: numerical (conj)
                        // 1: numerical (non-conj.),
                        // 0: pattern (with diag)
    int32_t *Perm,      // permutation for C=A(p,f)', or NULL
    int32_t *fset,      // a list of column indices in range 0:A->ncol-1
    size_t fsize,       // # of entries in fset
    // input/output:
    cholmod_sparse *C,  // output matrix, must be allocated on input
    cholmod_common *Common
) ;
int cholmod_l_transpose_unsym (cholmod_sparse *, int, int64_t *, int64_t *,
    size_t, cholmod_sparse *, cholmod_common *) ;
```

Purpose: Transposes and optionally permutes an unsymmetric sparse matrix. The output matrix must be preallocated before calling this routine. The `mode` parameter is the same as for `cholmod_transpose`.

Computes `F=A'`, `F=A(:,f)'` or `F=A(p,f)'`, except that the indexing by `f` does not work the same as the MATLAB notation (see below). `A->stype` must be zero on input, which denotes that the matrix is unsymmetric, with both the upper and lower triangular parts of A are present. `A` may be rectangular.

The integer vector `p` is a permutation of `0:m-1`, and `f` is a subset of `0:n-1`, where A is `m`-by-`n`. There can be no duplicate entries in `p` or `f`.
The set `f` is held in fset and fsize:

- `fset = NULL` means ":" in MATLAB. `fset` is ignored.
- `fset != NULL` means `f = fset [0..fsize-1]`.
- `fset != NULL` and `fsize = 0` means `f` is the empty set.

Columns not in the set `f` are considered to be zero. That is, if `A` is 5-by-10 then `F=A(:,[3 4])'` is not 2-by-5, but 10-by-5, and rows 3 and 4 of `F` are equal to columns 3 and 4 of `A` (the other rows of `F` are zero). More precisely, in MATLAB notation:

```
[m n] = size (A)
F = A
notf = ones (1,n)
notf (f) = 0
F (:, find (notf)) = 0
F = F'
```

If you want the MATLAB equivalent `F=A(p,f)` operation, use `cholmod_submatrix` instead (which does not compute the transpose). `F->nzmax` must be large enough to hold the matrix `F`. If `F->nz` is present then `F->nz [j]` is equal to the number of entries in column `j` of `F`. `A` can be sorted or unsorted, with packed or unpacked columns. If `f` is present and not sorted in ascending order, then `F` is unsorted (that is, it may contain columns whose row indices do not appear in ascending order). Otherwise, `F` is sorted (the row indices in each column of `F` appear in strictly ascending order).

`F` is returned in packed or unpacked form, depending on `F->packed` on input. If `F->packed` is `FALSE`, then `F` is returned in unpacked form (`F->nz` must be present). Each row `i` of `F` is large enough to hold all the entries in row `i` of `A`, even if `f` is provided. That is, `F->i` and `F->x [F->p [i] .. F->p [i] + F->nz [i] - 1]` contain all entries in `A(i,f)`, but `F->p [i+1] - F->p [i]` is equal to the number of nonzeros in `A (i,:)`, not just `A (i,f)`. The `cholmod_transpose_unsym` routine is the only operation in CHOLMOD that can produce an unpacked sparse matrix.

14.10 `cholmod_transpose_sym`: transpose/permute symmetric sparse matrix

```
int cholmod_transpose_sym
(
    // input:
    cholmod_sparse *A,  // input matrix
    int mode,           // 2: numerical (conj)
                        // 1: numerical (non-conj.),
                        // 0: pattern (with diag)
    int32_t *Perm,      // permutation for C=A(p,p)', or NULL
    // input/output:
    cholmod_sparse *C,  // output matrix, must be allocated on input
    cholmod_common *Common
) ;
int cholmod_l_transpose_sym (cholmod_sparse *, int, int64_t *, cholmod_sparse *,
    cholmod_common *) ;
```

Purpose: Computes `F = A'` or `F = A(p,p)'`, the transpose or permuted transpose, where `A->stype` is nonzero. `A` must be square and symmetric. If `A->stype` > 0, then `A` is a symmetric matrix where just the upper part of the matrix is stored. Entries in the lower triangular part may be present, but are ignored. If `A->stype` < 0, then `A` is a symmetric matrix where just the lower part of the matrix is stored. Entries in the upper triangular part may be present, but are ignored. If `F=A'`, then `F` is returned sorted; otherwise `F` is unsorted for the `F=A(p,p)'` case. There can be no duplicate entries in `p`.

The `mode` parameter is the same as for `cholmod_transpose`.

For `cholmod_transpose_unsym` and `cholmod_transpose_sym`, the output matrix `F` must already be pre-allocated by the caller, with the correct dimensions. If `F` is not valid or has the wrong

dimensions, it is not modified. Otherwise, if F is too small, the transpose is not computed; the contents of F->p contain the column pointers of the resulting matrix, where F->p [F->ncol] > F->nzmax. In this case, the remaining contents of F are not modified. F can still be properly freed with cholmod_free_sparse.

14.11 cholmod_ptranspose: transpose/permute sparse matrix

```
cholmod_sparse *cholmod_ptranspose      // return new sparse matrix C
(
    // input:
    cholmod_sparse *A,  // input matrix
    int mode,           // 2: numerical (conj)
                        // 1: numerical (non-conj.)
                        // 0: pattern (with diag)
    int32_t *Perm,      // permutation for C=A(p,f)', or NULL
    int32_t *fset,      // a list of column indices in range 0:A->ncol-1
    size_t fsize,       // # of entries in fset
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_ptranspose (cholmod_sparse *, int, int64_t *,
    int64_t *, size_t, cholmod_common *) ;
```

Purpose: Returns A' or A(p,p)' if A is symmetric. Returns A', A(:,f)', or A(p,f)' if A is unsymmetric. The mode parameter is the same as for cholmod_transpose. See cholmod_transpose_unsym for a discussion of how f is used; this usage deviates from the MATLAB notation. Can also return the array transpose.

14.12 cholmod_sort: sort columns of a sparse matrix

```
int cholmod_sort
(
    // input/output:
    cholmod_sparse *A,      // input/output matrix to sort
    cholmod_common *Common
) ;
int cholmod_l_sort (cholmod_sparse *, cholmod_common *) ;
```

Purpose: Sorts the columns of the matrix A. Returns A in packed form, even if it starts as unpacked. Removes entries in the ignored part of a symmetric matrix.

14.13 cholmod_band_nnz: count entries in a band of a sparse matrix

```
int64_t cholmod_band_nnz    // return # of entries in a band (-1 if error)
(
    // input:
    cholmod_sparse *A,      // matrix to examine
    int64_t k1,             // count entries in k1:k2 diagonals
    int64_t k2,
    bool ignore_diag,       // if true, exclude any diagonal entries
```

```
    cholmod_common *Common
) ;
int64_t cholmod_l_band_nnz (cholmod_sparse *, int64_t, int64_t, bool,
    cholmod_common *) ;
```

Purpose: This method has the same inputs as `cholmod_band`, except that it returns the count of entries instead of returning the new matrix. The `mode` parameter has no effect on this count.

14.14 `cholmod_band`: extract band of a sparse matrix

```
cholmod_sparse *cholmod_band    // return a new matrix C
(
    // input:
    cholmod_sparse *A,      // input matrix
    int64_t k1,             // count entries in k1:k2 diagonals
    int64_t k2,
    int mode,               // >0: numerical, 0: pattern, <0: pattern (no diag)
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_band (cholmod_sparse *, int64_t, int64_t, int,
    cholmod_common *) ;
```

Purpose: Returns `C = tril (triu (A,k1), k2)`. `C` is a matrix consisting of the diagonals of A from `k1` to `k2`. `k=0` is the main diagonal of `A`, `k=1` is the superdiagonal, `k=-1` is the subdiagonal, and so on. If `A` is `m`-by-`n`, then:

- `k1=-m` means `C = tril (A,k2)`
- `k2=n` means `C = triu (A,k1)`
- `k1=0` and `k2=0` means `C = diag(A)`, except `C` is a matrix, not a vector

Values of `k1` and `k2` less than `-m` are treated as `-m`, and values greater than `n` are treated as `n`.

`A` can be of any symmetry (upper, lower, or unsymmetric); `C` is returned in the same form, and packed. If `A->stype` > 0, entries in the lower triangular part of `A` are ignored, and the opposite is true if `A->stype` < 0. If `A` has sorted columns, then so does `C`. `C` has the same size as `A`.

The `mode` parameter determines how the numerical values are handled. `C` can be returned as a numerical valued matrix (if `A` has numerical values and `mode` > 0), as a pattern-only (`mode` $= 0$), or as a pattern-only but with the diagonal entries removed (`mode` < 0).

The xtype of `A` can be pattern or real. Complex or zomplex cases are supported only if `mode` is ≤ 0 (in which case the numerical values are ignored).

14.15 `cholmod_band_inplace`: extract band, in place

```
int cholmod_band_inplace
(
    // input:
    int64_t k1,             // count entries in k1:k2 diagonals
```

```
    int64_t k2,
    int mode,               // >0: numerical, 0: pattern, <0: pattern (no diag)
    // input/output:
    cholmod_sparse *A,      // input/output matrix
    cholmod_common *Common
) ;
int cholmod_l_band_inplace (int64_t, int64_t, int, cholmod_sparse *,
    cholmod_common *) ;
```

Purpose: Same as `cholmod_band`, except that it always operates in place. Only packed matrices can be converted in place.

14.16 `cholmod_aat`: compute AA^T

```
cholmod_sparse *cholmod_aat     // return sparse matrix C
(
    // input:
    cholmod_sparse *A,  // input matrix
    int32_t *fset,      // a list of column indices in range 0:A->ncol-1
    size_t fsize,       // # of entries in fset
    int mode,           // 2: numerical (conj)
                        // 1: numerical (non-conj.),
                        // 0: pattern (with diag)
                        // -1: pattern (remove diag),
                        // -2: pattern (remove diag; add ~50% extra space in C)
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_aat (cholmod_sparse *, int64_t *, size_t, int,
    cholmod_common *) ;
```

Purpose: Computes `C = A*A'` or `C = A(:,f)*A(:,f)'`. `A` can be packed or unpacked, sorted or unsorted, but must be stored with both upper and lower parts (`A->stype` of zero). `C` is returned as packed, `C->stype` of zero (both upper and lower parts present), and unsorted. See `cholmod_ssmult` in the `MatrixOps` Module for a more general matrix-matrix multiply. The xtype of `A` can be pattern or real. Complex or zomplex cases are supported only if `mode` is ≤ 0 (in which case the numerical values are ignored). You can trivially convert `C` to a symmetric upper/lower matrix by changing `C->stype` to 1 or -1, respectively, after calling this routine.

14.17 `cholmod_copy_sparse`: copy sparse matrix

```
cholmod_sparse *cholmod_copy_sparse  // return new sparse matrix
(
    // input:
    cholmod_sparse *A,     // sparse matrix to copy
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_copy_sparse (cholmod_sparse *, cholmod_common *) ;
```

Purpose: Returns an exact copy of the input sparse matrix `A`.

14.18 cholmod_copy: copy (and change) sparse matrix

```
cholmod_sparse *cholmod_copy_sparse  // return new sparse matrix
(
    // input:
    cholmod_sparse *A,     // sparse matrix to copy
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_copy_sparse (cholmod_sparse *, cholmod_common *) ;
cholmod_sparse *cholmod_copy        // return new sparse matrix
(
    // input:
    cholmod_sparse *A,  // input matrix, not modified
    int stype,          // stype of C
    int mode,           // 2: numerical (conj)
                        // 1: numerical (non-conj.)
                        // 0: pattern (with diag)
                        // -1: pattern (remove diag)
                        // -2: pattern (remove diag; add ~50% extra space in C)
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_copy (cholmod_sparse *, int, int, cholmod_common *) ;
```

Purpose: `C = A`, which allocates `C` and copies `A` into `C`, with possible change of `stype`. The diagonal can optionally be removed. The numerical entries can optionally be copied. This routine differs from `cholmod_copy_sparse`, which makes an exact copy of a sparse matrix.

`A` can be of any type (packed/unpacked, upper/lower/unsymmetric). `C` is packed and can be of any stype (upper/lower/unsymmetric), except that if `A` is rectangular `C` can only be unsymmetric. If the stype of A and C differ, then the appropriate conversion is made.
There are three cases for `A->stype`:

- < 0, lower: assume `A` is symmetric with just `tril(A)` stored; the rest of `A` is ignored
- 0, unsymmetric: assume `A` is unsymmetric; consider all entries in A
- > 0, upper: assume `A` is symmetric with just `triu(A)` stored; the rest of `A` is ignored

There are three cases for the requested symmetry of `C` (`stype` parameter):

- < 0, lower: return just `tril(C)`
- 0, unsymmetric: return all of `C`
- > 0, upper: return just `triu(C)`

This gives a total of nine combinations:

Equivalent MATLAB statements	Using cholmod_copy
C = A ;	A unsymmetric, C unsymmetric
C = tril (A) ;	A unsymmetric, C lower
C = triu (A) ;	A unsymmetric, C upper
U = triu (A) ; L = tril (U',-1) ; C = L+U ;	A upper, C unsymmetric
C = triu (A)' ;	A upper, C lower
C = triu (A) ;	A upper, C upper
L = tril (A) ; U = triu (L',1) ; C = L+U ;	A lower, C unsymmetric
C = tril (A) ;	A lower, C lower
C = tril (A)' ;	A lower, C upper

The mode parameter determines whether a pattern-only copy is made, or whether a numerical copy is make, and also how the transpose is done above for the complex case (conjugate matrix transpose, or non-conjugate array transpose).

14.19 cholmod_add: add sparse matrices

```
cholmod_sparse *cholmod_add     // return C = alpha*A + beta*B
(
    // input:
    cholmod_sparse *A,  // input matrix
    cholmod_sparse *B,  // input matrix
    double alpha [2],   // scale factor for A (two entires used if complex)
    double beta [2],    // scale factor for A (two entires used if complex)
    int mode,           // 2: numerical (conj) if A and/or B are symmetric,
                        // 1: numerical (non-conj.) if A and/or B are symmetric.
                        // 0: pattern
    int sorted,         // ignored; C is now always returned as sorted
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_add (cholmod_sparse *, cholmod_sparse *, double *,
    double *, int, int, cholmod_common *) ;
```

Purpose: Returns C = alpha*A + beta*B. If the stype of A and B match, then C has the same stype. Otherwise, C->stype is zero (C is unsymmetric). If the stype of any input matrix is nonzero, it must be converted to unsymmetric, controlled by the mode parameter.

14.20 cholmod_sparse_xtype: change sparse xtype

```
int cholmod_sparse_xtype
(
    // input:
    int to_xdtype,      // requested xtype and dtype
    // input/output:
    cholmod_sparse *A,  // sparse matrix to change
    cholmod_common *Common
) ;
int cholmod_l_sparse_xtype (int, cholmod_sparse *, cholmod_common *) ;
```

Purpose: Changes the `xtype` and/or `dtype` of a sparse matrix. The `xtype` can be changed to pattern, real, complex, or zomplex. The `dtype` can be changed to single or double. Changing from complex or zomplex to real discards the imaginary part.

The name of this method derives from an earlier version of CHOLMOD where all matrices had an implied `dtype` of `CHOLMOD_DOUBLE`. The method now supports changes to the `dtype`. For backward compatilibilty, the name of this method has not changed, and the `to_xdtype` parameter values of the prior versions are upwards compatible with this version of CHOLMOD.

15 Utility Module: cholmod_factor object

15.1 cholmod_factor object: a sparse Cholesky factorization

```
typedef struct cholmod_factor_struct
{
    size_t n ;      // L is n-by-n

    size_t minor ;  // If the factorization failed because of numerical issues
        // (the matrix being factorized is found to be singular or not positive
        // definte), then L->minor is the column at which it failed.  L->minor
        // = n means the factorization was successful.

    //--------------------------------------------------------------------------
    // symbolic ordering and analysis
    //--------------------------------------------------------------------------

    void *Perm ;    // int32/int64, size n, fill-reducing ordering
    void *ColCount ;// int32/int64, size n, # entries in each column of L
    void *IPerm ;   // int32/int64, size n, created by cholmod_solve2;
                    // containing the inverse of L->Perm

    //--------------------------------------------------------------------------
    // simplicial factorization (not supernodal)
    //--------------------------------------------------------------------------

    // The row indices of L(:,j) are held in L->i [L->p [j] ... L->p [j] +
    // L->nz [j] - 1].  The numeical values of L(:,j) are held in the same
    // positions in L->x (and L->z if L is zomplex).  L->next and L->prev hold
    // a link list of columns of L, that tracks the order they appear in the
    // arrays L->i, L->x, and L->z.  The head and tail of the list is n+1 and
    // n, respectively.

    size_t nzmax ;  // # of entries that L->i, L->x, and L->z can hold
    void *p ;       // int32/int64, size n+1, column pointers
    void *i ;       // int32/int64, size nzmax, row indices
    void *x ;       // float/double, size nzmax or 2*nzmax, numerical values
    void *z ;       // float/double, size nzmax or empty, imaginary values
    void *nz ;      // int32/int64, size ncol, # of entries in each column
    void *next ;    // int32/int64, size n+2
    void *prev ;    // int32/int64, size n+2

    //--------------------------------------------------------------------------
    // supernodal factorization (not simplicial)
    //--------------------------------------------------------------------------

    // L->x is shared with the simplicial structure above.  L->z is not used
    // for the supernodal case since a supernodal factor cannot be zomplex.

    size_t nsuper ;     // # of supernodes
    size_t ssize ;      // # of integers in L->s
    size_t xsize ;      // # of entries in L->x
    size_t maxcsize ;   // size of largest update matrix
    size_t maxesize ;   // max # of rows in supernodes, excl. triangular part
    // the following are int32/int64 and are size nsuper+1:
```

```
void *super ;       // first column in each supernode
void *pi ;          // index into L->s for integer part of a supernode
void *px ;          // index into L->x for numeric part of a supernode
// int32/int64, of size ssize:
void *s ;           // integer part of supernodes

//------------------------------------------------------------------------------
// type of the factorization
//------------------------------------------------------------------------------

int ordering ;      // the fill-reducing method used (CHOLMOD_NATURAL,
    // CHOLMOD_GIVEN, CHOLMOD_AMD, CHOLMOD_METIS, CHOLMOD_NESDIS,
    // CHOLMOD_COLAMD, or CHOLMOD_POSTORDERED).

int is_ll ;         // true: an LL' factorization; false: LDL' instead
int is_super ;      // true: supernodal; false: simplicial
int is_monotonic ;  // true: columns appear in order 0 to n-1 in L, for a
                    // simplicial factorization only

// Two boolean values above (is_ll, is_super) and L->xtype (pattern or
// otherwise, define eight types of factorizations, but only 6 are used:

// If L->xtype is CHOLMOD_PATTERN, then L is a symbolic factor:
//
// simplicial LDL': (is_ll false, is_super false).  Nothing is present
//      except Perm and ColCount.
//
// simplicial LL': (is_ll true, is_super false).  Identical to the
//      simplicial LDL', except for the is_ll flag.
//
// supernodal LL': (is_ll true, is_super true).  A supernodal symbolic
//      factorization.  The simplicial symbolic information is present
//      (Perm and ColCount), as is all of the supernodal factorization
//      except for the numerical values (x and z).
//
// If L->xtype is CHOLMOD_REAL, CHOLMOD_COMPLEX, or CHOLMOD_ZOMPLEX,
// then L is a numeric factor:
//
// simplicial LDL':  (is_ll false, is_super false).  Stored in compressed
//      column form, using the simplicial components above (nzmax, p, i,
//      x, z, nz, next, and prev).  The unit diagonal of L is not stored,
//      and D is stored in its place.  There are no supernodes.
//
// simplicial LL': (is_ll true, is_super false).  Uses the same storage
//      scheme as the simplicial LDL', except that D does not appear.
//      The first entry of each column of L is the diagonal entry of
//      that column of L.
//
// supernodal LL': (is_ll true, is_super true).  A supernodal factor,
//      using the supernodal components described above (nsuper, ssize,
//      xsize, maxcsize, maxesize, super, pi, px, s, x, and z).
//      A supernodal factorization is never zomplex.

int itype ; // integer type for L->Perm, L->ColCount, L->p, L->i, L->nz,
            // L->next, L->prev, L->super, L->pi, L->px, and L->s.
```

```
                // These are all int32 if L->itype is CHOLMOD_INT, or all int64
                // if L->itype is CHOLMOD_LONG.

    int xtype ; // pattern, real, complex, or zomplex
    int dtype ; // x and z are double or single

    int useGPU; // if true, symbolic factorization allows for use of the GPU

} cholmod_factor ;
```

Purpose: An $\mathbf{LL}^\mathsf{T}$ or $\mathbf{LDL}^\mathsf{T}$ factorization in simplicial or supernodal form. A simplicial factor is very similar to a `cholmod_sparse` matrix. For an $\mathbf{LDL}^\mathsf{T}$ factorization, the diagonal matrix $\mathbf{D}$ is stored as the diagonal of $\mathbf{L}$; the unit-diagonal of $\mathbf{L}$ is not stored.

15.2 `cholmod_free_factor`: free factor

```
int cholmod_free_factor
(
    // input/output:
    cholmod_factor **L,         // handle of sparse factorization to free
    cholmod_common *Common
) ;
int cholmod_l_free_factor (cholmod_factor **, cholmod_common *) ;
```

Purpose: Frees a factor.

15.3 `cholmod_allocate_factor`: allocate factor

```
cholmod_factor *cholmod_allocate_factor         // return the new factor L
(
    // input:
    size_t n,               // L is factorization of an n-by-n matrix
    cholmod_common *Common
) ;
cholmod_factor *cholmod_l_allocate_factor (size_t, cholmod_common *) ;
```

Purpose: Allocates a factor and sets it to identity.

15.4 `cholmod_alloc_factor`: allocate factor

```
cholmod_factor *cholmod_alloc_factor    // return the new factor L
(
    // input:
    size_t n,               // L is factorization of an n-by-n matrix
    int dtype,              // CHOLMOD_SINGLE or CHOLMOD_DOUBLE
    cholmod_common *Common
) ;
cholmod_factor *cholmod_l_alloc_factor (size_t, int, cholmod_common *) ;
```

Purpose: Allocates a factor and sets it to identity (`double` or `float`).

15.5 cholmod_reallocate_factor: reallocate factor

```
int cholmod_reallocate_factor
(
    // input:
    size_t nznew,       // new max # of nonzeros the factor matrix can hold
    // input/output:
    cholmod_factor *L,  // factor to reallocate
    cholmod_common *Common
) ;
int cholmod_l_reallocate_factor (size_t, cholmod_factor *, cholmod_common *) ;
```

Purpose: Reallocates a simplicial factor so that it can contain `nznew` entries.

15.6 cholmod_change_factor: change factor

```
int cholmod_change_factor
(
    // input:
    int to_xtype,       // CHOLMOD_PATTERN, _REAL, _COMPLEX, or _ZOMPLEX;
                        // L->dtype remains unchanged.
    int to_ll,          // if true: convert to LL'; else to LDL'
    int to_super,       // if true: convert to supernodal; else to simplicial
    int to_packed,      // if true: pack simplicial columns; else: do not pack
    int to_monotonic,   // if true, put simplicial columns in order
    // input/output:
    cholmod_factor *L,  // factor to change.
    cholmod_common *Common
) ;
int cholmod_l_change_factor (int, int, int, int, int, cholmod_factor *,
    cholmod_common *) ;
```

Purpose: Change the numeric or symbolic, $\mathbf{LL}^\mathsf{T}$ or $\mathbf{LDL}^\mathsf{T}$, simplicial or super, packed or unpacked, and monotonic or non-monotonic status of a `cholmod_factor` object.

There are four basic classes of factor types:

1. simplicial symbolic: Consists of two size-`n` arrays: the fill-reducing permutation (`L->Perm`) and the nonzero count for each column of L (`L->ColCount`). All other factor types also include this information. `L->ColCount` may be exact (obtained from the analysis routines), or it may be a guess. During factorization, and certainly after update/downdate, the columns of `L` can have a different number of nonzeros. `L->ColCount` is used to allocate space. `L->ColCount` is exact for the supernodal factorizations. The nonzero pattern of `L` is not kept.

2. simplicial numeric: These represent `L` in a compressed column form. The variants of this type are:

   - $\mathbf{LDL}^\mathsf{T}$: `L` is unit diagonal. Row indices in column `j` are located in `L->i [L->p [j] ... L->p [j] + L->nz [j]]`, and corresponding numeric values are in the same locations in `L->x`. The total number of entries is the sum of `L->nz [j]`. The unit diagonal is not

stored; D is stored on the diagonal of L instead. L->p may or may not be monotonic. The order of storage of the columns in L->i and L->x is given by a doubly-linked list (L->prev and L->next). L->p is of size n+1, but only the first n entries are used.

For the complex case, L->x is stored interleaved with real and imaginary parts, and is of size 2*lnz*sizeof(double) or 2*lnz*sizeof(float). For the zomplex case, L->x is of size lnz*sizeof(double) or lnz*sizeof(float) and holds the real part; L->z is the same size and holds the imaginary part.

- $\mathbf{LL}^\mathsf{T}$: This is identical to the $\mathbf{LDL}^\mathsf{T}$ form, except that the non-unit diagonal of L is stored as the first entry in each column of L.

3. supernodal symbolic: A representation of the nonzero pattern of the supernodes for a supernodal factorization. There are L->nsuper supernodes. Columns L->super [k] to L->super [k+1]-1 are in the kth supernode. The row indices for the kth supernode are in L->s [L->pi [k] ... L->pi [k+1]-1]. The numerical values are not allocated (L->x), but when they are they will be located in L->x [L->px [k] ... L->px [k+1]-1], and the L->px array is defined in this factor type.

   For the complex case, L->x is stored interleaved with real/imaginary parts, and is of size 2*L->xsize*sizeof(double) or 2*L->xsize*sizeof(float). The zomplex supernodal case is not supported, since it is not compatible with LAPACK and the BLAS.

4. supernodal numeric: Always an $\mathbf{LL}^\mathsf{T}$ factorization. L has a non-unit diagonal. L->x contains the numerical values of the supernodes, as described above for the supernodal symbolic factor. For the complex case, L->x is stored interleaved, and is of size 2*L->xsize*sizeof(double) or 2*L->xsize*sizeof(double). The zomplex supernodal case is not supported, since it is not compatible with LAPACK and the BLAS.

In all cases, the row indices in each column (L->i for simplicial L and L->s for supernodal L) are kept sorted from low indices to high indices. This means the diagonal of L (or D for a $\mathbf{LDL}^\mathsf{T}$ factorization) is always kept as the first entry in each column. The elimination tree is not kept. The parent of node j can be found as the second row index in the jth column. If column j has no off-diagonal entries then node j is a root of the elimination tree.

The cholmod_change_factor routine can do almost all possible conversions. It cannot do the following conversions:

- Simplicial numeric types cannot be converted to a supernodal symbolic type. This would simultaneously deallocate the simplicial pattern and numeric values and reallocate uninitialized space for the supernodal pattern. This isn't useful for the user, and not needed by CHOLMOD's own routines either.

- Only a symbolic factor (simplicial to supernodal) can be converted to a supernodal numeric factor.

Some conversions are meant only to be used internally by other CHOLMOD routines, and should not be performed by the end user. They allocate space whose contents are undefined:

- converting from simplicial symbolic to supernodal symbolic.

- converting any factor to supernodal numeric.

Supports all xtypes, except that there is no supernodal zomplex L.

The `to_xtype` parameter is used only when converting from symbolic to numeric or numeric to symbolic. It cannot be used to convert a numeric xtype (real, complex, or zomplex) to a different numeric `xtype`. It also cannot change the `dtype` of a factor (from double to single, for example). For those conversions, use `cholmod_factor_xtype` instead.

15.7 cholmod_pack_factor: pack the columns of a factor

```
int cholmod_pack_factor
(
    // input/output:
    cholmod_factor *L,      // factor to pack
    cholmod_common *Common
) ;
int cholmod_l_pack_factor (cholmod_factor *, cholmod_common *) ;
```

Purpose: Pack the columns of a simplicial $\mathbf{LDL}^\mathsf{T}$ or $\mathbf{LL}^\mathsf{T}$ factorization. This can be followed by a call to `cholmod_reallocate_factor` to reduce the size of `L` to the exact size required by the factor, if desired. Alternatively, you can leave the size of `L->i` and `L->x` the same, to allow space for future updates/rowadds. Each column is reduced in size so that it has at most `Common->grow2` free space at the end of the column. Does nothing and returns silently if given any other type of factor. Does not force the columns of `L` to be monotonic. It thus differs from

```
cholmod_change_factor (xtype, L->is_ll, FALSE, TRUE, TRUE, L, Common)
```

which packs the columns and ensures that they appear in monotonic order.

15.8 cholmod_reallocate_column: reallocate one column of a factor

```
int cholmod_reallocate_column
(
    // input:
    size_t j,                   // reallocate L(:,j)
    size_t need,                // space in L(:,j) for this # of entries
    // input/output:
    cholmod_factor *L,          // L factor modified, L(:,j) resized
    cholmod_common *Common
) ;
int cholmod_l_reallocate_column (size_t, size_t, cholmod_factor *,
    cholmod_common *) ;
```

Purpose: Reallocates the space allotted to a single column of **L**.

15.9 cholmod_factor_to_sparse: sparse matrix copy of a factor

```
cholmod_sparse *cholmod_factor_to_sparse    // return a new sparse matrix
(
    // input/output:
    cholmod_factor *L,  // input: factor to convert; output: L is converted
                        // to a simplicial symbolic factor
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_factor_to_sparse (cholmod_factor *,
    cholmod_common *) ;
```

Purpose: Returns a column-oriented sparse matrix containing the pattern and values of a simplicial or supernodal numerical factor, and then converts the factor into a simplicial symbolic factor. If `L` is already packed, monotonic, and simplicial (which is the case when `cholmod_factorize` uses the simplicial Cholesky factorization algorithm) then this routine requires only a small amount of time and memory, independent of `n`. It only operates on numeric factors (real, complex, or zomplex). It does not change `L->xtype` (the resulting sparse matrix has the same `xtype` as `L`). If this routine fails, `L` is left unmodified.

15.10 cholmod_copy_factor: copy factor

```
cholmod_factor *cholmod_copy_factor     // return a copy of the factor
(
    // input:
    cholmod_factor *L,      // factor to copy (not modified)
    cholmod_common *Common
) ;
cholmod_factor *cholmod_l_copy_factor (cholmod_factor *, cholmod_common *) ;
```

Purpose: Returns an exact copy of a factor.

15.11 cholmod_factor_xtype: change factor xtype

```
int cholmod_factor_xtype
(
    // input:
    int to_xdtype,      // requested xtype and dtype
    // input/output:
    cholmod_factor *L,  // factor to change
    cholmod_common *Common
) ;
int cholmod_l_factor_xtype (int, cholmod_factor *, cholmod_common *) ;
```

Purpose: Changes the `xtype` and/or `dtype` of a sparse factorization. The `xtype` can be changed to real, complex, or zomplex (not to pattern). The `dtype` can be changed to single or double. Changing from complex or zomplex to real discards the imaginary part. A supernodal factor cannot

be changed to the zomplex `xtype`. A factor cannot be converted to an `xtype` of pattern-only with this method; use `cholmod_change_factor` for that operation.

The name of this method derives from an earlier version of CHOLMOD where all matrices had an implied `dtype` of `CHOLMOD_DOUBLE`. The method now supports changes to the `dtype`. For backward compatilibilty, the name of this method has not changed, and the `to_xdtype` parameter values of the prior versions are upwards compatible with this version of CHOLMOD.

15.12 How many entries are in a CHOLMOD sparse Cholesky factorization?

This is a simple question perhaps a surprisingly complex question to answer. For a sparse Cholesky factorization in CHOLMOD, there are at least 6 ways to answer this question. First of all, the kind of factorization (LL' or LDL') doesn't affect this answer, if the n entries in unit diagonal of L in the LDL' factorization are excluded. So all of the discussion here includes this simplification, and "`L`" can be considered the same as "`LD`."

In strictly increasing order of size (except that definition 5 can be larger or smaller than 3 and/or 4):

1. The number of entries not numerically equal to zero. This is a bit ephemeral since it's affected by roundoff, denormals flushing to zero, and so on. Computing this requires converting a factor to sparse matrix with `cholmod_factor_to_sparse`, followed by `cholmod_drop` with a `tol` of zero, and then a call to `cholmod_nnz`. There is no method in CHOLMOD to compute this otherwise. This number is `nnz(L)` in MATLAB after the built-in `R=chol(A)` (which uses CHOLMOD and where `R=L'`) or in my MATLAB interface as `R=chol2(A)` or `L=lchol(A)`, since a valid MATLAB sparse matrix cannot include any explicit entries that are numerically equal to zero. However, such a matrix cannot be used in an update/downdate, nor in my internal solve routines, since those methods require the chordal property of `L`. That property is broken if entries are dropped because of exact numerical cancellation. Zero entries are not dropped by `LD=ldlchol(A)`, which works fine in practice but the matrix `LD` is thus technically not a valid MATLAB sparse matrix.

2. The number of entries in the filled graph of `L`. This is a very stable number, and doesn't depend on roundoff, data structures, or factorization method used. It only depends on the symbolic pattern of `A`, and the fill-reducing ordering. Some entries can be exactly zero because of round-off but this is ignored. This is returned as `Common->lnz` after the call to `cholmod_analyze`. The value is not saved in `L` itself because it can change with update/downdates. This can be computed in MATLAB via any of the following, where any fill-reducing ordering would first need to be applied to the input matrix `A`:

```
[count] = symbfact (A, ...) ;    % built-in
sum (count)

[count] = symbfact2 (A, ...) ;   % mine, but it's the same as symbfact
sum (count)

[count, h, parent, post, R] = symbfact (A, ...) ;    % built-in
nnz (R)
```

```
[count, h, parent, post, R] = symbfact2 (A, ...) ;   % mine, the same
nnz (R)

[count, h, parent, post, L] = symbfact2 (A, ..., 'L') ; % mine, with L not R
nnz (L)
```

Internally the built-in MATLAB `symbfact` function calls CHOLMOD. The `L` or `R` matrix returned above has the entire pattern, but where each entry is equal to one.

3. The number of entries in the supernodal `L`, which is like (2) but with added entries because of relaxed amalgamation. This affected by everything in (2), but also the relaxed amalgamation parameters. Entries in the upper trianglar part of each supernode are excluded, since mathematically, each supernode is lower trapezoidal (with a square lower triangular part stacked on top of a rectangular part). This number is computed internally during the computation to change a factor from a `cholmod_factor` to a `cholmod_sparse` object, in `cholmod_factor_to_sparse` and `cholmod_change_factor`, when `L` is converted to a simplicial form that is packed and monotonic.

4. The same as (3), but with entries in the upper triangular part included. This is the bare minimum space required to hold `L` in supernodal form. The value `L->xsize` is a tight bound on this number (almost always identical to this value, but I can't guarantee this will always be true).

5. The number of entries in `L` after any update/downdates. The factorization would no longer be supernodal, since doing any update/downdate converts a superondal `L` into a simplicial one (supernodes are exploited to reduce the time for each update/downdate, however, but this is done dynamically). A downdate can delete entries but I don't keep track of when this happens. I would need to keep `L` as a collection of multisets, and update/downdate the multiplicity of each entry. I actually did this for a while but it's costly. So instead I have a `cholmod_resymbol` method to prune these down to (2). This number can be computed by `sum(L->nz[0..n-1])`, if `L` is simplicial.

6. The size of the data structure to hold the entries of `L`. This number if `L->xsize` for the supernodal case, and `L->nzmax` otherwise. For a supernodal `L`, `L->xsize` treats each supernode as rectangular, not lower trapezoidal. For a simplicial `L`, after update/downdate, I can include gaps in the columns of `L` to allow for room for growth. This value can be obtained from any factor object, either supernodal or simplicial, and either symbolic or numeric, with the following expression:

   ```
   (L->is_super) ? L->xsize : L->nzmax
   ```

   If `L` is simplicial, this number can change when `L` converted from symbolic to numeric, depending on the `to_packed` parameter. However, at any stage this value does give the size of the `L->x` array (and the `L->z` array if zomplex) that holds the numerical values. If `L` is symbolic (supernodal or simplicial) these arrays are not yet allocated, and both `L->x` and `L->z` are `NULL`. This number is in terms of entires, not bytes, so to get the number of bytes, multiply by `sizeof(float)` if `L->dtype` is `CHOLMOD_SINGLE`, by `sizeof(double)` if `L->dtype`

is `CHOLMOD_DOUBLE`, and again by another factor of 2 if the matrix to factorize is complex or zomplex.

Except for the simple expressions for:

- (2) `Common->lnz`
- and (5) `(L->is_super) ? L->xsize : L->nzmax`,

there is no user-callable method in CHOLMOD to compute these values. I may consider adding such a method in a future release.

16 Utility Module: cholmod_dense object

16.1 cholmod_dense object: a dense matrix

```
typedef struct cholmod_dense_struct
{
    size_t nrow ;       // the matrix is nrow-by-ncol
    size_t ncol ;
    size_t nzmax ;      // maximum number of entries in the matrix
    size_t d ;          // leading dimension (d >= nrow must hold)
    void *x ;           // size nzmax or 2*nzmax, if present
    void *z ;           // size nzmax, if present
    int xtype ;         // pattern, real, complex, or zomplex
    int dtype ;         // x and z double or single

} cholmod_dense ;
```

Purpose: Contains a dense matrix.

16.2 cholmod_allocate_dense: allocate dense matrix

```
cholmod_dense *cholmod_allocate_dense
(
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
    size_t d,       // leading dimension
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (CHOLMOD_REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_allocate_dense (size_t, size_t, size_t, int,
    cholmod_common *) ;
```

Purpose: Allocates a dense matrix.

16.3 cholmod_free_dense: free dense matrix

```
int cholmod_free_dense
(
    // input/output:
    cholmod_dense **X,          // handle of dense matrix to free
    cholmod_common *Common
) ;
int cholmod_l_free_dense (cholmod_dense **, cholmod_common *) ;
```

Purpose: Frees a dense matrix.

16.4 cholmod_zeros: dense zero matrix

```
cholmod_dense *cholmod_zeros
(
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (CHOLMOD_REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_zeros (size_t, size_t, int, cholmod_common *) ;
```

Purpose: Returns an all-zero dense matrix.

16.5 cholmod_ones: dense matrix, all ones

```
cholmod_dense *cholmod_ones
(
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (_REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_ones (size_t, size_t, int, cholmod_common *) ;
```

Purpose: Returns a dense matrix with each entry equal to one.

16.6 cholmod_eye: dense identity matrix

```
cholmod_dense *cholmod_eye      // return a dense identity matrix
(
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (_REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_eye (size_t, size_t, int, cholmod_common *) ;
```

Purpose: Returns a dense identity matrix.

16.7 cholmod_ensure_dense: ensure dense matrix has a given size and type

```
cholmod_dense *cholmod_ensure_dense
(
    // input/output:
    cholmod_dense **X,  // matrix to resize as needed (*X may be NULL)
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
    size_t d,       // leading dimension
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (CHOLMOD_REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_ensure_dense (cholmod_dense **, size_t, size_t,
    size_t, int, cholmod_common *) ;
```

Purpose: Ensures a dense matrix has a given size and type.

16.8 cholmod_sparse_to_dense: dense matrix copy of a sparse matrix

```
cholmod_dense *cholmod_sparse_to_dense      // return a dense matrix
(
    // input:
    cholmod_sparse *A,      // input matrix
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_sparse_to_dense (cholmod_sparse *, cholmod_common *) ;
```

Purpose: Returns a dense copy of a sparse matrix.

16.9 cholmod_dense_nnz: number of nonzeros in a dense matrix

```
int64_t cholmod_dense_nnz       // return # of entries in the dense matrix
(
    // input:
    cholmod_dense *X,       // input matrix
    cholmod_common *Common
) ;
int64_t cholmod_l_dense_nnz (cholmod_dense *, cholmod_common *) ;
```

Purpose: Returns a count of the number of nonzero entries in a dense matrix.

16.10 cholmod_dense_to_sparse: sparse matrix copy of a dense matrix

```
cholmod_sparse *cholmod_dense_to_sparse         // return a sparse matrix C
(
    // input:
    cholmod_dense *X,       // input matrix
    int mode,               // 1: copy the values
                            // 0: C is pattern
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_dense_to_sparse (cholmod_dense *, int,
    cholmod_common *) ;
```

Purpose: Returns a sparse copy of a dense matrix.

16.11 cholmod_copy_dense: copy dense matrix

```
cholmod_dense *cholmod_copy_dense   // returns new dense matrix
(
    // input:
    cholmod_dense *X,   // input dense matrix
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_copy_dense (cholmod_dense *, cholmod_common *) ;
```

Purpose: Returns a copy of a dense matrix.

16.12 cholmod_copy_dense2: copy dense matrix (preallocated)

```
int cholmod_copy_dense2
(
    // input:
    cholmod_dense *X,   // input dense matrix
    // input/output:
    cholmod_dense *Y,   // output dense matrix (already allocated on input)
    cholmod_common *Common
) ;
int cholmod_l_copy_dense2 (cholmod_dense *, cholmod_dense *, cholmod_common *) ;
```

Purpose: Returns a copy of a dense matrix, placing the result in a preallocated matrix `Y`.

16.13 cholmod_dense_xtype: change dense matrix xtype

```
int cholmod_dense_xtype
(
    // input:
    int to_xdtype,      // requested xtype and dtype
    // input/output:
    cholmod_dense *X,   // dense matrix to change
    cholmod_common *Common
) ;
int cholmod_l_dense_xtype (int, cholmod_dense *, cholmod_common *) ;
```

Purpose: Changes the `xtype` and/or `dtype` of a dense matrix. The `xtype` can change to real, complex, or zomplex (not pattern-only). The `dtype` can change to single or double. Changing from complex or zomplex to real discards the imaginary part. A dense matrix cannot be converted to an `xtype` of pattern-only.

The name of this method derives from an earlier version of CHOLMOD where all matrices had an implied `dtype` of `CHOLMOD_DOUBLE`. The method now supports changes to the `dtype`. For backward compatilibilty, the name of this method has not changed, and the `to_xdtype` parameter values of the prior versions are upwards compatible with this version of CHOLMOD.

17 Utility Module: cholmod_triplet object

17.1 cholmod_triplet object: sparse matrix in triplet form

```
typedef struct cholmod_triplet_struct
{
    size_t nrow ;   // # of rows of the matrix
    size_t ncol ;   // # of colums of the matrix
    size_t nzmax ;  // max # of entries that can be held in the matrix
    size_t nnz ;    // current # of entries can be held in the matrix

    // int32 or int64 arrays (depending on T->itype)
    void *i ;   // i [0..nzmax-1], the row indices
    void *j ;   // j [0..nzmax-1], the column indices

    // double or float arrays:
    void *x ;   // size nzmax or 2*nzmax, or NULL
    void *z ;   // size nzmax, or NULL

    int stype ; // T->stype defines what parts of the matrix is held:
        // 0:  the matrix is unsymmetric with both lower and upper parts stored.
        // >0: the matrix is square and symmetric, where entries in the lower
        //      triangular part are transposed and placed in the upper
        //      triangular part of A if T is converted into a sparse matrix A.
        // <0: the matrix is square and symmetric, where entries in the upper
        //      triangular part are transposed and placed in the lower
        //      triangular part of A if T is converted into a sparse matrix A.
        //
        // Note that A->stype (for a sparse matrix) and T->stype (for a
        // triplet matrix) are handled differently.  In a triplet matrix T,
        // no entry is ever ignored.  For a sparse matrix A, if A->stype < 0
        // or A->stype > 0, then entries not in the correct triangular part
        // are ignored.

    int itype ; // T->itype defines the integers used for T->i and T->j.
        // if CHOLMOD_INT, these arrays are all of type int32_t.
        // if CHOLMOD_LONG, these arrays are all of type int64_t.
    int xtype ;     // pattern, real, complex, or zomplex
    int dtype ;     // x and z are double or single

} cholmod_triplet ;
```

Purpose: Contains a sparse matrix in triplet form.

17.2 cholmod_allocate_triplet: allocate triplet matrix

```
cholmod_triplet *cholmod_allocate_triplet       // return triplet matrix T
(
    // input:
    size_t nrow,    // # of rows
    size_t ncol,    // # of columns
    size_t nzmax,   // max # of entries the matrix can hold
    int stype,      // the stype of the matrix (unsym, tril, or triu)
```

```
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (CHOLMOD_PATTERN, _REAL, _COMPLEX, or _ZOMPLEX)
    cholmod_common *Common
) ;
cholmod_triplet *cholmod_l_allocate_triplet (size_t, size_t, size_t, int, int,
    cholmod_common *) ;
```

Purpose: Allocates a triplet matrix.

17.3 cholmod_free_triplet: free triplet matrix

```
int cholmod_free_triplet
(
    // input/output:
    cholmod_triplet **T,        // handle of triplet matrix to free
    cholmod_common *Common
) ;
int cholmod_l_free_triplet (cholmod_triplet **, cholmod_common *) ;
```

Purpose: Frees a triplet matrix.

17.4 cholmod_triplet_to_sparse: sparse matrix copy of a triplet matrix

```
cholmod_sparse *cholmod_triplet_to_sparse       // return sparse matrix A
(
    // input:
    cholmod_triplet *T,     // input triplet matrix
    size_t nzmax,           // allocate space for max(nzmax,nnz(A)) entries
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_triplet_to_sparse (cholmod_triplet *, size_t,
    cholmod_common *) ;
```

Purpose: Returns a sparse matrix copy of a triplet matrix. If the triplet matrix is symmetric with just the lower part present (`T->stype` < 0), then entries in the upper part are transposed and placed in the lower part when converting to a sparse matrix. Similarly, if the triplet matrix is symmetric with just the upper part present (`T->stype` > 0), then entries in the lower part are transposed and placed in the upper part when converting to a sparse matrix. Any duplicate entries are summed.

17.5 cholmod_reallocate_triplet: reallocate triplet matrix

```
int cholmod_reallocate_triplet
(
    // input:
    size_t nznew,       // new max # of nonzeros the triplet matrix can hold
    // input/output:
```

```
    cholmod_triplet *T, // triplet matrix to reallocate
    cholmod_common *Common
) ;
int cholmod_l_reallocate_triplet (size_t, cholmod_triplet *, cholmod_common *) ;
```

Purpose: Reallocates a triplet matrix so that it can hold `nznew` entries.

17.6 `cholmod_sparse_to_triplet`: triplet matrix copy of a sparse matrix

```
cholmod_triplet *cholmod_sparse_to_triplet
(
    // input:
    cholmod_sparse *A,      // matrix to copy into triplet form T
    cholmod_common *Common
) ;
cholmod_triplet *cholmod_l_sparse_to_triplet (cholmod_sparse *,
    cholmod_common *) ;
```

Purpose: Returns a triplet matrix copy of a sparse matrix.

17.7 `cholmod_copy_triplet`: copy triplet matrix

```
cholmod_triplet *cholmod_copy_triplet   // return new triplet matrix
(
    // input:
    cholmod_triplet *T,     // triplet matrix to copy
    cholmod_common *Common
) ;
cholmod_triplet *cholmod_l_copy_triplet (cholmod_triplet *, cholmod_common *) ;
```

Purpose: Returns an exact copy of a triplet matrix.

17.8 `cholmod_triplet_xtype`: change triplet xtype

```
int cholmod_triplet_xtype
(
    // input:
    int to_xdtype,      // requested xtype and dtype
    // input/output:
    cholmod_triplet *T, // triplet matrix to change
    cholmod_common *Common
) ;
int cholmod_l_triplet_xtype (int, cholmod_triplet *, cholmod_common *) ;
```

Purpose: Changes the `xtype` and/or `dtype` of a triplet matrix. The `xtype` can change to pattern, real, complex, or zomplex, and the `dtype` can change to single or double. Changing from complex or zomplex to real discards the imaginary part.

The name of this method derives from an earlier version of CHOLMOD where all matrices had an implied `dtype` of `CHOLMOD_DOUBLE`. The method now supports changes to the `dtype`. For backward compatilibilty, the name of this method has not changed, and the `to_xdtype` parameter values of the prior versions are upwards compatible with this version of CHOLMOD.

18 Utility Module: memory management

18.1 cholmod_malloc: allocate memory

```
void *cholmod_malloc    // return pointer to newly allocated memory
(
    // input:
    size_t n,           // number of items
    size_t size,        // size of each item
    cholmod_common *Common
) ;
void *cholmod_l_malloc (size_t, size_t, cholmod_common *) ;
```

Purpose: Allocates a block of memory of size `n*size`, using the `SuiteSparse_config.malloc_func` function pointer (default is to use the ANSI C `malloc` routine). A value of `n=0` is treated as `n=1`. If not successful, `NULL` is returned and `Common->status` is set to `CHOLMOD_OUT_OF_MEMORY`.

18.2 cholmod_calloc: allocate and clear memory

```
void *cholmod_calloc    // return pointer to newly allocated memory
(
    // input:
    size_t n,           // number of items
    size_t size,        // size of each item
    cholmod_common *Common
) ;
void *cholmod_l_calloc (size_t, size_t, cholmod_common *) ;
```

Purpose: Allocates a block of memory of size `n*size`, using the `SuiteSparse_config.calloc_func` function pointer (default is to use the ANSI C `calloc` routine). A value of `n=0` is treated as `n=1`. If not successful, `NULL` is returned and `Common->status` is set to `CHOLMOD_OUT_OF_MEMORY`.

18.3 cholmod_free: free memory

```
void *cholmod_free      // returns NULL to simplify its usage
(
    // input:
    size_t n,           // number of items
    size_t size,        // size of each item
    // input/output:
    void *p,            // memory to free
    cholmod_common *Common
) ;
void *cholmod_l_free (size_t, size_t, void *, cholmod_common *) ;
```

Purpose: Frees a block of memory of size `n*size`, using the `SuiteSparse_config.free_func` function pointer (default is to use the ANSI C `free` routine). The size of the block (`n` and `size`) is only required so that CHOLMOD can keep track of its current and peak memory usage. This is a useful statistic, and it can also help in tracking down memory leaks. After the call to

cholmod_finish, the count of allocated blocks (Common->malloc_count) should be zero, and the count of bytes in use (Common->memory_inuse) also should be zero. If you allocate a block with one size and free it with another, the Common->memory_inuse count will be wrong, but CHOLMOD will not have a memory leak.

18.4 cholmod_realloc: reallocate memory

```
void *cholmod_realloc   // return newly reallocated block of memory
(
    // input:
    size_t nnew,        // # of items in newly reallocate memory
    size_t size,        // size of each item
    // input/output:
    void *p,            // pointer to memory to reallocate (may be NULL)
    size_t *n,          // # of items in p on input; nnew on output if success
    cholmod_common *Common
) ;
void *cholmod_l_realloc (size_t, size_t, void *, size_t *, cholmod_common *) ;
```

Purpose: Reallocates a block of memory whose current size n*size, and whose new size will be nnew*size if successful, using the SuiteSparse_config.calloc_func function pointer (default is to use the ANSI C realloc routine). If the reallocation is not successful, p is returned unchanged and Common->status is set to CHOLMOD_OUT_OF_MEMORY. The value of n is set to nnew if successful, or left unchanged otherwise. A value of nnew=0 is treated as nnew=1.

18.5 cholmod_realloc_multiple: reallocate memory

```
int cholmod_realloc_multiple    // returns true if successful, false otherwise
(
    // input:
    size_t nnew,    // # of items in newly reallocate memory
    int nint,       // 0: do not allocate I_block or J_block, 1: just I_block,
                    // 2: both I_block and J_block
    int xdtype,     // xtype + dtype of the matrix:
                    // (CHOLMOD_DOUBLE, _SINGLE) +
                    // (CHOLMOD_PATTERN, _REAL, _COMPLEX, or _ZOMPLEX)
    // input/output:
    void **I_block,       // integer block of memory (int32_t or int64_t)
    void **J_block,       // integer block of memory (int32_t or int64_t)
    void **X_block,       // real or complex, double or single, block
    void **Z_block,       // zomplex only: double or single block
    size_t *n,      // current size of I_block, J_block, X_block, and/or Z_block
                    // on input,  changed to nnew on output, if successful
    cholmod_common *Common
) ;
int cholmod_l_realloc_multiple (size_t, int, int, void **, void **, void **,
    void **, size_t *, cholmod_common *) ;
```

Purpose: Reallocates multiple blocks of memory, all with the same number of items (but with different item sizes). Either all reallocations succeed, or all are returned to their original size.

19 Utility Module: version control

19.1 cholmod_version: return current CHOLMOD version

```
int cholmod_version     // returns CHOLMOD_VERSION, defined above
(
    // if version is not NULL, then cholmod_version returns its contents as:
    // version [0] = CHOLMOD_MAIN_VERSION
    // version [1] = CHOLMOD_SUB_VERSION
    // version [2] = CHOLMOD_SUBSUB_VERSION
    int version [3]
) ;
int cholmod_l_version (int version [3]) ;
```

Purpose: Returns the CHOLMOD version number, so that it can be tested at run time, even if the caller does not have access to the CHOLMOD include files. For example, for a CHOLMOD version 3.2.1, the `version` array will contain 3, 2, and 1, in that order. This function appears in CHOLMOD 2.1.1 and later. You can check if the function exists with the `CHOLMOD_HAS_VERSION_FUNCTION` macro, so that the following code fragment works in any version of CHOLMOD:

```
#ifdef CHOLMOD_HAS_VERSION_FUNCTION
v = cholmod_version (NULL) ;
#else
v = CHOLMOD_VERSION ;
#endif
```

Note that `cholmod_version` and `cholmod_l_version` have identical prototypes. Both use `int`'s. Unlike all other CHOLMOD functions, this function does not take the `Common` object as an input parameter.

20 Check Module routines

No CHOLMOD routines print anything, except for the cholmod_print_* routines in the Check Module, and the cholmod_error routine. The SuiteSparse_config.printf_function is a pointer to printf by default; you can redirect the output of CHOLMOD by redefining this pointer. If the function pointer is NULL, CHOLMOD does not print anything.

The Common->print parameter determines how much detail is printed. Each value of Common->print listed below also prints the items listed for smaller values of Common->print:

- 0: print nothing; check the data structures and return TRUE or FALSE.
- 1: print error messages.
- 2: print warning messages.
- 3: print a one-line summary of the object.
- 4: print a short summary of the object (first and last few entries).
- 5: print the entire contents of the object.

Values less than zero are treated as zero, and values greater than five are treated as five.

20.1 cholmod_check_common: check Common object

```
int cholmod_check_common
(
    cholmod_common *Common
) ;
int cholmod_l_check_common (cholmod_common *) ;
```

Purpose: Check if the Common object is valid.

20.2 cholmod_print_common: print Common object

```
int cholmod_print_common
(
    // input:
    const char *name,   // printed name of Common object
    cholmod_common *Common
) ;

int cholmod_l_print_common (const char *, cholmod_common *) ;
```

Purpose: Print the Common object and check if it is valid. This prints the CHOLMOD parameters and statistics.

20.3 cholmod_check_sparse: check sparse matrix

```
int cholmod_check_sparse
(
    // input:
    cholmod_sparse *A,  // sparse matrix to check
    cholmod_common *Common
) ;
int cholmod_l_check_sparse (cholmod_sparse *, cholmod_common *) ;
```

Purpose: Check if a sparse matrix is valid.

20.4 cholmod_print_sparse: print sparse matrix

```
int cholmod_print_sparse
(
    // input:
    cholmod_sparse *A,  // sparse matrix to print
    const char *name,   // printed name of sparse matrix
    cholmod_common *Common
) ;
int cholmod_l_print_sparse (cholmod_sparse *, const char *, cholmod_common *) ;
```

Purpose: Print a sparse matrix and check if it is valid.

20.5 cholmod_check_dense: check dense matrix

```
int cholmod_check_dense
(
    // input:
    cholmod_dense *X,   // dense matrix to check
    cholmod_common *Common
) ;
int cholmod_l_check_dense (cholmod_dense *, cholmod_common *) ;
```

Purpose: Check if a dense matrix is valid.

20.6 cholmod_print_dense: print dense matrix

```
int cholmod_print_dense
(
    // input:
    cholmod_dense *X,   // dense matrix to print
    const char *name,   // printed name of dense matrix
    cholmod_common *Common
) ;
int cholmod_l_print_dense (cholmod_dense *, const char *, cholmod_common *) ;
```

Purpose: Print a dense matrix and check if it is valid.

20.7 cholmod_check_factor: check factor

```
int cholmod_check_factor
(
    // input:
    cholmod_factor *L,  // factor to check
    cholmod_common *Common
) ;
int cholmod_l_check_factor (cholmod_factor *, cholmod_common *) ;
```

Purpose: Check if a factor is valid.

20.8 cholmod_print_factor: print factor

```
int cholmod_print_factor
(
    // input:
    cholmod_factor *L,  // factor to print
    const char *name,   // printed name of factor
    cholmod_common *Common
) ;
int cholmod_l_print_factor (cholmod_factor *, const char *, cholmod_common *) ;
```

Purpose: Print a factor and check if it is valid.

20.9 cholmod_check_triplet: check triplet matrix

```
int cholmod_check_triplet
(
    // input:
    cholmod_triplet *T, // triplet matrix to check
    cholmod_common *Common
) ;
int cholmod_l_check_triplet (cholmod_triplet *, cholmod_common *) ;
```

Purpose: Check if a triplet matrix is valid.

20.10 cholmod_print_triplet: print triplet matrix

```
int cholmod_print_triplet
(
    // input:
    cholmod_triplet *T, // triplet matrix to print
    const char *name,   // printed name of triplet matrix
    cholmod_common *Common
) ;
int cholmod_l_print_triplet (cholmod_triplet *, const char *,
    cholmod_common *) ;
```

Purpose: Print a triplet matrix and check if it is valid.

20.11 cholmod_check_subset: check subset

```
int cholmod_check_subset
(
    // input:
    int32_t *Set,       // Set [0:len-1] is a subset of 0:n-1.  Duplicates OK
    int64_t len,        // size of Set (an integer array)
    size_t n,           // 0:n-1 is valid range
    cholmod_common *Common
) ;
int cholmod_l_check_subset (int64_t *, int64_t, size_t, cholmod_common *) ;
```

Purpose: Check if a subset is valid.

20.12 cholmod_print_subset: print subset

```
int cholmod_print_subset
(
    // input:
    int32_t *Set,       // Set [0:len-1] is a subset of 0:n-1.  Duplicates OK
    int64_t len,        // size of Set (an integer array)
    size_t n,           // 0:n-1 is valid range
    const char *name,   // printed name of Set
    cholmod_common *Common
) ;
int cholmod_l_print_subset (int64_t *, int64_t, size_t, const char *,
    cholmod_common *) ;
```

Purpose: Print a subset and check if it is valid.

20.13 cholmod_check_perm: check permutation

```
int cholmod_check_perm
(
    // input:
    int32_t *Perm,      // Perm [0:len-1] is a permutation of subset of 0:n-1
    size_t len,         // size of Perm (an integer array)
    size_t n,           // 0:n-1 is valid range
    cholmod_common *Common
) ;
int cholmod_l_check_perm (int64_t *, size_t, size_t, cholmod_common *) ;
```

Purpose: Check if a permutation is valid.

20.14 cholmod_print_perm: print permutation

```
int cholmod_print_perm
(
    // input:
    int32_t *Perm,      // Perm [0:len-1] is a permutation of subset of 0:n-1
    size_t len,         // size of Perm (an integer array)
    size_t n,           // 0:n-1 is valid range
    const char *name,   // printed name of Perm
    cholmod_common *Common
) ;
int cholmod_l_print_perm (int64_t *, size_t, size_t, const char *,
    cholmod_common *) ;
```

Purpose: Print a permutation and check if it is valid.

20.15 cholmod_check_parent: check elimination tree

```
int cholmod_check_parent
(
    // input:
    int32_t *Parent,    // Parent [0:n-1] is an elimination tree
    size_t n,           // size of Parent
    cholmod_common *Common
) ;
int cholmod_l_check_parent (int64_t *, size_t, cholmod_common *) ;
```

Purpose: Check if an elimination tree is valid.

20.16 cholmod_print_parent: print elimination tree

```
int cholmod_print_parent
(
    // input:
    int32_t *Parent,    // Parent [0:n-1] is an elimination tree
    size_t n,           // size of Parent
    const char *name,   // printed name of Parent
    cholmod_common *Common
) ;
int cholmod_l_print_parent (int64_t *, size_t, const char *, cholmod_common *) ;
```

Purpose: Print an elimination tree and check if it is valid.

20.17 cholmod_read_triplet: read triplet matrix from file

```
cholmod_triplet *cholmod_read_triplet   // return triplet matrix (double)
(
    // input:
    FILE *f,        // file to read from, must already be open
    cholmod_common *Common
) ;
cholmod_triplet *cholmod_l_read_triplet (FILE *, cholmod_common *) ;
```

Purpose: Read a sparse matrix in triplet form, using the the `coord` Matrix Market format (http://www.nist.gov/MatrixMarket). Skew-symmetric and complex symmetric matrices are returned with both upper and lower triangular parts present (an stype of zero). Real symmetric and complex Hermitian matrices are returned with just their upper or lower triangular part, depending on their stype. The Matrix Market `array` data type for dense matrices is not supported (use `cholmod_read_dense` for that case).

If the first line of the file starts with `%%MatrixMarket`, then it is interpreted as a file in Matrix Market format. The header line is optional. If present, this line must have the following format:

`%%MatrixMarket matrix coord` *type storage*

where *type* is one of: `real`, `complex`, `pattern`, or `integer`, and *storage* is one of: `general`, `hermitian`, `symmetric`, or `skew-symmetric`. In CHOLMOD, these roughly correspond to the `xtype` (pattern, real, complex, or zomplex) and `stype` (unsymmetric, symmetric/upper, and symmetric/lower). The strings are case-insensitive. Only the first character (or the first two for skew-symmetric) is significant. The `coord` token can be replaced with `array` in the Matrix Market format, but this format not supported by `cholmod_read_triplet`. The `integer` type is converted to real. The *type* is ignored; the actual type (real, complex, or pattern) is inferred from the number of tokens in each line of the file (2: pattern, 3: real, 4: complex). This is compatible with the Matrix Market format.

The matrix is read in `double` precision. To read a matrix in either double or single precision (`double` or `float`), use `cholmod_read_triplet2`.

A storage of `general` implies an stype of zero (see below). A storage of `symmetric` and `hermitian` imply an stype of -1. Skew-symmetric and complex symmetric matrices are returned with an stype of 0. Blank lines, any other lines starting with "`%`" are treated as comments, and are ignored.

The first non-comment line contains 3 or 4 integers:

nrow ncol nnz stype

where *stype* is optional (stype does not appear in the Matrix Market format). The matrix is *nrow*-by-*ncol*. The following *nnz* lines (excluding comments) each contain a single entry. Duplicates are permitted, and are summed in the output matrix.

If stype is present, it denotes the storage format for the matrix.

- stype = 0 denotes an unsymmetric matrix (same as Matrix Market `general`).
- stype = -1 denotes a symmetric or Hermitian matrix whose lower triangular entries are stored. Entries may be present in the upper triangular part, but these are ignored (same as Matrix Market `symmetric` for the real case, `hermitian` for the complex case).
- stype = 1 denotes a symmetric or Hermitian matrix whose upper triangular entries are stored. Entries may be present in the lower triangular part, but these are ignored. This format is not available in the Matrix Market format.

If neither the stype nor the Matrix Market header are present, then the stype is inferred from the rest of the data. If the matrix is rectangular, or has entries in both the upper and lower triangular parts, then it is assumed to be unsymmetric (stype=0). If only entries in the lower triangular part

are present, the matrix is assumed to have stype = -1. If only entries in the upper triangular part are present, the matrix is assumed to have stype = 1.

Each nonzero consists of one line with 2, 3, or 4 entries. All lines must have the same number of entries. The first two entries are the row and column indices of the nonzero. If 3 entries are present, the 3rd entry is the numerical value, and the matrix is real. If 4 entries are present, the 3rd and 4th entries in the line are the real and imaginary parts of a complex value.

The matrix can be either 0-based or 1-based. It is first assumed to be one-based (compatible with Matrix Market), with row indices in the range 1 to ncol and column indices in the range 1 to nrow. If a row or column index of zero is found, the matrix is assumed to be zero-based (with row indices in the range 0 to ncol-1 and column indices in the range 0 to nrow-1). This test correctly determines that all Matrix Market matrices are in 1-based form.

For symmetric pattern-only matrices, the kth diagonal (if present) is set to one plus the degree of the row k or column k (whichever is larger), and the off-diagonals are set to -1. A symmetric pattern-only matrix with a zero-free diagonal is thus converted into a symmetric positive definite matrix. All entries are set to one for an unsymmetric pattern-only matrix. This differs from the MatrixMarket format (`A = mmread ('file')` returns a binary pattern for A for symmetric pattern-only matrices). To return a binary format for all pattern-only matrices, use `A = mread('file',1)`.

Example matrices that follow this format can be found in the `CHOLMOD/Demo/Matrix` and `CHOLMOD/Tcov/Matrix` directories. You can also try any of the matrices in the Matrix Market collection at http://www.nist.gov/MatrixMarket.

20.18 `cholmod_read_triplet2`: read triplet matrix from file

```
cholmod_triplet *cholmod_read_triplet2  // return triplet matrix (double/single)
(
    // input:
    FILE *f,        // file to read from, must already be open
    int dtype,      // CHOLMOD_DOUBLE or CHOLMOD_SINGLE
    cholmod_common *Common
) ;
cholmod_triplet *cholmod_l_read_triplet2 (FILE *, int, cholmod_common *) ;
```

Purpose: Identical to `cholmod_read_triplet`, except that the dtype can be specified (`CHOLMOD_DOUBLE` or `CHOLMOD_SINGLE`).

20.19 `cholmod_read_sparse`: read sparse matrix from file

```
cholmod_sparse *cholmod_read_sparse     // return sparse matrix (double)
(
    // input:
    FILE *f,        // file to read from, must already be open
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_read_sparse (FILE *, cholmod_common *) ;
```

Purpose: Read a sparse matrix in triplet form from a file (using `cholmod_read_triplet`) and convert to a CHOLMOD sparse matrix. The Matrix Market format is used. If `Common->prefer_upper`

is TRUE (the default case), a symmetric matrix is returned stored in upper-triangular form (A->stype is 1). Otherwise, it is left in its original form, either upper or lower. The matrix is read in double precision. To read a matrix in either double or single precision (double or float), use cholmod_read_sparse2.

20.20 cholmod_read_sparse2: read sparse matrix from file

```
cholmod_sparse *cholmod_read_sparse2    // return sparse matrix (double/single)
(
    // input:
    FILE *f,        // file to read from, must already be open
    int dtype,      // CHOLMOD_DOUBLE or CHOLMOD_SINGLE
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_read_sparse2 (FILE *, int, cholmod_common *) ;
```

Purpose: Identical to cholmod_read_sparse, except that the dtype can be specified (CHOLMOD_DOUBLE or CHOLMOD_SINGLE).

20.21 cholmod_read_dense: read dense matrix from file

```
cholmod_dense *cholmod_read_dense   // return dense matrix (double)
(
    // input:
    FILE *f,        // file to read from, must already be open
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_read_dense (FILE *, cholmod_common *) ;
```

Purpose: Read a dense matrix from a file, using the array Matrix Market format (http://www.nist.gov/MatrixMarket). The matrix is read in double precision. To read a matrix in either double or single precision (double or float), use cholmod_read_dense2.

20.22 cholmod_read_dense2: read dense matrix from file

```
cholmod_dense *cholmod_read_dense2  // return dense matrix (double/single)
(
    // input:
    FILE *f,        // file to read from, must already be open
    int dtype,      // CHOLMOD_DOUBLE or CHOLMOD_SINGLE
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_read_dense2 (FILE *, int, cholmod_common *) ;
```

Purpose: Identical to cholmod_read_dense, except that the dtype can be specified (CHOLMOD_DOUBLE or CHOLMOD_SINGLE).

20.23 cholmod_read_matrix: read a matrix from file

```
void *cholmod_read_matrix   // return sparse/triplet/double matrix (double)
(
    // input:
    FILE *f,        // file to read from, must already be open
    int prefer,     // If 0, a sparse matrix is always return as a
                    // cholmod_triplet form.  It can have any stype
                    // (symmetric-lower, unsymmetric, or symmetric-upper).
                    // If 1, a sparse matrix is returned as an unsymmetric
                    // cholmod_sparse form (A->stype == 0), with both upper and
                    // lower triangular parts present.  This is what the MATLAB
                    // mread mexFunction does, since MATLAB does not have an
                    // stype.
                    // If 2, a sparse matrix is returned with an stype of 0 or
                    // 1 (unsymmetric, or symmetric with upper part stored).
                    // This argument has no effect for dense matrices.
    // output:
    int *mtype,     // CHOLMOD_TRIPLET, CHOLMOD_SPARSE or CHOLMOD_DENSE
    cholmod_common *Common
) ;
void *cholmod_l_read_matrix (FILE *, int, int *, cholmod_common *) ;
```

Purpose: Read a sparse or dense matrix from a file, in Matrix Market format. Returns a `void` pointer to either a `cholmod_triplet`, `cholmod_sparse`, or `cholmod_dense` object. The matrix is read in `double` precision. To read a matrix in either double or single precision (`double` or `float`), use `cholmod_read_matrix2`.

20.24 cholmod_read_matrix2: read a matrix from file

```
void *cholmod_read_matrix2  // sparse/triplet/double matrix (double/single)
(
    // input:
    FILE *f,        // file to read from, must already be open
    int prefer,     // see cholmod_read_matrix
    int dtype,      // CHOLMOD_DOUBLE or CHOLMOD_SINGLE
    // output:
    int *mtype,     // CHOLMOD_TRIPLET, CHOLMOD_SPARSE or CHOLMOD_DENSE
    cholmod_common *Common
) ;
void *cholmod_l_read_matrix2 (FILE *, int, int, int *, cholmod_common *) ;
```

Purpose: Identical to `cholmod_read_matrix`, except that the dtype can be specified (`CHOLMOD_DOUBLE` or `CHOLMOD_SINGLE`).

20.25 cholmod_write_sparse: write a sparse matrix to a file

```
int cholmod_write_sparse    // see above, or -1 on error
(
    // input:
```

```
    FILE *f,                // file to write to, must already be open
    cholmod_sparse *A,      // matrix to print
    cholmod_sparse *Z,      // optional matrix with pattern of explicit zeros
    const char *comments,   // optional filename of comments to include
    cholmod_common *Common
) ;
int cholmod_l_write_sparse (FILE *, cholmod_sparse *, cholmod_sparse *,
    const char *c, cholmod_common *) ;
```

Purpose: Write a sparse matrix to a file in Matrix Market format. Optionally include comments, and print explicit zero entries given by the pattern of the `Z` matrix. If not NULL, the `Z` matrix must have the same dimensions and stype as `A`.

Returns the symmetry in which the matrix was printed (1 to 7) or -1 on failure. See the `cholmod_symmetry` function for a description of the return codes.

If `A` and `Z` are sorted on input, and either unsymmetric (stype = 0) or symmetric-lower (stype < 0), and if `A` and `Z` do not overlap, then the triplets are sorted, first by column and then by row index within each column, with no duplicate entries. If all the above holds except stype > 0, then the triplets are sorted by row first and then column.

20.26 `cholmod_write_dense`: write a dense matrix to a file

```
int cholmod_write_dense     // CHOLMOD_MM_UNSYMMETRIC or _RECTANGULAR, or
(                           // -1 on error
    // input:
    FILE *f,                // file to write to, must already be open
    cholmod_dense *X,       // matrix to print
    const char *comments,   // optional filename of comments to include
    cholmod_common *Common
) ;
int cholmod_l_write_dense (FILE *, cholmod_dense *, const char *,
    cholmod_common *) ;
```

Purpose: Write a dense matrix to a file in Matrix Market format. Optionally include comments. Returns > 0 if successful, -1 otherwise (1 if rectangular, 2 if square). A dense matrix is written in "general" format; symmetric formats in the Matrix Market standard are not exploited.

21 Cholesky Module routines

21.1 cholmod_analyze: symbolic factorization

```
cholmod_factor *cholmod_analyze     // returns symbolic factor L
(
    // input:
    cholmod_sparse *A,      // matrix to order and analyze
    cholmod_common *Common
) ;
cholmod_factor *cholmod_l_analyze (cholmod_sparse *, cholmod_common *) ;
```

Purpose: Orders and analyzes a matrix (either simplicial or supernodal), in preparation for numerical factorization via `cholmod_factorize` or via the "expert" routines `cholmod_rowfac` and `cholmod_super_numeric`.

In the symmetric case, `A` or `A(p,p)` is analyzed, where `p` is the fill-reducing ordering. In the unsymmetric case, `A*A'` or `A(p,:)*A(p,:)'` is analyzed. The `cholmod_analyze_p` routine can be given a user-provided permutation `p` (see below).

The default ordering strategy is to first try AMD. The ordering quality is analyzed, and if AMD obtains an ordering where `nnz(L)` is greater than or equal to `5*nnz(tril(A))` (or `5*nnz(tril(A*A'))` if `A` is unsymmetric) and the floating-point operation count for the subsequent factorization is greater than or equal to `500*nnz(L)`, then METIS is tried (if installed). For `cholmod_analyze_p`, the user-provided ordering is also tried. This default behavior is obtained when `Common->nmethods` is zero. In this case, methods 0, 1, and 2 in `Common->method[...]` are reset to user-provided, AMD, and METIS, respectively. The ordering with the smallest `nnz(L)` is kept.

If `Common->default_nesdis` is true (nonzero), then CHOLMOD's nested dissection (NESDIS) is used for the default strategy described above, in place of METIS.

Other ordering options can be requested. These include:

1. natural: A is not permuted to reduce fill-in.
2. user-provided: a permutation can be provided to `cholmod_analyze_p`.
3. AMD: approximate minimum degree (AMD for the symmetric case, COLAMD for the `A*A'` case).
4. METIS: nested dissection with `METIS_NodeND`
5. `NESDIS`: CHOLMOD's nested dissection using `METIS_NodeComputeSeparator`, followed by a constrained minimum degree (CAMD or CSYMAMD for the symmetric case, CCOLAMD for the `A*A'` case). This is typically slower than METIS, but typically provides better orderings.

Multiple ordering options can be tried (up to 9 of them), and the best one is selected (the one that gives the smallest number of nonzeros in the simplicial factor L). If one method fails, `cholmod_analyze` keeps going, and picks the best among the methods that succeeded. This routine fails (and returns `NULL`) if either the initial memory allocation fails, all ordering methods fail, or the supernodal analysis (if requested) fails. Change `Common->nmethods` to the number of methods you wish to try. By default, the 9 methods available are:

1. user-provided permutation (only for `cholmod_analyze_p`).
2. AMD with default parameters.
3. METIS with default parameters.
4. `NESDIS` with default parameters: stopping the partitioning when the graph is of size `nd_small` = 200 or less, remove nodes with more than `max (16, prune_dense * sqrt (n))` nodes where `prune_dense` = 10, and follow partitioning with constrained minimum degree ordering (CAMD for the symmetric case, CCOLAMD for the unsymmetric case).
5. natural ordering (with weighted postorder).
6. NESDIS, `nd_small` = 20000, `prune_dense` = 10.
7. NESDIS, `nd_small` = 4, `prune_dense` = 10, no constrained minimum degree.
8. NESDIS, `nd_small` = 200, `prune_dense` = 0.
9. COLAMD for `A*A'` or AMD for `A`

You can modify these 9 methods and the number of methods tried by changing parameters in the `Common` argument. If you know the best ordering for your matrix, set `Common->nmethods` to 1 and set `Common->method[0].ordering` to the requested ordering method. Parameters for each method can also be modified (refer to the description of `cholmod_common` for details).

Note that it is possible for METIS to terminate your program if it runs out of memory. This is not the case for any CHOLMOD or minimum degree ordering routine (AMD, COLAMD, CAMD, CCOLAMD, or CSYMAMD). Since `NESDIS` relies on METIS, it too can terminate your program.

The selected ordering is followed by a weighted postorder of the elimination tree by default (see `cholmod_postorder` for details), unless `Common->postorder` is set to `FALSE`. The postorder does not change the number of nonzeros in $\mathbf{L}$ or the floating-point operation count. It does improve performance, particularly for the supernodal factorization. If you truly want the natural ordering with no postordering, you must set `Common->postorder` to `FALSE`.

The factor `L` is returned as simplicial symbolic if `Common->supernodal` is `CHOLMOD_SIMPLICIAL` (zero) or as supernodal symbolic if `Common->supernodal` is `CHOLMOD_SUPERNODAL` (two). If `Common->supernodal` is `CHOLMOD_AUTO` (one), then `L` is simplicial if the flop count per nonzero in `L` is less than `Common->supernodal_switch` (default: 40), and supernodal otherwise. In both cases, `L->xtype` is `CHOLMOD_PATTERN`. A subsequent call to `cholmod_factorize` will perform a simplicial or supernodal factorization, depending on the type of `L`.

For the simplicial case, `L` contains the fill-reducing permutation (`L->Perm`) and the counts of nonzeros in each column of `L` (`L->ColCount`). For the supernodal case, `L` also contains the nonzero pattern of each supernode.

If a simplicial factorization is selected, it will be $\mathbf{LDL}^{\mathsf{T}}$ by default, since this is the kind required by the `Modify` Module. CHOLMOD does not include a supernodal $\mathbf{LDL}^{\mathsf{T}}$ factorization, so if a supernodal factorization is selected, it will be in the form $\mathbf{LL}^{\mathsf{T}}$. The $\mathbf{LDL}^{\mathsf{T}}$ method can be used to factorize positive definite matrices and indefinite matrices whose leading minors are well-conditioned (2-by-2 pivoting is not supported). The $\mathbf{LL}^{\mathsf{T}}$ method is restricted to positive definite matrices. To factorize a large indefinite matrix, set `Common->supernodal` to `CHOLMOD_SIMPLICIAL`,

and the simplicial $\mathbf{LDL}^{\mathsf{T}}$ method will always be used. This will be significantly slower than a supernodal $\mathbf{LL}^{\mathsf{T}}$ factorization, however.

Refer to `cholmod_transpose_unsym` for a description of `f`.

21.2 `cholmod_factorize`: numeric factorization

```
int cholmod_factorize       // simplicial or superodal Cholesky factorization
(
    // input:
    cholmod_sparse *A,  // matrix to factorize
    // input/output:
    cholmod_factor *L,  // resulting factorization
    cholmod_common *Common
) ;
int cholmod_l_factorize (cholmod_sparse *, cholmod_factor *, cholmod_common *) ;
```

Purpose: Computes the numerical factorization of a symmetric matrix. The inputs to this routine are a sparse matrix `A` and the symbolic factor `L` from `cholmod_analyze` or a prior numerical factor `L`. If `A` is symmetric, this routine factorizes `A(p,p)`. where p is the fill-reducing permutation (`L->Perm`). If `A` is unsymmetric, `A(p,:)*A(p,:)'` is factorized. The nonzero pattern of the matrix `A` must be the same as the matrix passed to `cholmod_analyze` for the supernodal case. For the simplicial case, it can be different, but it should be the same for best performance.

A simplicial factorization or supernodal factorization is chosen, based on the type of the factor `L`. If `L->is_super` is `TRUE`, a supernodal $\mathbf{LL}^{\mathsf{T}}$ factorization is computed. Otherwise, a simplicial numeric factorization is computed, either $\mathbf{LL}^{\mathsf{T}}$ or $\mathbf{LDL}^{\mathsf{T}}$, depending on `Common->final_ll` (the default for the simplicial case is to compute an $\mathbf{LDL}^{\mathsf{T}}$ factorization).

Once the factorization is complete, it can be left as is or optionally converted into any simplicial numeric type, depending on the `Common->final_*` parameters. If converted from a supernodal to simplicial type, and `Common->final_resymbol` is `TRUE`, then numerically zero entries in `L` due to relaxed supernodal amalgamation are removed from the simplicial factor (they are always left in the supernodal form of `L`). Entries that are numerically zero but present in the simplicial symbolic pattern of `L` are left in place (the graph of `L` remains chordal). This is required for the update/downdate/rowadd/rowdel routines to work properly.

If the matrix is not positive definite the routine returns `TRUE`, but `Common->status` is set to `CHOLMOD_NOT_POSDEF` and `L->minor` is set to the column at which the failure occurred. Columns `L->minor` to `L->n-1` are set to zero.

Supports any xtype (pattern, real, complex, or zomplex), except that the input matrix `A` cannot be pattern-only and any dtype (double or single). If `L` is simplicial, its numeric xtype matches `A` on output. If `L` is supernodal, its xtype is real if `A` is real, or complex if `A` is complex or zomplex. CHOLMOD does not provide a supernodal zomplex factor, since it is incompatible with how complex numbers are stored in LAPACK and the BLAS.

If the `L` factor on input is a purely symbolic factorizatin (with `L->xtype` of `CHOLMOD_PATTERN`, then it is converted to a numeric factorization with same dtype as A. Otherwise, the dtypes of L and A must match.

21.3 cholmod_analyze_p: symbolic factorization, given permutation

```
cholmod_factor *cholmod_analyze_p   // returns symbolic factor L
(
    // input:
    cholmod_sparse *A,  // matrix to order and analyze
    int32_t *UserPerm,  // user-provided permutation, size A->nrow
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    cholmod_common *Common
) ;
cholmod_factor *cholmod_l_analyze_p (cholmod_sparse *, int64_t *, int64_t *,
    size_t, cholmod_common *) ;
cholmod_factor *cholmod_analyze_p2
(
    // input:
    int for_whom,       // FOR_SPQR     (0): for SPQR but not GPU-accelerated
                        // FOR_CHOLESKY (1): for Cholesky (GPU or not)
                        // FOR_SPQRGPU  (2): for SPQR with GPU acceleration
    cholmod_sparse *A,  // matrix to order and analyze
    int32_t *UserPerm,  // user-provided permutation, size A->nrow
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    cholmod_common *Common
) ;
cholmod_factor *cholmod_l_analyze_p2 (int, cholmod_sparse *, int64_t *,
    int64_t *, size_t, cholmod_common *) ;
```

Purpose: Identical to `cholmod_analyze`, except that a user-provided permutation `p` can be provided, and the set `f` for the unsymmetric case can be provided. The matrices `A(:,f)*A(:,f)'` or `A(p,f)*A(p,f)'` can be analyzed in the the unsymmetric case.

21.4 cholmod_factorize_p: numeric factorization, given permutation

```
int cholmod_factorize_p
(
    // input:
    cholmod_sparse *A,  // matrix to factorize
    double beta [2],    // factorize beta*I+A or beta*I+A'*A
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    // input/output:
    cholmod_factor *L,  // resulting factorization
    cholmod_common *Common
) ;
int cholmod_l_factorize_p (cholmod_sparse *, double *, int64_t *, size_t,
    cholmod_factor *, cholmod_common *) ;
```

Purpose: Identical to `cholmod_factorize`, but with additional options. The set `f` can be provided for the unsymmetric case; `A(p,f)*A(p,f)'` is factorized. The term `beta*I` can be added to the matrix before it is factorized, where `beta` is real. Only the real part, `beta[0]`, is used.

21.5 cholmod_solve: solve a linear system

```
cholmod_dense *cholmod_solve    // returns the solution X
(
    // input:
    int sys,            // system to solve
    cholmod_factor *L,  // factorization to use
    cholmod_dense *B,   // right-hand-side
    cholmod_common *Common
) ;
cholmod_dense *cholmod_l_solve (int, cholmod_factor *, cholmod_dense *,
    cholmod_common *) ;
```

Purpose: Returns a solution `X` that solves one of the following systems:

system	`sys` parameter	system	`sys` parameter
$\mathbf{Ax} = \mathbf{b}$	0: `CHOLMOD_A`		
$\mathbf{LDL}^{\mathsf{T}}\mathbf{x} = \mathbf{b}$	1: `CHOLMOD_LDLt`	$\mathbf{L}^{\mathsf{T}}\mathbf{x} = \mathbf{b}$	5: `CHOLMOD_Lt`
$\mathbf{LDx} = \mathbf{b}$	2: `CHOLMOD_LD`	$\mathbf{Dx} = \mathbf{b}$	6: `CHOLMOD_D`
$\mathbf{DL}^{\mathsf{T}}\mathbf{x} = \mathbf{b}$	3: `CHOLMOD_DLt`	$\mathbf{x} = \mathbf{Pb}$	7: `CHOLMOD_P`
$\mathbf{Lx} = \mathbf{b}$	4: `CHOLMOD_L`	$\mathbf{x} = \mathbf{P}^{\mathsf{T}}\mathbf{b}$	8: `CHOLMOD_Pt`

The factorization can be simplicial $\mathbf{LDL}^{\mathsf{T}}$, simplicial $\mathbf{LL}^{\mathsf{T}}$, or supernodal $\mathbf{LL}^{\mathsf{T}}$. For an $\mathbf{LL}^{\mathsf{T}}$ factorization, $\mathbf{D}$ is the identity matrix. Thus `CHOLMOD_LD` and `CHOLMOD_L` solve the same system if an $\mathbf{LL}^{\mathsf{T}}$ factorization was performed, for example. This is one of the few routines in CHOLMOD for which the xtype of the input arguments need not match. If both `L` and `B` are real, then `X` is returned real. If either is complex or zomplex, `X` is returned as either complex or zomplex, depending on the `Common->prefer_zomplex` parameter (default is complex).

This routine does not check to see if the diagonal of $\mathbf{L}$ or $\mathbf{D}$ is zero, because sometimes a partial solve can be done with an indefinite or singular matrix. If you wish to check in your own code, test `L->minor`. If `L->minor == L->n`, then the matrix has no zero diagonal entries. If `k = L->minor < L->n`, then `L(k,k)` is zero for an $\mathbf{LL}^{\mathsf{T}}$ factorization, or `D(k,k)` is zero for an $\mathbf{LDL}^{\mathsf{T}}$ factorization.

Iterative refinement is not performed, but this can be easily done with the `MatrixOps` Module. See `Demo/cholmod_demo.c` for an example.

The dtypes of `L` and `B` must match, and `X` is returned with the same dtype.

21.6 cholmod_spsolve: solve a linear system

```
cholmod_sparse *cholmod_spsolve            // returns the sparse solution X
(
    // input:
    int sys,            // system to solve
    cholmod_factor *L,  // factorization to use
    cholmod_sparse *B,  // right-hand-side
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_spsolve (int, cholmod_factor *, cholmod_sparse *,
    cholmod_common *) ;
```

Purpose: Identical to `cholmod_solve`, except that `B` and `X` are sparse. This function converts `B` to full format, solves the system, and then converts `X` back to sparse. If you want to solve with a sparse `B` and get just a partial solution back in `X` (corresponding to the pattern of `B`), use `cholmod_solve2` below.

21.7 `cholmod_solve2`: solve a linear system, reusing workspace

```
int cholmod_solve2     // returns TRUE on success, FALSE on failure
(
    // input:
    int sys,                        // system to solve
    cholmod_factor *L,              // factorization to use
    cholmod_dense *B,               // right-hand-side
    cholmod_sparse *Bset,
    // output:
    cholmod_dense **X_Handle,       // solution, allocated if need be
    cholmod_sparse **Xset_Handle,
    // workspace:
    cholmod_dense **Y_Handle,       // workspace, or NULL
    cholmod_dense **E_Handle,       // workspace, or NULL
    cholmod_common *Common
) ;
int cholmod_l_solve2 (int, cholmod_factor *, cholmod_dense *, cholmod_sparse *,
    cholmod_dense **, cholmod_sparse **, cholmod_dense **, cholmod_dense **,
    cholmod_common *) ;
```

Purpose: Solve a linear system, optionally reusing workspace from a prior call to `cholmod_solve2`.

The inputs to this function are the same as `cholmod_solve`, with the addition of three parameters: `X`, `Y`, and `E`. The dense matrix `X` is the solution on output. On input, `&X` can point to a NULL matrix, or be the wrong size. If that is the case, it is freed and allocated to be the proper size. If `X` has the right size and type on input, then the allocation is skipped. In contrast, the `cholmod_solve` function always allocates its output `X`. This `cholmod_solve2` function allows you to reuse the memory space of a prior `X`, thereby saving time.

The two workspace matrices `Y` and `E` can also be reused between calls. You must free `X` `Y`, and `E` yourself, when your computations are done. Below is an example of usage. Note that `X` `Y`, and `E` must be defined on input (either NULL, or valid dense matrices).

```
cholmod_dense *X = NULL, *Y = NULL, *E = NULL ;
...
cholmod_l_solve2 (sys, L, B1, NULL, &X, NULL, &Y, &E, Common) ;
cholmod_l_solve2 (sys, L, B2, NULL, &X, NULL, &Y, &E, Common) ;
cholmod_l_solve2 (sys, L, B3, NULL, &X, NULL, &Y, &E, Common) ;
cholmod_l_free_dense (&X, Common) ;
cholmod_l_free_dense (&Y, Common) ;
cholmod_l_free_dense (&E, Common) ;
```

The equivalent when using `cholmod_solve` is:

```
cholmod_dense *X = NULL, *Y = NULL, *E = NULL ;
...
X = cholmod_l_solve (sys, L, B1, Common) ;
cholmod_l_free_dense (&X, Common) ;
X = cholmod_l_solve (sys, L, B2, Common) ;
cholmod_l_free_dense (&X, Common) ;
X = cholmod_l_solve (sys, L, B3, Common) ;
cholmod_l_free_dense (&X, Common) ;
```

Both methods work fine, but in the second method with `cholmod_solve`, the internal workspaces (`Y` and `E`) and the solution (`X`) are allocated and freed on each call.

The `cholmod_solve2 function` can also solve for a subset of the solution vector `X`, if the optional `Bset` parameter is non-NULL. The right-hand-side `B` must be a single column vector, and its complexity (real, complex, zomplex) must match that of `L`. The vector `B` is dense, but it is assumed to be zero except for row indices specified in `Bset`. The vector `Bset` must be a sparse column vector, of dimension the same as `B`. Only the pattern of `Bset` is used. The solution `X` (a dense column vector) is modified on output, but is defined only in the rows defined by the sparse vector `Xset`. `Bset` is ignored when solving with `sys` equal to `CHOLMOD_P` or `CHOLMOD_Pt`). The entries in `Bset` are a subset of `Xset` (except if `sys` is `CHOLMOD_P` or `CHOLMOD_Pt`).

No memory allocations are done if the outputs and internal workspaces (`X`, `Xset`, `Y`, and `E`) have been allocated by a prior call (or if allocated by the user). To let `cholmod_solve2` allocate these outputs and workspaces for you, simply initialize them to NULL (as in the example above). Since it is possible for this function to reallocate these 4 arrays, you should always re-acquire the pointers to their internal data (`X->x` for example) after calling `cholmod_solve2`, since they may change. They normally will not change except in the first call to this function.

On the first call to `cholmod_solve2` when `Bset` is NULL, the factorization is converted from supernodal to simplicial, if needed. The inverse permutation is also computed and stored in the factorization object, `L`. This can take a modest amount of time. Subsequent calls to `cholmod_solve2` with a small `Bset` are very fast (both asymptotically and in practice).

The dtypes of all parameters must match.

You can find an example of how to use `cholmod_solve2` in the four demo programs, `cholmod_*_demo`.

21.8 `cholmod_etree`: find elimination tree

```
int cholmod_etree
(
    // input:
    cholmod_sparse *A,
    // output:
    int32_t *Parent,      // size ncol.  Parent [j] = p if p is the parent of j
    cholmod_common *Common
) ;
int cholmod_l_etree (cholmod_sparse *, int64_t *, cholmod_common *) ;
```

Purpose: Computes the elimination tree of `A` or `A'*A`. In the symmetric case, the upper triangular part of `A` is used. Entries not in this part of the matrix are ignored. Computing the etree of a

symmetric matrix from just its lower triangular entries is not supported. In the unsymmetric case, all of `A` is used, and the etree of `A'*A` is computed. Refer to [20] for a discussion of the elimination tree and its use in sparse Cholesky factorization.

21.9 `cholmod_rowcolcounts`: nonzeros counts of a factor

```
int cholmod_rowcolcounts
(
    // input:
    cholmod_sparse *A,  // matrix to analyze
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    int32_t *Parent,    // size nrow.  Parent [i] = p if p is the parent of i
    int32_t *Post,      // size nrow.  Post [k] = i if i is the kth node in
                        // the postordered etree.
    // output:
    int32_t *RowCount,  // size nrow. RowCount [i] = # entries in the ith
                        // row of L, including the diagonal.
    int32_t *ColCount,  // size nrow. ColCount [i] = # entries in the ith
                        // column of L, including the diagonal.
    int32_t *First,     // size nrow.  First [i] = k is the least
                        // postordering of any descendant of i.
    int32_t *Level,     // size nrow.  Level [i] is the length of the path
                        // from i to the root, with Level [root] = 0.
    cholmod_common *Common
) ;
int cholmod_l_rowcolcounts (cholmod_sparse *, int64_t *, size_t, int64_t *,
    int64_t *, int64_t *, int64_t *, int64_t *, int64_t *, cholmod_common *) ;
```

Purpose: Computes the row and column counts of the Cholesky factor `L` of the matrix `A` or `A*A'`. The etree and its postordering must already be computed (see `cholmod_etree` and `cholmod_postorder`) and given as inputs to this routine. For the symmetric case ($\mathbf{LL}^{\mathsf{T}} = \mathbf{A}$), `A` must be stored in symmetric/lower form (`A->stype = -1`). In the unsymmetric case, `A*A'` or `A(:,f)*A(:,f)'` can be analyzed. The fundamental floating-point operation count is returned in `Common->fl` (this excludes extra flops due to relaxed supernodal amalgamation). Refer to `cholmod_transpose_unsym` for a description of `f`. The algorithm is described in [13, 15].

21.10 `cholmod_analyze_ordering`: analyze a permutation

```
int cholmod_analyze_ordering
(
    // input:
    cholmod_sparse *A,  // matrix to analyze
    int ordering,       // ordering method used
    int32_t *Perm,      // size n, fill-reducing permutation to analyze
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    // output:
    int32_t *Parent,    // size n, elimination tree
    int32_t *Post,      // size n, postordering of elimination tree
    int32_t *ColCount,  // size n, nnz in each column of L
```

```
    // workspace:
    int32_t *First,     // size n workspace for cholmod_postorder
    int32_t *Level,     // size n workspace for cholmod_postorder
    cholmod_common *Common
) ;
int cholmod_l_analyze_ordering (cholmod_sparse *, int, int64_t *, int64_t *,
    size_t, int64_t *, int64_t *, int64_t *, int64_t *, int64_t *,
    cholmod_common *) ;
```

Purpose: Given a matrix `A` and its fill-reducing permutation, compute the elimination tree, its (non-weighted) postordering, and the number of nonzeros in each column of `L`. Also computes the flop count, the total nonzeros in `L`, and the nonzeros in `tril(A)` (`Common->fl`, `Common->lnz`, and `Common->anz`). In the unsymmetric case, `A(p,f)*A(p,f)'` is analyzed, and `Common->anz` is the number of nonzero entries in the lower triangular part of the product, not in `A` itself.

Refer to `cholmod_transpose_unsym` for a description of `f`.

The column counts of `L`, flop count, and other statistics from `cholmod_rowcolcounts` are not computed if `ColCount` is `NULL`.

21.11 `cholmod_amd`: interface to AMD

```
int cholmod_amd
(
    // input:
    cholmod_sparse *A,  // matrix to order
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    // output:
    int32_t *Perm,      // size A->nrow, output permutation
    cholmod_common *Common
) ;
int cholmod_l_amd (cholmod_sparse *, int64_t *, size_t, int64_t *,
    cholmod_common *) ;
```

Purpose: CHOLMOD interface to the AMD ordering package. Orders `A` if the matrix is symmetric. On output, `Perm [k] = i` if row/column `i` of `A` is the `k`th row/column of `P*A*P'`. This corresponds to `A(p,p)` in MATLAB notation. If A is unsymmetric, `cholmod_amd` orders `A*A'` or `A(:,f)*A(:,f)'`. On output, `Perm [k] = i` if row/column `i` of `A*A'` is the `k`th row/column of `P*A*A'*P'`. This corresponds to `A(p,:)*A(p,:)'` in MATLAB notation. If `f` is present, `A(p,f)*A(p,f)'` is the permuted matrix. Refer to `cholmod_transpose_unsym` for a description of `f`.

Computes the flop count for a subsequent $\mathbf{LL}^{\mathsf{T}}$ factorization, the number of nonzeros in `L`, and the number of nonzeros in the matrix ordered (`A`, `A*A'` or `A(:,f)*A(:,f)'`). These statistics are returned in `Common->fl`, `Common->lnz`, and `Common->anz`, respectively.

21.12 `cholmod_colamd`: interface to COLAMD

```
int cholmod_colamd
(
```

```
    // input:
    cholmod_sparse *A,  // matrix to order
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    int postorder,      // if TRUE, follow with a coletree postorder
    // output:
    int32_t *Perm,      // size A->nrow, output permutation
    cholmod_common *Common
) ;
int cholmod_l_colamd (cholmod_sparse *, int64_t *, size_t, int, int64_t *,
    cholmod_common *) ;
```

Purpose: CHOLMOD interface to the COLAMD ordering package. Finds a permutation `p` such that the Cholesky factorization of `P*A*A'*P'` is sparser than `A*A'`, using COLAMD. If the `postorder` input parameter is `TRUE`, the column elimination tree is found and postordered, and the COLAMD ordering is then combined with its postordering (COLAMD itself does not perform this postordering). `A` must be unsymmetric (`A->stype = 0`).

21.13 `cholmod_rowfac`: row-oriented Cholesky factorization

```
int cholmod_rowfac
(
    // input:
    cholmod_sparse *A,  // matrix to factorize
    cholmod_sparse *F,  // used for A*A' case only. F=A' or A(:,f)'
    double beta [2],    // factorize beta*I+A or beta*I+AA'
    size_t kstart,      // first row to factorize
    size_t kend,        // last row to factorize is kend-1
    // input/output:
    cholmod_factor *L,
    cholmod_common *Common
) ;
int cholmod_l_rowfac (cholmod_sparse *, cholmod_sparse *, double *, size_t,
    size_t, cholmod_factor *, cholmod_common *) ;
```

Purpose: Full or incremental numerical $\mathbf{LDL}^\mathsf{T}$ or $\mathbf{LL}^\mathsf{T}$ factorization (simplicial, not supernodal). `cholmod_factorize` is the "easy" wrapper for this code, but it does not provide access to incremental factorization. The algorithm is the row-oriented, up-looking method described in [5]. See also [19]. No 2-by-2 pivoting (or any other pivoting) is performed.

`cholmod_rowfac` computes the full or incremental $\mathbf{LDL}^\mathsf{T}$ or $\mathbf{LL}^\mathsf{T}$ factorization of `A+beta*I` (where `A` is symmetric) or `A*F+beta*I` (where `A` and `F` are unsymmetric and only the upper triangular part of `A*F+beta*I` is used). It computes `L` (and `D`, for $\mathbf{LDL}^\mathsf{T}$) one row at a time. The input scalar `beta` is real; only the real part (`beta[0]`) is used.

`L` can be a simplicial symbolic or numeric (`L->is_super` must be `FALSE`). A symbolic factor is converted immediately into a numeric factor containing the identity matrix.

For a full factorization, use `kstart = 0` and `kend = nrow`. The existing nonzero entries (numerical values in `L->x` and `L->z` for the zomplex case, and indices in `L->i`) are overwritten.

To compute an incremental factorization, select `kstart` and `kend` as the range of rows of `L` you wish to compute. Rows `kstart` to `kend-1` of `L` will be computed. A correct factorization will be

computed only if all descendants of all nodes `kstart` to `kend-1` in the elimination tree have been factorized by a prior call to this routine, and if rows `kstart` to `kend-1` have not been factorized. This condition is **not** checked on input.

In the symmetric case, `A` must be stored in upper form (`A->stype` is greater than zero). The matrix `F` is not accessed and may be `NULL`. Only columns `kstart` to `kend-1` of `A` are accessed.

In the unsymmetric case, the typical case is `F=A'`. Alternatively, if `F=A(:,f)'`, then this routine factorizes the matrix `S = beta*I + A(:,f)*A(:,f)'`. The product `A*F` is assumed to be symmetric; only the upper triangular part of `A*F` is used. `F` must be of size `A->ncol` by `A->nrow`.

If the `L` factor on input is a purely symbolic factorization (with `L->xtype` of `CHOLMOD_PATTERN`), then it is converted to a numeric factorization with same dtype as A. Otherwise, the dtypes of L and A must match.

21.14 cholmod_row_subtree: pattern of row of a factor

```
int cholmod_row_subtree
(
    // input:
    cholmod_sparse *A,  // matrix to analyze
    cholmod_sparse *F,  // used for A*A' case only. F=A' or A(:,f)'
    size_t krow,        // row k of L
    int32_t *Parent,    // elimination tree
    // output:
    cholmod_sparse *R,  // pattern of L(k,:), 1-by-n with R->nzmax >= n
    cholmod_common *Common
) ;
int cholmod_l_row_subtree (cholmod_sparse *, cholmod_sparse *, size_t,
    int64_t *, cholmod_sparse *, cholmod_common *) ;
```

Purpose: Compute the nonzero pattern of the solution to the lower triangular system

```
L(0:k-1,0:k-1) * x = A (0:k-1,k)
```

if `A` is symmetric, or

```
L(0:k-1,0:k-1) * x = A (0:k-1,:) * A (:,k)'
```

if `A` is unsymmetric. This gives the nonzero pattern of row `k` of `L` (excluding the diagonal). The pattern is returned postordered, according to the subtree of the elimination tree rooted at node `k`.

The symmetric case requires `A` to be in symmetric-upper form.

The result is returned in `R`, a pre-allocated sparse matrix of size `nrow`-by-1, with `R->nzmax >= nrow`. `R` is assumed to be packed (`Rnz [0]` is not updated); the number of entries in `R` is given by `Rp [0]`.

21.15 cholmod_row_lsubtree: pattern of row of a factor

```
int cholmod_row_lsubtree
(
    // input:
    cholmod_sparse *A,  // matrix to analyze
```

```
    int32_t *Fi,        // nonzero pattern of kth row of A', not required
                        // for the symmetric case.  Need not be sorted.
    size_t fnz,         // size of Fi
    size_t krow,        // row k of L
    cholmod_factor *L,  // the factor L from which parent(i) is derived
    // output:
    cholmod_sparse *R,  // pattern of L(k,:), n-by-1 with R->nzmax >= n
    cholmod_common *Common
) ;
int cholmod_l_row_lsubtree (cholmod_sparse *, int64_t *, size_t, size_t,
    cholmod_factor *, cholmod_sparse *, cholmod_common *) ;
```

Purpose: Identical to `cholmod_row_subtree`, except the elimination tree is found from `L` itself, not `Parent`. Also, `F=A'` is not provided; the nonzero pattern of the `k`th column of `F` is given by `Fi` and `fnz` instead.

21.16 `cholmod_resymbol`: re-do symbolic factorization

```
int cholmod_resymbol    // recompute symbolic pattern of L
(
    // input:
    cholmod_sparse *A,  // matrix to analyze
    int *fset,          // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    int pack,           // if TRUE, pack the columns of L
    // input/output:
    cholmod_factor *L,  // factorization, entries pruned on output
    cholmod_common *Common
) ;
int cholmod_l_resymbol (cholmod_sparse *, int64_t *, size_t, int,
    cholmod_factor *, cholmod_common *) ;
```

Purpose: Recompute the symbolic pattern of `L`. Entries not in the symbolic pattern of the factorization of `A(p,p)` or `F*F'`, where `F=A(p,f)` or `F=A(:,f)`, are dropped, where `p = L->Perm` is used to permute the input matrix `A`.

Refer to `cholmod_transpose_unsym` for a description of `f`.

If an entry in `L` is kept, its numerical value does not change.

This routine is used after a supernodal factorization is converted into a simplicial one, to remove zero entries that were added due to relaxed supernode amalgamation. It can also be used after a series of downdates to remove entries that would no longer be present if the matrix were factorized from scratch. A downdate (`cholmod_updown`) does not remove any entries from `L`.

21.17 `cholmod_resymbol_noperm`: re-do symbolic factorization

```
int cholmod_resymbol_noperm
(
    // input:
    cholmod_sparse *A,  // matrix to analyze
    int32_t *fset,      // subset of 0:(A->ncol)-1
```

```
    size_t fsize,       // size of fset
    int pack,           // if TRUE, pack the columns of L
    // input/output:
    cholmod_factor *L,  // factorization, entries pruned on output
    cholmod_common *Common
) ;
int cholmod_l_resymbol_noperm (cholmod_sparse *, int64_t *, size_t, int,
    cholmod_factor *, cholmod_common *) ;
```

Purpose: Identical to `cholmod_resymbol`, except that the fill-reducing ordering `L->Perm` is not used.

21.18 `cholmod_postorder`: tree postorder

```
int32_t cholmod_postorder   // return # of nodes postordered
(
    // input:
    int32_t *Parent,    // size n. Parent [j] = p if p is the parent of j
    size_t n,
    int32_t *Weight,    // size n, optional. Weight [j] is weight of node j
    // output:
    int32_t *Post,      // size n. Post [k] = j is kth in postordered tree
    cholmod_common *Common
) ;
int64_t cholmod_l_postorder (int64_t *, size_t, int64_t *, int64_t *,
    cholmod_common *) ;
```

Purpose: Postorder a tree. The tree is either an elimination tree (the output from `cholmod_etree`) or a component tree (from `cholmod_nested_dissection`).

An elimination tree is a complete tree of `n` nodes with `Parent [j] > j` or `Parent [j] = -1` if j is a root. On output `Post [0..n-1]` is a complete permutation vector; `Post [k] = j` if node `j` is the `k`th node in the postordered elimination tree, where `k` is in the range 0 to `n-1`.

A component tree is a subset of `0:n-1`. `Parent [j] = -2` if node `j` is not in the component tree. `Parent [j] = -1` if `j` is a root of the component tree, and `Parent [j]` is in the range 0 to `n-1` if `j` is in the component tree but not a root. On output, `Post [k]` is defined only for nodes in the component tree. `Post [k] = j` if node `j` is the `k`th node in the postordered component tree, where `k` is in the range 0 to the number of components minus 1. Node `j` is ignored and not included in the postorder if `Parent [j] < -1`. As a result, `cholmod_check_parent (Parent, ...)` and `cholmod_check_perm (Post, ...)` fail if used for a component tree and its postordering.

An optional node weight can be given. When starting a postorder at node `j`, the children of `j` are ordered in decreasing order of their weight. If no weights are given (`Weight` is `NULL`) then children are ordered in decreasing order of their node number. The weight of a node must be in the range 0 to `n-1`. Weights outside that range are silently converted to that range (weights < 0 are treated as zero, and weights $\geq$ `n` are treated as `n-1`).

21.19 `cholmod_rcond`: reciprocal condition number

```
double cholmod_rcond        // return rcond estimate
(
    // input:
    cholmod_factor *L,      // factorization to query; not modified
    cholmod_common *Common
) ;
double cholmod_l_rcond (cholmod_factor *, cholmod_common *) ;
```

Purpose: Returns a rough estimate of the reciprocal of the condition number: the minimum entry on the diagonal of `L` (or absolute entry of `D` for an $\mathbf{LDL}^{\mathsf{T}}$ factorization) divided by the maximum entry. `L` can be real, complex, or zomplex. Returns -1 on error, 0 if the matrix is singular or has a zero or NaN entry on the diagonal of `L`, 1 if the matrix is 0-by-0, or `min(diag(L))/max(diag(L))` otherwise. Never returns NaN; if `L` has a NaN on the diagonal it returns zero instead.

22 Modify Module routines

22.1 cholmod_updown: update/downdate

```
int cholmod_updown          // update/downdate
(
    // input:
    int update,         // TRUE for update, FALSE for downdate
    cholmod_sparse *C,  // the incoming sparse update
    // input/output:
    cholmod_factor *L,  // factor to modify
    cholmod_common *Common
) ;
int cholmod_l_updown (int, cholmod_sparse *, cholmod_factor *,
    cholmod_common *) ;
```

Purpose: Updates/downdates the $\mathbf{LDL}^\mathsf{T}$ factorization (symbolic, then numeric), by computing a new factorization of

$$\overline{\mathbf{LDL}}^\mathsf{T} = \mathbf{LDL}^\mathsf{T} \pm \mathbf{CC}^\mathsf{T}$$

where $\overline{\mathbf{L}}$ denotes the new factor. `C` must be sorted. It can be either packed or unpacked. As in all CHOLMOD routines, the columns of `L` are sorted on input, and also on output. If `L` does not contain a simplicial numeric $\mathbf{LDL}^\mathsf{T}$ factorization, it is converted into one. Thus, a supernodal $\mathbf{LL}^\mathsf{T}$ factorization can be passed to `cholmod_updown`. A symbolic `L` is converted into a numeric identity matrix. If the initial conversion fails, the factor is returned unchanged.

If memory runs out during the update, the factor is returned as a simplicial symbolic factor. That is, everything is freed except for the fill-reducing ordering and its corresponding column counts (typically computed by `cholmod_analyze`).

Note that the fill-reducing permutation `L->Perm` is not used. The row indices of `C` refer to the rows of `L`, not `A`. If your original system is $\mathbf{LDL}^\mathsf{T} = \mathbf{PAP}^\mathsf{T}$ (where $\mathbf{P}$ = `L->Perm`), and you want to compute the $\mathbf{LDL}^\mathsf{T}$ factorization of $\mathbf{A} + \mathbf{CC}^\mathsf{T}$, then you must permute $\mathbf{C}$ first. That is, if

$$\mathbf{PAP}^\mathsf{T} = \mathbf{LDL}^\mathsf{T}$$

is the initial factorization, then

$$\mathbf{P}(\mathbf{A} + \mathbf{CC}^\mathsf{T})\mathbf{P}^\mathsf{T} = \mathbf{PAP}^\mathsf{T} + \mathbf{PCC}^\mathsf{T}\mathbf{P}^\mathsf{T} = \mathbf{LDL}^\mathsf{T} + (\mathbf{PC})(\mathbf{PC})^\mathsf{T} = \mathbf{LDL}^\mathsf{T} + \overline{\mathbf{CC}}^\mathsf{T}$$

where $\overline{\mathbf{C}} = \mathbf{PC}$. You can use the `cholmod_submatrix` routine in the `MatrixOps` Module to permute `C`, with:

```
Cnew = cholmod_submatrix (C, L->Perm, L->n, NULL, -1, TRUE, TRUE, Common) ;
```

Note that the `sorted` input parameter to `cholmod_submatrix` must be `TRUE`, because `cholmod_updown` requires `C` with sorted columns. Only real matrices are supported (double or single). The algorithms are described in [8, 9].

22.2 cholmod_updown_solve: update/downdate of a factorization and a solution

```
int cholmod_updown_solve
(
    // input:
    int update,         // TRUE for update, FALSE for downdate
    cholmod_sparse *C,  // the incoming sparse update
    // input/output:
    cholmod_factor *L,  // factor to modify
    cholmod_dense *X,   // solution to Lx=b (size n-by-1)
    cholmod_dense *DeltaB,  // change in b, zero on output
    cholmod_common *Common
) ;
int cholmod_l_updown_solve (int, cholmod_sparse *, cholmod_factor *,
    cholmod_dense *, cholmod_dense *, cholmod_common *) ;
```

Purpose: Identical to `cholmod_updown`, except the system $\mathbf{Lx} = \mathbf{b}$ is also updated/downdated. The new system is $\overline{\mathbf{L}}\overline{\mathbf{x}} = \mathbf{b} + \Delta\mathbf{b}$. The old solution $\mathbf{x}$ is overwritten with $\overline{\mathbf{x}}$. Note that as in the update/downdate of $\mathbf{L}$ itself, the fill-reducing permutation `L->Perm` is not used. The vectors $\mathbf{x}$ and $\mathbf{b}$ are in the permuted ordering.

22.3 cholmod_rowadd: add row to factor

```
int cholmod_rowadd      // add a row to an LDL' factorization
(
    // input:
    size_t k,           // row/column index to add
    cholmod_sparse *R,  // row/column of matrix to factorize (n-by-1)
    // input/output:
    cholmod_factor *L,  // factor to modify
    cholmod_common *Common
) ;
int cholmod_l_rowadd (size_t, cholmod_sparse *, cholmod_factor *,
    cholmod_common *) ;
```

Purpose: Adds a row and column to an $\mathbf{LDL}^{\mathsf{T}}$ factorization. The `k`th row and column of `L` must be equal to the `k`th row and column of the identity matrix on input. Only real matrices are supported (double or single). The dtypes of all matrices must match, except when `L` is symbolic (with an xtype of `CHOLMOD_PATTERN`). In this case, `L` is converted to the dtype of `R`. The algorithm is described in [10].

22.4 cholmod_rowadd_solve: add row to factor and update a solution

```
int cholmod_rowadd_solve
(
    // input:
    size_t k,           // row/column index to add
    cholmod_sparse *R,  // row/column of matrix to factorize (n-by-1)
    double bk [2],      // kth entry of the right-hand-side b
```

```
    // input/output:
    cholmod_factor *L,  // factor to modify
    cholmod_dense *X,   // solution to Lx=b (size n-by-1)
    cholmod_dense *DeltaB,  // change in b, zero on output
    cholmod_common *Common
) ;
int cholmod_l_rowadd_solve (size_t, cholmod_sparse *, double *,
    cholmod_factor *, cholmod_dense *, cholmod_dense *, cholmod_common *) ;
```

Purpose: Identical to `cholmod_rowadd`, except the system $\mathbf{Lx} = \mathbf{b}$ is also updated/downdated, just like `cholmod_updown_solve`.

22.5 `cholmod_rowdel`: delete row from factor

```
int cholmod_rowdel      // delete a rw from an LDL' factorization
(
    // input:
    size_t k,           // row/column index to delete
    cholmod_sparse *R,  // NULL, or the nonzero pattern of kth row of L
    // input/output:
    cholmod_factor *L,  // factor to modify
    cholmod_common *Common
) ;
int cholmod_l_rowdel (size_t, cholmod_sparse *, cholmod_factor *,
    cholmod_common *) ;
```

Purpose: Deletes a row and column from an $\mathbf{LDL}^{\mathsf{T}}$ factorization. The `k`th row and column of `L` is equal to the `k`th row and column of the identity matrix on output. The dtypes of all matrices must match. Only real matrices are supported (double or single).

22.6 `cholmod_rowdel_solve`: delete row from factor and update a solution

```
int cholmod_rowdel_solve
(
    // input:
    size_t k,           // row/column index to delete
    cholmod_sparse *R,  // NULL, or the nonzero pattern of kth row of L
    double yk [2],      // kth entry in the solution to A*y=b
    // input/output:
    cholmod_factor *L,  // factor to modify
    cholmod_dense *X,   // solution to Lx=b (size n-by-1)
    cholmod_dense *DeltaB,  // change in b, zero on output
    cholmod_common *Common
) ;
int cholmod_l_rowdel_solve (size_t, cholmod_sparse *, double *,
    cholmod_factor *, cholmod_dense *, cholmod_dense *, cholmod_common *) ;
```

Purpose: Identical to `cholmod_rowdel`, except the system $\mathbf{Lx} = \mathbf{b}$ is also updated/downdated, just like `cholmod_updown_solve`. When row/column k of $\mathbf{A}$ is deleted from the system $\mathbf{Ay} = \mathbf{b}$, this can induce a change to $\mathbf{x}$, in addition to changes arising when $\mathbf{L}$ and $\mathbf{b}$ are modified. If this is the case, the kth entry of $\mathbf{y}$ is required as input (`yk`). The algorithm is described in [10].

23 MatrixOps Module routines

23.1 cholmod_drop: drop small entries

```
int cholmod_drop
(
    // input:
    double tol,         // keep entries with absolute value > tol
    // input/output:
    cholmod_sparse *A,  // matrix to drop entries from
    cholmod_common *Common
) ;
int cholmod_l_drop (double, cholmod_sparse *, cholmod_common *) ;
```

Purpose: Drop small entries from A, and entries in the ignored part of A if A is symmetric. No CHOLMOD routine drops small numerical entries from a matrix, except for this one. NaN's and Inf's are kept.

23.2 cholmod_norm_dense: dense matrix norm

```
double cholmod_norm_dense       // returns norm (X)
(
    // input:
    cholmod_dense *X,   // matrix to compute the norm of
    int norm,           // type of norm: 0: inf. norm, 1: 1-norm, 2: 2-norm
    cholmod_common *Common
) ;
double cholmod_l_norm_dense (cholmod_dense *, int, cholmod_common *) ;
```

Purpose: Returns the infinity-norm, 1-norm, or 2-norm of a dense matrix. Can compute the 2-norm only for a dense column vector.

23.3 cholmod_norm_sparse: sparse matrix norm

```
double cholmod_norm_sparse          // returns norm (A)
(
    // input:
    cholmod_sparse *A,  // matrix to compute the norm of
    int norm,           // type of norm: 0: inf. norm, 1: 1-norm
    cholmod_common *Common
) ;
double cholmod_l_norm_sparse (cholmod_sparse *, int, cholmod_common *) ;
```

Purpose: Returns the infinity-norm or 1-norm of a sparse matrix.

23.4 cholmod_scale: scale sparse matrix

```
#define CHOLMOD_SCALAR 0    /* A = s*A               */
#define CHOLMOD_ROW    1    /* A = diag(s)*A         */
#define CHOLMOD_COL    2    /* A = A*diag(s)         */
#define CHOLMOD_SYM    3    /* A = diag(s)*A*diag(s) */

int cholmod_scale
(
    // input:
    cholmod_dense *S,   // scale factors (scalar or vector)
    int scale,          // type of scaling to compute
    // input/output:
    cholmod_sparse *A,  // matrix to scale
    cholmod_common *Common
) ;
int cholmod_l_scale (cholmod_dense *, int, cholmod_sparse *, cholmod_common *) ;
```

Purpose: Scales a matrix: `A = diag(s)*A`, `A*diag(s)`, `s*A`, or `diag(s)*A*diag(s)`.

`A` can be of any type (packed/unpacked, upper/lower/unsymmetric). The symmetry of `A` is ignored; all entries in the matrix are modified.

If `A` is `m`-by-`n` unsymmetric but scaled symmetrically, the result is

```
A = diag (s (1:m)) * A * diag (s (1:n))
```

Row or column scaling of a symmetric matrix still results in a symmetric matrix, since entries are still ignored by other routines. For example, when row-scaling a symmetric matrix where just the upper triangular part is stored (and lower triangular entries ignored) `A = diag(s)*triu(A)` is performed, where the result `A` is also symmetric-upper. This has the effect of modifying the implicit lower triangular part. In MATLAB notation:

```
U = diag(s)*triu(A) ;
L = tril (U',-1)
A = L + U ;
```

The scale parameter determines the kind of scaling to perform and the size of `S`:

scale	operation	size of S
CHOLMOD_SCALAR	s[0]*A	1
CHOLMOD_ROW	diag(s)*A	nrow-by-1 or 1-by-nrow
CHOLMOD_COL	A*diag(s)	ncol-by-1 or 1-by-ncol
CHOLMOD_SYM	diag(s)*A*diag(s)	max(nrow,ncol)-by-1, or 1-by-max(nrow,ncol)

23.5 cholmod_sdmult: sparse-times-dense matrix

```
int cholmod_sdmult
(
    // input:
    cholmod_sparse *A,  // sparse matrix to multiply
    int transpose,      // use A if 0, otherwise use A'
```

```
    double alpha [2],   // scale factor for A
    double beta [2],    // scale factor for Y
    cholmod_dense *X,   // dense matrix to multiply
    // input/output:
    cholmod_dense *Y,   // resulting dense matrix
    cholmod_common *Common
) ;
int cholmod_l_sdmult (cholmod_sparse *, int, double *, double *,
    cholmod_dense *, cholmod_dense *Y, cholmod_common *) ;
```

Purpose: Sparse matrix times dense matrix: `Y = alpha*(A*X) + beta*Y` or `Y = alpha*(A'*X) + beta*Y`, where `A` is sparse and `X` and `Y` are dense. When using `A`, `X` has `A->ncol` rows and `Y` has `A->nrow` rows. When using `A'`, `X` has `A->nrow` rows and `Y` has `A->ncol` rows. If `transpose = 0`, then `A` is used; otherwise, `A'` is used (the complex conjugate transpose).

The `transpose` parameter is ignored if the matrix is symmetric or Hermitian. Supports real, complex, and zomplex matrices, but the xtypes and dtypes of `A`, `X`, and `Y` must all match.

23.6 cholmod_ssmult: sparse-times-sparse matrix

```
cholmod_sparse *cholmod_ssmult      // return C=A*B
(
    // input:
    cholmod_sparse *A,  // left matrix to multiply
    cholmod_sparse *B,  // right matrix to multiply
    int stype,          // requested stype of C
    int mode,           // 2: numerical (conj) if A and/or B are symmetric,
                        // 1: numerical (non-conj.) if A and/or B are symmetric.
                        // 0: pattern
    int sorted,         // if TRUE then return C with sorted columns
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_ssmult (cholmod_sparse *, cholmod_sparse *, int, int,
    int, cholmod_common *) ;
```

Purpose: Computes `C = A*B`; multiplying two sparse matrices. `C` is returned as packed, and either unsorted or sorted, depending on the `sorted` input parameter. If `C` is returned sorted, then either `C = (B'*A')'` or `C = (A*B)''` is computed, depending on the number of nonzeros in `A`, `B`, and `C`. The stype of `C` is determined by the `stype` parameter. The xtypes and dtypes of `A` and `B` must match, unless `mode` is 0. If `A` and/or `B` are symmetric, a temporary unsymmetric copy is made, and the conversion is controlled by the `mode` parameter.

23.7 cholmod_submatrix: sparse submatrix

```
cholmod_sparse *cholmod_submatrix   // return C = A (rset,cset)
(
    // input:
    cholmod_sparse *A,  // matrix to subreference
    int32_t *rset,      // set of row indices, duplicates OK
    int64_t rsize,      // size of rset, or -1 for ":"
```

```
    int32_t *cset,      // set of column indices, duplicates OK
    int64_t csize,      // size of cset, or -1 for ":"
    int mode,           // 2: numerical (conj) if A and/or B are symmetric,
                        // 1: numerical (non-conj.) if A and/or B are symmetric.
                        // 0: pattern
    int sorted,         // if TRUE then return C with sorted columns
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_submatrix (cholmod_sparse *, int64_t *,
    int64_t, int64_t *, int64_t, int, int, cholmod_common *) ;
```

Purpose: Returns `C = A (rset,cset)`, where `C` becomes `length(rset)`-by-`length(cset)` in dimension. `rset` and `cset` can have duplicate entries. `A` must be unsymmetric. `C` unsymmetric and is packed. If `sorted` is `TRUE` on input, or `rset` is sorted and `A` is sorted, then `C` is sorted; otherwise `C` is unsorted. If `A` and/or `B` are symmetric, a temporary unsymmetric copy is made, and the conversion is controlled by the `mode` parameter.

If `rset` is `NULL`, it means "`[ ]`" in MATLAB notation, the empty set. The number of rows in the result `C` will be zero if `rset` is `NULL`. Likewise if `cset` means the empty set; the number of columns in the result `C` will be zero if `cset` is `NULL`. If `rsize` or `csize` is negative, it denotes "`:`" in MATLAB notation. Thus, if both `rsize` and `csize` are negative `C = A(:,:) = A` is returned.

For permuting a matrix, this routine is an alternative to `cholmod_ptranspose` (which permutes and transposes a matrix and can work on symmetric matrices).

The time taken by this routine is O(`A->nrow`) if the `Common` workspace needs to be initialized, plus O(`C->nrow + C->ncol + nnz (A (:,cset))`). Thus, if `C` is small and the workspace is not initialized, the time can be dominated by the call to `cholmod_allocate_work`. However, once the workspace is allocated, subsequent calls take less time.

23.8 `cholmod_horzcat`: horizontal concatenation

```
cholmod_sparse *cholmod_horzcat     // returns C = [A B]
(
    // input:
    cholmod_sparse *A,  // left matrix to concatenate
    cholmod_sparse *B,  // right matrix to concatenate
    int mode,           // 2: numerical (conj) if A and/or B are symmetric,
                        // 1: numerical (non-conj.) if A and/or B are symmetric.
                        // 0: pattern
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_horzcat (cholmod_sparse *, cholmod_sparse *, int,
    cholmod_common *) ;
```

Purpose: Horizontal concatenation, returns `C = [A,B]` in MATLAB notation. `A` and `B` can have any stype. `C` is returned unsymmetric and packed. `A` and `B` must have the same number of rows. `C` is sorted if both `A` and `B` are sorted. `A` and `B` must have the same xtype and dtype, unless `mode` is 0. If `A` and/or `B` are symmetric, a temporary unsymmetric copy is made, and the conversion is controlled by the `mode` parameter.

23.9 cholmod_vertcat: vertical concatenation

```
cholmod_sparse *cholmod_vertcat     // returns C = [A ; B]
(
    // input:
    cholmod_sparse *A,  // top matrix to concatenate
    cholmod_sparse *B,  // bottom matrix to concatenate
    int mode,           // 2: numerical (conj) if A and/or B are symmetric,
                        // 1: numerical (non-conj.) if A and/or B are symmetric
                        // 0: pattern
    cholmod_common *Common
) ;
cholmod_sparse *cholmod_l_vertcat (cholmod_sparse *, cholmod_sparse *, int,
    cholmod_common *) ;
```

Purpose: Vertical concatenation, returns `C = [A;B]` in MATLAB notation. `A` and `B` can have any stype. `C` is returned unsymmetric and packed. `A` and `B` must have the same number of columns. `C` is sorted if both `A` and `B` are sorted. `A` and `B` must have the same xtype and dtype, unless `mode` is 0. If `A` and/or `B` are symmetric, a temporary unsymmetric copy is made, and the conversion is controlled by the `mode` parameter.

23.10 cholmod_symmetry: compute the symmetry of a matrix

```
int cholmod_symmetry            // returns the matrix symmetry (see above)
(
    // input:
    cholmod_sparse *A,
    int option,                 // option 0, 1, or 2
    // output:
    int32_t *xmatched,          // # of matched numerical entries
    int32_t *pmatched,          // # of matched entries in pattern
    int32_t *nzoffdiag,         // # of off diagonal entries
    int32_t *nzdiag,            // # of diagonal entries
    cholmod_common *Common
) ;
int cholmod_l_symmetry (cholmod_sparse *, int, int64_t *, int64_t *, int64_t *,
    int64_t *, cholmod_common *) ;
```

Purpose: Determines if a sparse matrix is rectangular, unsymmetric, symmetric, skew-symmetric, or Hermitian. It does so by looking at its numerical values of both upper and lower triangular parts of a CHOLMOD "unsymmetric" matrix, where `A->stype` == 0. The transpose of A is NOT constructed.

If not unsymmetric, it also determines if the matrix has a diagonal whose entries are all real and positive (and thus a candidate for sparse Cholesky if `A->stype` is changed to a nonzero value).

Note that a Matrix Market "general" matrix is either rectangular or unsymmetric.

The row indices in the column of each matrix MUST be sorted for this function to work properly (A-¿sorted must be TRUE). This routine returns EMPTY if A-¿stype is not zero, or if A-¿sorted is FALSE. The exception to this rule is if A is rectangular.

If option == 0, then this routine returns immediately when it finds a non-positive diagonal entry (or one with nonzero imaginary part). If the matrix is not a candidate for sparse Cholesky, it returns the value `CHOLMOD_MM_UNSYMMETRIC`, even if the matrix might in fact be symmetric or Hermitian.

This routine is useful inside the MATLAB backslash, which must look at an arbitrary matrix (`A->stype` == 0) and determine if it is a candidate for sparse Cholesky. In that case, option should be 0.

This routine is also useful when writing a MATLAB matrix to a file in Rutherford/Boeing or Matrix Market format. Those formats require a determination as to the symmetry of the matrix, and thus this routine should not return upon encountering the first non-positive diagonal. In this case, option should be 1.

If option is 2, this function can be used to compute the numerical and pattern symmetry, where 0 is a completely unsymmetric matrix, and 1 is a perfectly symmetric matrix. This option is used when computing the following statistics for the matrices in the SuiteSparse Matrix Collection.

- **numerical symmetry**: number of matched off-diagonal nonzeros over the total number of off-diagonal entries. A real entry a_{ij}, $i \neq j$, is matched if $a_{ji} = a_{ij}$, but this is only counted if both a_{ji} and a_{ij} are nonzero. This does not depend on `Z`. (If A is complex, then the above test is modified; a_{ij} is matched if conj$(a_{ji}) = a_{ij}$).

  Then numeric symmetry = xmatched / nzoffdiag, or 1 if nzoffdiag = 0.

- **pattern symmetry**: number of matched offdiagonal entries over the total number of offdiagonal entries. An entry a_{ij}, $i \neq j$, is matched if a_{ji} is also an entry.

  Then pattern symmetry = pmatched / nzoffdiag, or 1 if nzoffdiag = 0.

The symmetry of a matrix with no offdiagonal entries is equal to 1.

Summary of return values:

`EMPTY (-1)`	out of memory, stype not zero, A not sorted
`CHOLMOD_MM_RECTANGULAR 1`	A is rectangular
`CHOLMOD_MM_UNSYMMETRIC 2`	A is unsymmetric
`CHOLMOD_MM_SYMMETRIC 3`	A is symmetric, but with non-pos. diagonal
`CHOLMOD_MM_HERMITIAN 4`	A is Hermitian, but with non-pos. diagonal
`CHOLMOD_MM_SKEW_SYMMETRIC 5`	A is skew symmetric
`CHOLMOD_MM_SYMMETRIC_POSDIAG 6`	A is symmetric with positive diagonal
`CHOLMOD_MM_HERMITIAN_POSDIAG 7`	A is Hermitian with positive diagonal

See also the `spsym` mexFunction, which is a MATLAB interface for this code. If the matrix is a candidate for sparse Cholesky, it will return a result `CHOLMOD_MM_SYMMETRIC_POSDIAG` if real, or `CHOLMOD_MM_HERMITIAN_POSDIAG` if complex. Otherwise, it will return a value less than this. This is true regardless of the value of the option parameter.

24 Supernodal Module routines

Normally these methods are not called directly, but are accessed via the `cholmod_analyze`, `cholmod_factorize`, and `cholmod_solve` methods.

24.1 cholmod_super_symbolic: supernodal symbolic factorization

```
int cholmod_super_symbolic
(
    // input:
    cholmod_sparse *A,  // matrix to analyze
    cholmod_sparse *F,  // F = A' or A(:,f)'
    int32_t *Parent,    // elimination tree
    // input/output:
    cholmod_factor *L,  // simplicial symbolic on input,
                        // supernodal symbolic on output
    cholmod_common *Common
) ;
int cholmod_l_super_symbolic (cholmod_sparse *, cholmod_sparse *, int64_t *,
    cholmod_factor *, cholmod_common *) ;
```

Purpose: Supernodal symbolic analysis of the $\mathbf{LL}^{\mathsf{T}}$ factorization of `A`, `A*A'`, or `A(:,f)*A(:,f)'`. This routine must be preceded by a simplicial symbolic analysis (`cholmod_rowcolcounts`). See `Cholesky/cholmod_analyze.c` for an example of how to use this routine. The user need not call this directly; `cholmod_analyze` is a "simple" wrapper for this routine. `A` can be symmetric (upper), or unsymmetric. The symmetric/lower form is not supported. In the unsymmetric case `F` is the normally transpose of `A`. Alternatively, if `F=A(:,f)'` then `F*F'` is analyzed. Requires `Parent` and `L->ColCount` to be defined on input; these are the simplicial `Parent` and `ColCount` arrays as computed by `cholmod_rowcolcounts`. Does not use `L->Perm`; the input matrices `A` and `F` must already be properly permuted. Allocates and computes the supernodal pattern of `L` (`L->super`, `L->pi`, `L->px`, and `L->s`). Does not allocate the real part (`L->x`).

24.2 cholmod_super_numeric: supernodal numeric factorization

```
int cholmod_super_numeric
(
    // input:
    cholmod_sparse *A,  // matrix to factorize
    cholmod_sparse *F,  // F = A' or A(:,f)'
    double beta [2],    // beta*I is added to diagonal of matrix to factorize
    // input/output:
    cholmod_factor *L,  // factorization
    cholmod_common *Common
) ;
int cholmod_l_super_numeric (cholmod_sparse *, cholmod_sparse *, double *,
    cholmod_factor *, cholmod_common *) ;
```

Purpose: Computes the numerical Cholesky factorization of `A+beta*I` or `A*F+beta*I`. Only the lower triangular part of `A+beta*I` or `A*F+beta*I` is accessed. The matrices `A` and `F` must already be

permuted according to the fill-reduction permutation `L->Perm`. `cholmod_factorize` is an "easy" wrapper for this code which applies that permutation. The input scalar `beta` is real; only the real part (`beta[0]`) is used.

Symmetric case: `A` is a symmetric (lower) matrix. `F` is not accessed and may be `NULL`. With a fill-reducing permutation, `A(p,p)` should be passed for `A`, where is `p` is `L->Perm`.

Unsymmetric case: `A` is unsymmetric, and `F` must be present. Normally, `F=A'`. With a fill-reducing permutation, `A(p,f)` and `A(p,f)'` should be passed as the parameters `A` and `F`, respectively, where `f` is a list of the subset of the columns of `A`.

The input factorization `L` must be supernodal (`L->is_super` is `TRUE`). It can either be symbolic or numeric. In the first case, `L` has been analyzed by `cholmod_analyze` or `cholmod_super_symbolic`, but the matrix has not yet been numerically factorized. The numerical values are allocated here and the factorization is computed. In the second case, a prior matrix has been analyzed and numerically factorized, and a new matrix is being factorized. The numerical values of `L` are replaced with the new numerical factorization.

`L->is_ll` is ignored on input, and set to `TRUE` on output. This routine always computes an $\mathbf{LL}^\mathsf{T}$ factorization. Supernodal $\mathbf{LDL}^\mathsf{T}$ factorization is not supported.

If the matrix is not positive definite the routine returns `TRUE`, but sets `Common->status` to `CHOLMOD_NOT_POSDEF` and `L->minor` is set to the column at which the failure occurred. Columns `L->minor` to `L->n-1` are set to zero.

If `L` is supernodal symbolic on input, it is converted to a supernodal numeric factor on output, with an xtype of real if `A` is real, or complex if `A` is complex or zomplex. If `L` is supernodal numeric on input, its xtype must match `A` (except that `L` can be complex and `A` zomplex). The xtype of `A` and `F` must match.

24.3 `cholmod_super_lsolve`: supernodal forward solve

```
int cholmod_super_lsolve
(
    // input:
    cholmod_factor *L,  // factor to use for the forward solve
    // input/output:
    cholmod_dense *X,   // b on input, solution to Lx=b on output
    // workspace:
    cholmod_dense *E,   // workspace of size nrhs*(L->maxesize)
    cholmod_common *Common
) ;
int cholmod_l_super_lsolve (cholmod_factor *, cholmod_dense *, cholmod_dense *,
    cholmod_common *) ;
```

Purpose: Solve $\mathbf{Lx} = \mathbf{b}$ for a supernodal factorization. This routine does not apply the permutation `L->Perm`. See `cholmod_solve` for a more general interface that performs that operation. Only real and complex xtypes are supported. `L`, `X`, and `E` must have the same xtype.

24.4 `cholmod_super_ltsolve`: supernodal backsolve

```
int cholmod_super_ltsolve
(
```

```
    // input:
    cholmod_factor *L,  // factor to use for the backsolve
    // input/output:
    cholmod_dense *X,   // b on input, solution to L'x=b on output
    // workspace:
    cholmod_dense *E,   // workspace of size nrhs*(L->maxesize)
    cholmod_common *Common
) ;
int cholmod_l_super_ltsolve (cholmod_factor *, cholmod_dense *, cholmod_dense *,
    cholmod_common *) ;
```

Purpose: Solve $\mathbf{L}^{\mathsf{T}}\mathbf{x} = \mathbf{b}$ for a supernodal factorization. This routine does not apply the permutation `L->Perm`. See `cholmod_solve` for a more general interface that performs that operation. Only real and complex xtypes are supported. `L`, `X`, and `E` must have the same xtype.

25 Partition Module routines

25.1 cholmod_nested_dissection: nested dissection ordering

```
int64_t cholmod_nested_dissection  // returns # of components, or -1 if error
(
    // input:
    cholmod_sparse *A,  // matrix to order
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    // output:
    int32_t *Perm,      // size A->nrow, output permutation
    int32_t *CParent,   // size A->nrow.  On output, CParent [c] is the parent
                        // of component c, or EMPTY if c is a root, and where
                        // c is in the range 0 to # of components minus 1
    int32_t *Cmember,   // size A->nrow.  Cmember [j] = c if node j of A is
                        // in component c
    cholmod_common *Common
) ;
int64_t cholmod_l_nested_dissection (cholmod_sparse *, int64_t *, size_t,
    int64_t *, int64_t *, int64_t *, cholmod_common *) ;
```

Purpose: CHOLMOD's nested dissection algorithm: using its own compression and connected-components algorithms, an external graph partitioner (METIS), and a constrained minimum degree ordering algorithm (CAMD, CCOLAMD, or CSYMAMD). Typically gives better orderings than `METIS_NodeND` (about 5% to 10% fewer nonzeros in `L`).

This method uses a node bisector, applied recursively (but using a non-recursive implementation). Once the graph is partitioned, it calls a constrained minimum degree code (CAMD or CSYMAMD for `A+A'`, and CCOLAMD for `A*A'`) to order all the nodes in the graph - but obeying the constraints determined by the separators. This routine is similar to `METIS_NodeND`, except for how it treats the leaf nodes. `METIS_NodeND` orders the leaves of the separator tree with `MMD`, ignoring the rest of the matrix when ordering a single leaf. This routine orders the whole matrix with CAMD, CSYMAMD, or CCOLAMD, all at once, when the graph partitioning is done.

25.2 cholmod_metis: interface to METIS nested dissection

```
int cholmod_metis
(
    // input:
    cholmod_sparse *A,  // matrix to order
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    int postorder,      // if TRUE, follow with etree or coletree postorder
    // output:
    int32_t *Perm,      // size A->nrow, output permutation
    cholmod_common *Common
) ;
int cholmod_l_metis (cholmod_sparse *, int64_t *, size_t, int, int64_t *,
    cholmod_common *) ;
```

Purpose: CHOLMOD wrapper for the `METIS_NodeND` ordering routine. Creates `A+A'`, `A*A'` or `A(:,f)*A(:,f)'` and then calls `METIS_NodeND` on the resulting graph. This routine is comparable to `cholmod_nested_dissection`, except that it calls `METIS_NodeND` directly, and it does not return the separator tree.

25.3 `cholmod_camd`: interface to CAMD

```
int cholmod_camd
(
    // input:
    cholmod_sparse *A,  // matrix to order
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    int32_t *Cmember,   // size nrow.  see cholmod_ccolamd.c for description.
    // output:
    int32_t *Perm,      // size A->nrow, output permutation
    cholmod_common *Common
) ;
int cholmod_l_camd (cholmod_sparse *, int64_t *, size_t, int64_t *, int64_t *,
    cholmod_common *) ;
```

Purpose: CHOLMOD interface to the CAMD ordering routine. Finds a permutation `p` such that the Cholesky factorization of `A(p,p)` is sparser than `A`. If `A` is unsymmetric, `A*A'` is ordered. If `Cmember[i]=c` then node `i` is in set `c`. All nodes in set 0 are ordered first, followed by all nodes in set 1, and so on.

25.4 `cholmod_ccolamd`: interface to CCOLAMD

```
int cholmod_ccolamd
(
    // input:
    cholmod_sparse *A,  // matrix to order
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    int32_t *Cmember,   // size A->nrow.  Cmember [i] = c if row i is in the
                        // constraint set c.  c must be >= 0.  The # of
                        // constraint sets is max (Cmember) + 1.  If Cmember is
                        // NULL, then it is interpretted as Cmember [i] = 0 for
                        // all i.
    // output:
    int32_t *Perm,      // size A->nrow, output permutation
    cholmod_common *Common
) ;
int cholmod_l_ccolamd (cholmod_sparse *, int64_t *, size_t, int64_t *,
    int64_t *, cholmod_common *) ;
```

Purpose: CHOLMOD interface to the CCOLAMD ordering routine. Finds a permutation `p` such that the Cholesky factorization of `A(p,:)*A(p,:)'` is sparser than `A*A'`. The column elimination is found and postordered, and the CCOLAMD ordering is then combined with its postordering. `A`

must be unsymmetric. If `Cmember[i]=c` then node `i` is in set `c`. All nodes in set 0 are ordered first, followed by all nodes in set 1, and so on.

25.5 cholmod_csymamd: interface to CSYMAMD

```
int cholmod_csymamd
(
    // input:
    cholmod_sparse *A,  // matrix to order
    // output:
    int32_t *Cmember,   // size nrow.  see cholmod_ccolamd.c for description
    int32_t *Perm,      // size A->nrow, output permutation
    cholmod_common *Common
) ;
int cholmod_l_csymamd (cholmod_sparse *, int64_t *, int64_t *,
    cholmod_common *) ;
```

Purpose: CHOLMOD interface to the CSYMAMD ordering routine. Finds a permutation `p` such that the Cholesky factorization of `A(p,p)` is sparser than `A`. The elimination tree is found and postordered, and the CSYMAMD ordering is then combined with its postordering. If `A` is unsymmetric, `A+A'` is ordered (`A` must be square). If `Cmember[i]=c` then node `i` is in set `c`. All nodes in set 0 are ordered first, followed by all nodes in set 1, and so on.

25.6 cholmod_bisect: graph bisector

```
int64_t cholmod_bisect  // returns # of nodes in separator
(
    // input:
    cholmod_sparse *A,  // matrix to bisect
    int32_t *fset,      // subset of 0:(A->ncol)-1
    size_t fsize,       // size of fset
    int compress,       // if TRUE, compress the graph first
    // output:
    int32_t *Partition, // size A->nrow.  Node i is in the left graph if
                        // Partition [i] = 0, the right graph if 1, and in the
                        // separator if 2.
    cholmod_common *Common
) ;
int64_t cholmod_l_bisect (cholmod_sparse *, int64_t *, size_t, int, int64_t *,
    cholmod_common *) ;
```

Purpose: Finds a node bisector of `A`, `A*A'`, `A(:,f)*A(:,f)'`: a set of nodes that partitions the graph into two parts. Compresses the graph first, ensures the graph is symmetric with no diagonal entries, and then calls METIS.

25.7 cholmod_metis_bisector: interface to METIS node bisector

```
int64_t cholmod_metis_bisector      // returns separator size
(
    // input:
    cholmod_sparse *A,  // matrix to bisect
    int32_t *Anw,       // size A->nrow, node weights, can be NULL,
                        // which means the graph is unweighted.
    int32_t *Aew,       // size nz, edge weights (silently ignored).
                        // This option was available with METIS 4, but not
                        // in METIS 5.  This argument is now unused, but
                        // it remains for backward compatibilty, so as not
                        // to change the API for cholmod_metis_bisector.
    // output:
    int32_t *Partition, // size A->nrow
    cholmod_common *Common
) ;
int64_t cholmod_l_metis_bisector (cholmod_sparse *, int64_t *, int64_t *,
    int64_t *, cholmod_common *) ;
```

Purpose: Finds a set of nodes that bisects the graph of `A` or `A*A'` (a direct interface to `METIS_NodeComputeSeparator`). The input matrix `A` must be square, symmetric (with both upper and lower parts present) and with no diagonal entries. These conditions are not checked. Use `cholmod_bisect` to check these conditions.

25.8 `cholmod_collapse_septree`: prune a separator tree

```
int64_t cholmod_collapse_septree
(
    // input:
    size_t n,           // # of nodes in the graph
    size_t ncomponents, // # of nodes in the separator tree (must be <= n)
    double nd_oksep,    // collapse if #sep >= nd_oksep * #nodes in subtree
    size_t nd_small,    // collapse if #nodes in subtree < nd_small
    // output:
    int32_t *CParent,   // size ncomponents; from cholmod_nested_dissection
    int32_t *Cmember,   // size n; from cholmod_nested_dissection
    cholmod_common *Common
) ;
int64_t cholmod_l_collapse_septree (size_t, size_t, double, size_t, int64_t *,
    int64_t *, cholmod_common *) ;
```

Purpose: Prunes a separator tree obtained from `cholmod_nested_dissection`.

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