

The nilpotent Lie rings of order p^k for $k \leq 7$

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1 Introduction

We give presentations for the nilpotent Lie rings of order p^k for $k \leq 7$, valid for all $p > 3$. The presentations give a complete classification of these Lie rings. The class 2 presentations also give a classification for the class 2 Lie rings of order 3^k for $k \leq 7$. (On a historical note, the classification was originally obtained with the aim of classifying the groups of order p^k for $k \leq 7$ via the Baker-Campbell-Hausdorff formula. See [5] and [7]. The Baker-Campbell-Hausdorff formula only applies for p -groups of class at most $p - 1$, and so computing the nilpotent Lie rings of order 2^k was of no use in this project, and computing the nilpotent Lie rings of order 3^k was only useful in this project for classifying 3-groups of nilpotency class 2.)

Our method of classifying nilpotent Lie rings of order p^n closely follows the p -group generation algorithm (see Newman [4] and O'Brien [6]). A Lie ring L is an abelian group under $+$ (addition) together with a bilinear product which satisfies

$$\begin{aligned}aa &= 0 \text{ for all } a \in L, \\(ab)c + (bc)a + (ca)b &= 0 \text{ for all } a, b, c \in L.\end{aligned}$$

(We use ab to denote the Lie product of a and b , rather than the more familiar $[a, b]$.) Note that the axiom $aa = 0$ together with bilinearity implies that $ba = -ab$. The identity $(ab)c + (bc)a + (ca)b = 0$ is the Jacobi identity. Since the Lie product is not associative the bracketing of a product is significant. We adopt the left-normed convention whereby

$$a_1 a_2 \dots a_n = (\dots ((a_1 a_2) a_3) \dots a_{n-1}) a_n.$$

For a Lie ring L , we define the *lower p -central series*

$$L = L_1 \geq L_2 \geq L_3 \geq \dots \geq L_c \geq \dots$$

in an analogous way to groups. We set $L_1 = L$, $L_2 = L^2 + pL$, and for $c > 1$ we set $L_{c+1} = L_c L + pL_c$. (Here $L^2 = \langle ab \mid a, b \in L \rangle$, and $L_c L = \langle ab \mid a \in L_c, b \in L \rangle$.) The ideal L_c consists of all linear combinations of terms of the form

$$a_1 a_2 \dots a_c, p a_1 a_2 \dots a_{c-1}, p^2 a_1 a_2 \dots a_{c-2}, \dots, p^{c-1} a_1,$$

with $a_i \in L$. We say that L has p -class c if $L_{c+1} = \{0\}$, $L_c \neq \{0\}$. If L is a nilpotent Lie ring with finite order p^n for some prime p , then L_{c+1} will equal $\{0\}$ for some c . In fact if L is nilpotent of class k , and if the exponent of L as a finite abelian group is p^m then L has p -class c for some c with $k \leq c < k + m$.

If L and M are two finite nilpotent Lie rings with prime-power order, then L is a *descendant* of M if $L/L_c \cong M$ for some $c \geq 2$. If $L/L_c \cong M$ and L has p -class c (so that $L_c \neq \{0\}$, $L_{c+1} = \{0\}$) then L is an *immediate descendant* of M . Note that if L is a descendant of M then $L/L_2 \cong M/M_2$, so that L and M have the same generator number.

The key idea for calculating the nilpotent Lie rings L with order p^n is as follows. If L has p -class 1 then L is the direct sum of n copies of \mathbb{Z}_p ; call L *elementary abelian*. If L has p -class $c > 1$, then L is an immediate descendant of L/L_c , which is a nilpotent Lie ring with order p^m for some $m < n$. To compute the nilpotent Lie rings of order p^n , first compute the nilpotent Lie rings of order p^k for $k < n$, and for each of these Lie rings compute their immediate descendants of order p^n . A method for doing this is described in [5]. Our presentations are grouped according to their immediate ancestors.

In their simplest form, the presentations below are fairly standard. For example, here is a presentation for one of the nilpotent Lie rings of order p^4 :

$$\langle a, b \mid bab, pa, pb - baa, \text{class } 3 \rangle. \quad (4.11)$$

Note that we specify the lower p -central class of the Lie ring as well as giving generators and relations. So if L is the Lie ring with presentation $\langle a, b \mid bab, pa, pb - baa \rangle$, algebra 4.11 is L/L_4 . One key property of these presentations is that the prime p is symbolic. If we substitute the prime 5 for p in this presentation then we get a presentation for a nilpotent Lie ring of order 5^4 . And if we substitute 7 for p in this presentation, then we get a presentation for a nilpotent Lie ring of order 7^4 . However many of the presentations are more complicated than this, and involve parameters. For example algebra 4.12 has presentation

$$\langle a, b \mid bab, pa, pb - \omega baa, \text{class } 3 \rangle. \quad (4.12)$$

The parameter ω occurs in hundreds of the presentations below, and is assumed throughout to be a primitive element modulo p . If you want to use the presentations to generate a complete and irredundant list of the nilpotent Lie rings of order p^k for some $k \leq 7$ and some specific prime p , then it is important that you choose a primitive element modulo p at the outset, and use this one value for ω throughout. Actually, this is not strictly necessary, but if you use different values for ω in different presentations, then no guarantees are given!

It is perhaps worthwhile to take a moment to explore algebras 4.11 and 4.12 in more detail. Consider a presentation

$$\langle a, b \mid bab, pa, pb - xbaa, \text{class } 3 \rangle$$

where x is an integer. Then (for any given p) this presentation defines a nilpotent Lie ring L_x of order p^4 . Since L_x has class 3, and since $baa \in L_3$, the element baa has additive order p , and so L_x is isomorphic to L_y whenever $x = y \pmod{p}$. It turns out (in this example) that if x and y are non-zero modulo p , then L_x is isomorphic to L_y if and only if $\frac{x}{y}$ is a square modulo p . So (for $p > 2$) there are two isomorphism classes of algebras L_x with $x \not\equiv 0 \pmod{p}$: one where x is a square modulo p , and one where x is not a square modulo p . Taking $x = 1, \omega$ we obtain representatives for these two isomorphism classes. (In this case, ω does not need to be a primitive element modulo p , and any integer which is not a square modulo p would do.)

Many presentations involve parameters other than ω . Here are two simple examples for Lie rings of order p^6 and p^7 :

$$\langle a, b, c \mid pa - ba, pb - cb, pc - xba - ca, \text{class } 2 \rangle \quad (6.114)$$

$$\langle a, b \mid baaaa, baab - baaab, babb - baaab, pa - xbaaab, pb - ybaaab, \text{class } 5 \rangle \quad (7.351)$$

Presentation 6.114 has a single parameter x , and since x appears as the coefficient of ba , and since the algebra has class 2, it is clear that two parameters x and x' give isomorphic algebras if $x = x' \pmod{p}$. As we will see below, we often restrict the values that parameters can take. But in this case there are no restrictions on the parameter x , and so there are p distinct isomorphism classes of algebras with presentation 6.114, with x and x' giving isomorphic algebras if and only if $x = x' \pmod{p}$. This means that you can obtain p non-isomorphic algebras by taking $x = 0, 1, \dots, p-1$. However, sometimes you might want to make a different choice of parameters. For example, it might be that the algebra is terminal unless $x = -1 \pmod{p}$. In that case you might want to take $x = -1, 0, 1, \dots, p-2$ as a representative set of parameters. Similarly, in presentation 7.351 we have two parameters x, y , with no restriction on their values. So this family of presentations gives p^2 non-isomorphic algebras. Parameters x, y and x', y' give isomorphic algebras if and only if $x = x' \pmod{p}$ and $y = y' \pmod{p}$.

Would that things were always this simple! Often the parameters are given a limited range, such as “ $x \neq 0$ ” or “ $1 + 4x$ not a square”. Here $x \neq 0$ is to be understood to mean $x \not\equiv 0 \pmod{p}$, and $1 + 4x$ not a square is to be taken to mean $1 + 4x$ not a square mod p . More generally, if we have a one parameter family of Lie rings L_x then we define an equivalence relation \sim on the parameters x by setting $x \sim y$ if L_x is isomorphic to L_y . It is to be understood that $x \sim y$ whenever $x = y \pmod{p}$, but the comments associated with the presentations below often define slightly more complex equivalence relations. To save space the comments are rather cryptic. Here are some examples:

$$x \sim ax \text{ if } a^3 = 1,$$

$$x \sim ax \text{ if } a^4 = 1,$$

$$x \sim -x,$$

$$x \neq 0, x \sim x^{-1}.$$

In all these examples “mod p ” is to be understood. Generally, if we have a parameter x and we replace it by x' where $x = x' \bmod p$ then we obtain an isomorphic Lie ring. Actually, in most cases (as in 6.114 above) we obtain an identically equal Lie ring. The rare exceptions are when the parameter is the coefficient of an element which does not have order p . But even in those cases we obtain an isomorphic Lie ring. The same applies to presentations with more than one parameter. In the first example above x and x' give isomorphic Lie rings if and only if $x' = ax \bmod p$ for some a with $a^3 = 1 \bmod p$. The second example is similar. In the third example the cryptic comment $x \sim -x$ is intended to mean that $x \sim y$ if and only if $x = \pm y \bmod p$. In the fourth example, we restrict to parameters x with $x \neq 0 \bmod p$, and $x \sim y$ if and only if $x = y \bmod p$ or $xy = 1 \bmod p$. The simplest way of thinking about these equivalence relations is to think of them as being defined on subsets of the field \mathbb{Z}_p , and if you if you want to compute a set of representatives for the equivalence classes for a particular prime p , it is often easiest to compute in \mathbb{Z}_p . But nevertheless, the parameters take values in the integers. This can be confusing, but the same applies in the p -group generation algorithm — the isomorphism problem for the immediate descendants of a finite p -group is solved over \mathbb{Z}_p , but the presentations for the descendants have integer exponents.

We need to be careful when dealing with presentations for the descendants of a Lie ring L_x which involves a parameter x . For example 6.98 has presentation

$$\langle a, b, c \mid cb, pb - ba, pc - xca, \text{class } 2 \rangle (x \neq 0, x \sim x^{-1})$$

Here the parameter x can take all integer values such that $x \neq 0 \bmod p$. If we let L_x be the Lie ring defined by the presentation then L_x is isomorphic to L_y if $x = y \bmod p$ or $xy = 1 \bmod p$. This defines an equivalence relation on the set of possible parameters x which we denote by $x \sim x^{-1}$. If we pick a set S of representatives for the equivalence classes then $\{L_x \mid x \in S\}$ gives a complete and irredundant list of representatives for the isomorphism classes of Lie rings 6.98. The descendants of 6.98 have presentations 7.2931 – 7.2943, and some of these presentations (7.2935 – 7.2939) contain the parameter x . If you want a complete and irredundant list of the descendants of 6.98 then it is important to choose the same set S of representatives x for each of 7.2935 – 7.2939. This is because the equivalence relation $x \sim x^{-1}$ does not correspond to isomorphisms of the individual Lie rings 7.2935 – 7.2939, but rather to a permutation of those Lie rings. To be precise, if we let A_x, B_x, C_x, D_x, E_x be the Lie rings defined by the presentations 7.2935 – 7.2939, and if $xy = 1 \bmod p$ then

$$A_x \cong A_y, B_x \cong B_y, C_x \cong D_y, D_x \cong C_y, E_x \cong E_y.$$

Similar considerations apply to the descendants of 6.108.

The presentations for the nilpotent Lie rings of order p^k for $k \leq 4$ do not involve parameters, apart from the parameter ω which is taken to be a primitive element modulo p whenever it appears. The presentations for the nilpotent Lie rings of order p^5 involve at most one parameter other than ω . But many of the presentations for the nilpotent Lie rings of order p^6 and p^7 involve two or more parameters other than ω . Just as with a single parameter, we associate a comment with these presentations to describe the restrictions on the values that the parameters can take, and to describe the equivalence relation \sim associated with the isomorphism problem. Here are a few examples – two for presentations involving parameters x and y , and one for a three parameter presentation:

$$\begin{aligned} (x, y) &\sim (y, x), \\ x \neq 0, y \neq 0, 1, -1, (x, y) &\sim (x, y^{-1}), \\ x \neq 0, (x, y, z) &\sim (ax, y, a^2z) \text{ if } a^3 = 1, \end{aligned}$$

The first of these examples indicates that the parameters (x, y) and (y, x) give isomorphic Lie rings. The second example indicates that if $x \neq 0 \bmod p$ and if $y \neq 0, 1, -1 \bmod p$ and if $yz = 1 \bmod p$ then (x, y) and (x, z) give isomorphic Lie rings.

In the last of these three examples we have a presentation with three parameters x, y, z . In this example, (x, y, z) and (x', y', z') give isomorphic Lie rings if and only if $x' = ax \bmod p$, $y' = y \bmod p$, $z' = a^2z \bmod p$, for some a with $a^3 = 1 \bmod p$.

In a few cases I have been unable to give simply defined equivalence relations on the parameter sets, and I need to give more complicated definitions. See for example algebra 6.62 below.

There is one further type of comment attached to many of the presentations — some of the presentations only apply for certain residue classes modulo p . A comment “ $p = 1 \bmod 3$ ”, for example, implies that the presentation should only be included for the prime p if $p = 1 \bmod 3$.

The classification given here has been criticized on the grounds that to produce a complete and irredundant set of Lie rings for any given p you have to do some computations “on the fly”. Now even for groups and nilpotent Lie rings of order p^4 , if you want an irredundant list of presentations for any given p then you need to find an integer which is not a square modulo p . This requires a separate calculation for each p , albeit an easy one. (See the remarks above about algebras 4.11 and 4.12.) The database of groups of order p^5 in both GAP [2] and MAGMA [1] is based on Boris Gornat’s classification of the groups of order p^5 [3]. (Although Gornat’s classification was only published in 2018, it appeared in his 2003 Masters thesis written under the supervision of Bettina Eick.) The format is different, but Gornat’s presentation for his group G11 corresponds to the presentation

$$\langle a, b \mid pa - baa, pb - xbab, \text{class } 3 \rangle (x \neq 0, x \sim x^{-1})$$

for algebra 5.42 below. Gornat solves the isomorphism problem by letting x take the values ω^k for $k = 1, 2, \dots, \frac{p-1}{2}$, where ω is an integer which is a primitive element modulo p . (Gornat covers the case $x = 1$ with his group G7.) This is certainly an elegant way of writing down a list of representatives for the equivalence classes of x under the equivalence relation $x \sim x^{-1}$, but if you wanted a list of presentations for any given p you would want to compute the values $\omega^k \bmod p$. Again, this involves a separate calculation for each prime. Furthermore the comment $x \sim x^{-1}$ gives isomorphism information which is not explicit in Gornat’s presentation for G11.

So I think the criticism is that there are some presentations with several parameters, and quite complicated equivalence relations. There are

$$3p^2 + 39p + 344 + 24 \gcd(p - 1, 3) + 11 \gcd(p - 1, 4) + 2 \gcd(p - 1, 5)$$

nilpotent Lie rings of order p^6 for every prime $p \geq 5$, and so there have to be some presentations for the Lie rings of order p^6 involving at least two parameters (as well as possibly involving the parameter ω). The presentations for the Lie rings of order p^6 involve at most 2 parameters (other than ω) in all except one case. The one exception is algebra 6.178 which has a presentation involving four parameters. Actually one of the four parameters can be restricted to values 0, 1, and so this four parameter presentation quickly reduces to two three parameter presentations. For $p \geq 5$ the number of nilpotent Lie rings of order p^7 is

$$\begin{aligned} & 3p^5 + 12p^4 + 44p^3 + 170p^2 + 707p + 2455 \\ & + (4p^2 + 44p + 291) \gcd(p - 1, 3) + (p^2 + 19p + 135) \gcd(p - 1, 4) \\ & + (3p + 31) \gcd(p - 1, 5) + 4 \gcd(p - 1, 7) + 5 \gcd(p - 1, 8) + \gcd(p - 1, 9), \end{aligned}$$

and so there have to be presentations for Lie rings of order p^7 involving at least 5 parameters. The ideal, I suppose, would be a number of presentations with at most five parameters, and with no restrictions on the parameters. There is no reason to suppose that this ideal is achievable, and even if it were it would still take p^5 work to list all the different algebras given by a five parameter presentation. If you had a five parameter presentation with anything but the simplest restrictions on the parameters, and anything but the simplest equivalence relation solving the isomorphism problem, it could easily take more than p^5 work to list them all.

So the real issue is the amount of work required to produce a complete list of algebras of order p^7 for a given p . Ideally we would not need more than p^5 work, and perhaps that is achievable. But I have been unable to reach that goal in every case. The amount of work required to produce a complete list of algebras for any given prime p for any of the parametrized presentations in the classification is of order at most p^5 , except in one case. The one exception is presentation 7.3285. This is a 12 parameter presentation for (some) immediate descendants of algebra 4.1. The equivalence relation defined on these parameters to solve the isomorphism problem is the most complicated of all the conditions appearing in this classification. It is fairly easy to see that there are 11 orbits for the values of the first 6 parameters, so that the problem of finding a set of representatives for the 12 parameters quickly reduces to 11 problems with 6 parameters. I have a program which computes a set of representatives for the orbits with work of order somewhere between p^6 and p^7 . For primes up to 23 the time taken to run the program is roughly proportional to $p^{6.2}$. It would be nice to do better than this, as this particular presentation creates a serious bottleneck in producing a complete list of Lie rings of order p^7 for any p greater than 5 or 7.

2 Order p

$$\langle a \mid \text{class } 1 \rangle \tag{1.0}$$

3 Order p^2

$$\langle a \mid \text{class 2} \rangle \quad (2.0)$$

$$\langle a, b \mid \text{class 1} \rangle \quad (2.1)$$

4 Order p^3

$$\langle a \mid \text{class 3} \rangle \quad (3.0)$$

$$\langle a, b, c \mid \text{class 1} \rangle \quad (3.1)$$

$$\langle a, b \mid pa, pb, \text{class 2} \rangle \quad (3.2)$$

$$\langle a, b \mid pa - ba, pb, \text{class 2} \rangle \quad (3.3)$$

$$\langle a, b \mid ba, pb, \text{class 2} \rangle \quad (3.4)$$

5 Order p^4

$$\langle a \mid \text{class 4} \rangle \quad (4.0)$$

$$\langle a, b, c, d \mid \text{class 1} \rangle \quad (4.1)$$

$$\langle a, b, c \mid ba, ca, cb, pb, pc, \text{class 2} \rangle \quad (4.2)$$

$$\langle a, b, c \mid ca, cb, pa, pb, pc, \text{class 2} \rangle \quad (4.3)$$

$$\langle a, b, c \mid ca, cb, pa - ba, pb, pc, \text{class 2} \rangle \quad (4.4)$$

$$\langle a, b, c \mid ca, cb, pa, pb, pc - ba, \text{class 2} \rangle \quad (4.5)$$

$$\langle a, b \mid ba, \text{class 2} \rangle \quad (4.6)$$

$$\langle a, b \mid pb, \text{class 2} \rangle \quad (4.7)$$

$$\langle a, b \mid pb - ba, \text{class 2} \rangle \quad (4.8)$$

$$\langle a, b \mid bab, pa, pb, \text{class 3} \rangle \quad (4.9)$$

$$\langle a, b \mid bab, pa - baa, pb, \text{class 3} \rangle \quad (4.10)$$

$$\langle a, b \mid bab, pa, pb - baa, \text{class 3} \rangle \quad (4.11)$$

$$\langle a, b \mid bab, pa, pb - \omega baa, \text{class 3} \rangle \quad (4.12)$$

$$\langle a, b \mid ba, pb, \text{class } 3 \rangle \quad (4.13)$$

$$\langle a, b \mid ba - p^2a, pb, \text{class } 3 \rangle \quad (4.14)$$

6 Order p^5

There are $2p + 61 + 2 \gcd(p-1, 3) + \gcd(p-1, 4)$ nilpotent Lie rings of order p^5 for $p \geq 5$. We first list the one generator and five generator Lie rings of order p^5 , and then we group the rest according to their immediate ancestors.

$$\langle a \mid \text{class } 5 \rangle \quad (5.0)$$

$$\langle a, b, c, d, e \mid \text{class } 1 \rangle \quad (5.1)$$

6.1 Descendants of 4.1

$$\langle a, b, c, d \mid ba, ca, da, cb, db, dc, pb, pc, pd, \text{class } 2 \rangle \quad (5.2)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pa, pb, pc, pd, \text{class } 2 \rangle \quad (5.3)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pa - ba, pb, pc, pd, \text{class } 2 \rangle \quad (5.4)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pa, pb, pc - ba, pd, \text{class } 2 \rangle \quad (5.5)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pa, pb, pc, pd, \text{class } 2 \rangle \quad (5.6)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pa - ba, pb, pc, pd, \text{class } 2 \rangle \quad (5.7)$$

6.2 Descendants of 3.1

$$\langle a, b, c \mid ba, ca, cb, pc, \text{class } 2 \rangle \quad (5.8)$$

$$\langle a, b, c \mid ca, cb, pb, pc, \text{class } 2 \rangle \quad (5.9)$$

$$\langle a, b, c \mid ca, cb, pb - ba, pc, \text{class } 2 \rangle \quad (5.10)$$

$$\langle a, b, c \mid ca, cb, pb, pc - ba, \text{class } 2 \rangle \quad (5.11)$$

$$\langle a, b, c \mid ca, cb, pa, pb, \text{class } 2 \rangle \quad (5.12)$$

$$\langle a, b, c \mid ca, cb, pa - ba, pb, \text{class } 2 \rangle \quad (5.13)$$

$$\langle a, b, c \mid cb, pa, pb, pc, \text{class } 2 \rangle \quad (5.14)$$

$$\langle a, b, c \mid cb, pa - ba, pb, pc, \text{class } 2 \rangle \quad (5.15)$$

$$\langle a, b, c \mid cb, pa, pb - ba, pc, \text{class } 2 \rangle \quad (5.16)$$

$$\langle a, b, c \mid cb, pa - ca, pb - ba, pc, \text{class } 2 \rangle \quad (5.17)$$

$$\langle a, b, c \mid cb, pa, pb - ca, pc, \text{class } 2 \rangle \quad (5.18)$$

$$\langle a, b, c \mid cb, pa - ba, pb - ca, pc, \text{class } 2 \rangle \quad (5.19)$$

$$\langle a, b, c \mid cb, pa, pb - ba, pc - xca, \text{class } 2 \rangle (x \neq 0, x \sim x^{-1}) \quad (5.20)$$

$$\langle a, b, c \mid cb, pa, pb - ba - ca, pc - ca, \text{class } 2 \rangle \quad (5.21)$$

$$\langle a, b, c \mid cb, pa, pb - \omega ca, pc - ba, \text{class } 2 \rangle \quad (5.22)$$

$$\langle a, b, c \mid cb, pa, pb - xca, pc - ba - ca, \text{class } 2 \rangle (1 + 4x \text{ not a square}) \quad (5.23)$$

6.3 Descendants of 4.2

$$\langle a, b, c \mid ba, ca, cb, pb, pc, \text{class } 3 \rangle \quad (5.24)$$

$$\langle a, b, c \mid ba, ca, cb - p^2a, pb, pc, \text{class } 3 \rangle \quad (5.25)$$

$$\langle a, b, c \mid ba - p^2a, ca, cb, pb, pc, \text{class } 3 \rangle \quad (5.26)$$

6.4 Descendants of 4.3

$$\langle a, b, c \mid ca, cb, bab, pa, pb, pc, \text{class } 3 \rangle \quad (5.27)$$

$$\langle a, b, c \mid ca, cb, bab, pa - baa, pb, pc, \text{class } 3 \rangle \quad (5.28)$$

$$\langle a, b, c \mid ca, cb, bab, pa, pb - baa, pc, \text{class } 3 \rangle \quad (5.29)$$

$$\langle a, b, c \mid ca, cb, bab, pa, pb - \omega baa, pc, \text{class } 3 \rangle \quad (5.30)$$

$$\langle a, b, c \mid ca, cb, bab, pa, pb, pc - baa, \text{class } 3 \rangle \quad (5.31)$$

$$\langle a, b, c \mid ca, cb - baa, bab, pa, pb, pc, \text{class } 3 \rangle \quad (5.32)$$

$$\langle a, b, c \mid ca, cb - baa, bab, pa - baa, pb, pc, \text{class } 3 \rangle \quad (5.33)$$

$$\langle a, b, c \mid ca, cb - baa, bab, pa, pb - baa, pc, \text{class } 3 \rangle \quad (5.34)$$

$$\langle a, b, c \mid ca, cb - baa, bab, pa, pb - \omega baa, pc, \text{class } 3 \rangle \quad (5.35)$$

$$\langle a, b, c \mid ca, cb - baa, bab, pa, pb, pc - baa, \text{class } 3 \rangle \quad (5.36)$$

6.5 Descendants of 4.4

Algebra 4.4 is terminal.

6.6 Descendants of 4.5

Algebra 4.5 is terminal.

6.7 Descendants of 2.1

$$\langle a, b \mid \text{class } 2 \rangle \quad (5.37)$$

6.8 Descendants of 3.2

$$\langle a, b \mid pa, pb, \text{class } 3 \rangle \quad (5.38)$$

$$\langle a, b \mid pa - bab, pb, \text{class } 3 \rangle \quad (5.39)$$

$$\langle a, b \mid pa - \omega bab, pb, \text{class } 3 \rangle \quad (5.40)$$

$$\langle a, b \mid pa - baa, pb, \text{class } 3 \rangle \quad (5.41)$$

$$\langle a, b \mid pa - baa, pb - xbab, \text{class } 3 \rangle \quad (x \neq 0, x \sim x^{-1}) \quad (5.42)$$

$$\langle a, b \mid pa - baa - bab, pb - bab, \text{class } 3 \rangle \quad (5.43)$$

$$\langle a, b \mid pa - baa - \omega bab, pb - bab, \text{class } 3 \rangle \quad (5.44)$$

$$\langle a, b \mid pa - \omega bab, pb - baa, \text{class } 3 \rangle \quad (5.45)$$

$$\langle a, b \mid pa - xbab, pb - baa - bab, \text{class } 3 \rangle \quad (1 + 4x \text{ not a square}) \quad (5.46)$$

6.9 Descendants of 3.3

Algebra 3.3 is terminal.

6.10 Descendants of 3.4

Algebra 3.4 is terminal.

6.11 Descendants of 4.6

$$\langle a, b \mid ba, p^2b, \text{class } 3 \rangle \quad (5.47)$$

$$\langle a, b \mid ba - p^2a, p^2b, \text{class } 3 \rangle \quad (5.48)$$

6.12 Descendants of 4.7

$$\langle a, b \mid baa, bab, pb, \text{class } 3 \rangle \quad (5.49)$$

$$\langle a, b \mid bab, p^2a, pb, \text{class } 3 \rangle \quad (5.50)$$

$$\langle a, b \mid bab, p^2a, pb - baa, \text{class } 3 \rangle \quad (5.51)$$

$$\langle a, b \mid bab, p^2a, pb - \omega baa, \text{class } 3 \rangle \quad (5.52)$$

$$\langle a, b \mid bab, p^2a - baa, pb, \text{class } 3 \rangle \quad (5.53)$$

$$\langle a, b \mid baa, p^2a, pb, \text{class } 3 \rangle \quad (5.54)$$

$$\langle a, b \mid baa, p^2a, pb - bab, \text{class } 3 \rangle \quad (5.55)$$

$$\langle a, b \mid baa, p^2a - bab, pb, \text{class } 3 \rangle \quad (5.56)$$

$$\langle a, b \mid baa, p^2a - \omega bab, pb, \text{class } 3 \rangle \quad (5.57)$$

6.13 Descendants of 4.8

$$\langle a, b \mid baa, pb - ba, \text{class } 3 \rangle \quad (5.58)$$

$$\langle a, b \mid p^2a, pb - ba, \text{class } 3 \rangle \quad (5.59)$$

6.14 Descendants of 4.9

$$\langle a, b \mid bab, pa, pb, \text{class } 4 \rangle \quad (5.60)$$

$$\langle a, b \mid bab, pa - baaa, pb, \text{class } 4 \rangle \quad (5.61)$$

$$\langle a, b \mid bab, pa, pb - baaa, \text{class } 4 \rangle \quad (5.62)$$

$$\langle a, b \mid bab, pa, pb - \omega baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (5.63)$$

$$\langle a, b \mid bab, pa, pb - \omega^2 baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (5.64)$$

$$\langle a, b \mid bab - baaa, pa, pb, \text{class } 4 \rangle \quad (5.65)$$

$$\langle a, b \mid bab - baaa, pa - baaa, pb, \text{class } 4 \rangle \quad (5.66)$$

$$\langle a, b \mid bab - baaa, pa - \omega baaa, pb, \text{class } 4 \rangle \quad (5.67)$$

$$\langle a, b \mid bab - baaa, pa - \omega^2 baaa, pb, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (5.68)$$

$$\langle a, b \mid bab - baaa, pa - \omega^3 baaa, pb, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (5.69)$$

$$\langle a, b \mid bab - baaa, pa, pb - baaa, \text{class } 4 \rangle \quad (5.70)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (5.71)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega^2 baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (5.72)$$

6.15 Descendants of 4.10

Algebra 4.10 is terminal.

6.16 Descendants of 4.11 and 4.12

Algebras 4.11 and 4.12 are terminal.

6.17 Descendants of 4.13

$$\langle a, b \mid ba, pb, \text{class } 4 \rangle \quad (5.73)$$

$$\langle a, b \mid ba - p^3a, pb, \text{class } 4 \rangle \quad (5.74)$$

6.18 Descendants of 4.14

Algebra 4.14 is terminal.

7 Order p^6

There are

$$3p^2 + 39p + 344 + 24 \gcd(p-1, 3) + 11 \gcd(p-1, 4) + 2 \gcd(p-1, 5)$$

nilpotent Lie rings of order p^6 for every prime $p \geq 5$.

The following table lists the nilpotent Lie rings of order at most p^5 which have immediate descendants of order p^6 .

3.1	$\langle a, b, c \mid \text{class 1} \rangle$
4.1	$\langle a, b, c, d \mid \text{class 1} \rangle$
4.3	$\langle a, b, c \mid ca, cb, pa, pb, pc, \text{class 2} \rangle$
4.6	$\langle a, b \mid ba, \text{class 2} \rangle$
4.7	$\langle a, b \mid pb, \text{class 2} \rangle$
4.8	$\langle a, b \mid pb - ba, \text{class 2} \rangle$
5.0	$\langle a \mid \text{class 5} \rangle$
5.1	$\langle a, b, c, d, e \mid \text{class 1} \rangle$
5.2	$\langle a, b, c, d \mid ba, ca, da, cb, db, dc, pb, pc, pd, \text{class 2} \rangle$
5.3	$\langle a, b, c, d \mid ca, cb, da, db, dc, pa, pb, pc, pd, \text{class 2} \rangle$
5.8	$\langle a, b, c \mid ba, ca, cb, pc, \text{class 2} \rangle$
5.9	$\langle a, b, c \mid ca, cb, pb, pc, \text{class 2} \rangle$
5.10	$\langle a, b, c \mid ca, cb, pb - ba, pc, \text{class 2} \rangle$
5.11	$\langle a, b, c \mid ca, cb, pb, pc - ba, \text{class 2} \rangle$
5.12	$\langle a, b, c \mid ca, cb, pa, pb, \text{class 2} \rangle$
5.13	$\langle a, b, c \mid ca, cb, pa - ba, pb, \text{class 2} \rangle$
5.14	$\langle a, b, c \mid cb, pa, pb, pc, \text{class 2} \rangle$
5.15	$\langle a, b, c \mid cb, pa - ba, pb, pc, \text{class 2} \rangle$
5.16	$\langle a, b, c \mid cb, pa, pb - ba, pc, \text{class 2} \rangle$
5.18	$\langle a, b, c \mid cb, pa, pb - ca, pc, \text{class 2} \rangle$
5.19	$\langle a, b, c \mid cb, pa - ba, pb - ca, pc, \text{class 2} \rangle$
5.24	$\langle a, b, c \mid ba, ca, cb, pb, pc, \text{class 3} \rangle$
5.27	$\langle a, b, c \mid bab, ca, cb, pa, pb, pc, \text{class 3} \rangle$
5.32	$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, pc, \text{class 3} \rangle$
5.37	$\langle a, b \mid \text{class 2} \rangle$
5.38	$\langle a, b \mid pa, pb, \text{class 3} \rangle$
5.39	$\langle a, b \mid pa - bab, pb, \text{class 3} \rangle$
5.40	$\langle a, b \mid pa - \omega bab, pb, \text{class 3} \rangle$
5.41	$\langle a, b \mid pa - baa, pb, \text{class 3} \rangle$
5.42	$\langle a, b \mid pa - baa, pb + bab, \text{class 3} \rangle$
5.45	$\langle a, b \mid pa + bab, pb + \omega baa, \text{class 3} \rangle$
5.47	$\langle a, b \mid ba, p^2b, \text{class 3} \rangle$
5.48	$\langle a, b \mid ba - p^2a, p^2b, \text{class 3} \rangle$
5.49	$\langle a, b \mid baa, bab, pb, \text{class 3} \rangle$
5.50	$\langle a, b \mid bab, p^2a, pb, \text{class 3} \rangle$
5.51	$\langle a, b \mid bab, p^2a, pb - baa, \text{class 3} \rangle$
5.52	$\langle a, b \mid bab, p^2a, pb - \omega baa, \text{class 3} \rangle$
5.54	$\langle a, b \mid baa, p^2a, pb, \text{class 3} \rangle$
5.58	$\langle a, b \mid baa, pb - ba, \text{class 3} \rangle$
5.60	$\langle a, b \mid bab, pa, pb, \text{class 4} \rangle$
5.65	$\langle a, b \mid bab - baaa, pa, pb, \text{class 4} \rangle$
5.73	$\langle a, b \mid ba, pb, \text{class 4} \rangle$

We first give the cyclic and 6 generator Lie rings of order p^6 , and then list the remainder, grouped according to their immediate ancestors.

$$\langle a \mid \text{class 6} \rangle \quad (6.0)$$

$$\langle a, b, c, d, e, f \mid \text{class 1} \rangle \quad (6.1)$$

7.1 Descendants of 3.1

$3p + 27$ algebras

$$\langle a, b, c \mid ba, ca, cb, \text{class 2} \rangle \quad (6.85)$$

$$\langle a, b, c \mid ca, cb, pc, \text{class 2} \rangle \quad (6.86)$$

$$\langle a, b, c \mid ca, cb, pc - ba, \text{class 2} \rangle \quad (6.87)$$

$$\langle a, b, c \mid ca, cb, pa, \text{class 2} \rangle \quad (6.88)$$

$$\langle a, b, c \mid ca, cb, pa - ba, \text{class 2} \rangle \quad (6.89)$$

$$\langle a, b, c \mid cb, pb, pc, \text{class 2} \rangle \quad (6.90)$$

$$\langle a, b, c \mid cb, pa, pc, \text{class 2} \rangle \quad (6.91)$$

$$\langle a, b, c \mid cb, pb - ba, pc, \text{class 2} \rangle \quad (6.92)$$

$$\langle a, b, c \mid cb, pb - ca, pc, \text{class 2} \rangle \quad (6.93)$$

$$\langle a, b, c \mid cb, pa, pc - ba, \text{class 2} \rangle \quad (6.94)$$

$$\langle a, b, c \mid cb, pa, pc - ca, \text{class 2} \rangle \quad (6.95)$$

$$\langle a, b, c \mid cb, pa - ba, pc, \text{class 2} \rangle \quad (6.96)$$

$$\langle a, b, c \mid cb, pa - ca, pc, \text{class 2} \rangle \quad (6.97)$$

$$\langle a, b, c \mid cb, pb - ba, pc - xca, \text{class 2} \rangle (x \neq 0, x \sim x^{-1}) \quad (6.98)$$

$$\langle a, b, c \mid cb, pb - ba - ca, pc - ca, \text{class 2} \rangle \quad (6.99)$$

$$\langle a, b, c \mid cb, pb - \omega ca, pc - ba, \text{class 2} \rangle \quad (6.100)$$

$$\langle a, b, c \mid cb, pb - xca, pc - ba - ca, \text{class 2} \rangle (1 + 4x \text{ not a square}) \quad (6.101)$$

$$\langle a, b, c \mid cb, pa - ca, pc - ba, \text{class } 2 \rangle \quad (6.102)$$

$$\langle a, b, c \mid cb, pa - ba, pc - ca, \text{class } 2 \rangle \quad (6.103)$$

$$\langle a, b, c \mid pa, pb, pc, \text{class } 2 \rangle \quad (6.104)$$

$$\langle a, b, c \mid pa - cb, pb, pc, \text{class } 2 \rangle \quad (6.105)$$

$$\langle a, b, c \mid pa - ba, pb, pc, \text{class } 2 \rangle \quad (6.106)$$

$$\langle a, b, c \mid pa - ca, pb - xcb, pc, \text{class } 2 \rangle \quad (x \neq 0, x \sim x^{-1}) \quad (6.108)$$

$$\langle a, b, c \mid pa - ca - cb, pb - cb, pc, \text{class } 2 \rangle \quad (6.109)$$

$$\langle a, b, c \mid pa - \omega cb, pb - ca, pc, \text{class } 2 \rangle \quad (6.110)$$

$$\langle a, b, c \mid pa - xcb, pb - ca - cb, pc, \text{class } 2 \rangle \quad (1 + 4x \text{ not a square}) \quad (6.111)$$

$$\langle a, b, c \mid pa - ba, pb - ca, pc, \text{class } 2 \rangle \quad (6.112)$$

$$\langle a, b, c \mid pa - ba, pb - cb, pc, \text{class } 2 \rangle \quad (6.113)$$

$$\langle a, b, c \mid pa - ba, pb - cb, pc - xba - ca, \text{class } 2 \rangle \quad (6.114)$$

$$\langle a, b, c \mid pa - ba, pb - ca, pc - cb, \text{class } 2 \rangle \quad (6.115)$$

$$\langle a, b, c \mid pa - ba, pb - ca, pc + cb, \text{class } 2 \rangle \quad (6.116)$$

$$\langle a, b, c \mid pa - ba, pb - ca, pc - \omega ba + cb, \text{class } 2 \rangle \quad (6.117)$$

7.2 Descendants of 4.1

$4p + 48$ algebras

$$\langle a, b, c, d \mid ba, ca, da, cb, db, dc, pc, pd, \text{class } 2 \rangle \quad (6.9)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pa, pb, pc - ba, \text{class } 2 \rangle \quad (6.10)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pb, pc, pd, \text{class } 2 \rangle \quad (6.11)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pb - ba, pc, pd, \text{class } 2 \rangle \quad (6.12)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pb, pc - ba, pd, \text{class } 2 \rangle \quad (6.13)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pa, pb, pd, \text{class } 2 \rangle \quad (6.14)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pa - ba, pb, pd, \text{class } 2 \rangle \quad (6.15)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pb, pc, pd, \text{class } 2 \rangle \quad (6.16)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pb - ba, pc, pd, \text{class } 2 \rangle \quad (6.17)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pb, pc - ba, pd, \text{class } 2 \rangle \quad (6.18)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb, pc, pd, \text{class } 2 \rangle \quad (6.19)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb, pc, pd, \text{class } 2 \rangle \quad (6.20)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ba, pc, pd, \text{class } 2 \rangle \quad (6.21)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa - ca, pb - ba, pc, pd, \text{class } 2 \rangle \quad (6.22)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ca, pc, pd, \text{class } 2 \rangle \quad (6.23)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb - ca, pc, pd, \text{class } 2 \rangle \quad (6.24)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ba, pc - xca, pd, \text{class } 2 \rangle (x \neq 0, x \sim x^{-1}) \quad (6.25)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ba - ca, pc - ca, pd, \text{class } 2 \rangle \quad (6.26)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb - \omega ca, pc - ba, pd, \text{class } 2 \rangle \quad (6.27)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb - xca, pc - ba - ca, pd, \text{class } 2 \rangle (1 + 4x \text{ not a square}) \quad (6.28)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb, pc, pd - ca, \text{class } 2 \rangle \quad (6.29)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb, pc, pd - ca, \text{class } 2 \rangle \quad (6.30)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ba, pc, pd - ca, \text{class } 2 \rangle \quad (6.31)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pb, pc - ba, pd - ca, \text{class } 2 \rangle \quad (6.32)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa, pb, pc, pd, \text{class } 2 \rangle \quad (6.33)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba, pb, pc, pd, \text{class } 2 \rangle \quad (6.34)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa, pb - ba, pc, pd, \text{class } 2 \rangle \quad (6.35)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba, pb - ba, pc, pd, \text{ class } 2 \rangle \quad (6.36)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba - ca, pb, pc, pd, \text{ class } 2 \rangle \quad (6.37)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba - ca, pb - ba - ca, pc, pd, \text{ class } 2 \rangle \quad (6.38)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba, pb - ba, pc - ca, pd - ca, \text{ class } 2 \rangle \quad (6.39)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba, pb - ba, pc, pd - ca, \text{ class } 2 \rangle \quad (6.39B)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba + ca, pb - ba, pc, pd - ca, \text{ class } 2 \rangle \quad (6.40)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa + ca, pb - ba, pc, pd - ca, \text{ class } 2 \rangle \quad (6.41)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa, pb - ba, pc, pd - ca, \text{ class } 2 \rangle \quad (6.42)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba - ca, pb - ba - xca, pc, pd, \text{ class } 2 \rangle \quad (x \neq 0, 1) \quad (6.43)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba - ca, pb - ba, pc, pd, \text{ class } 2 \rangle \quad (6.44)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba - ca, pb - ca, pc, pd, \text{ class } 2 \rangle \quad (6.45)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ba, pb - ca, pc, pd, \text{ class } 2 \rangle \quad (6.46)$$

$$\langle a, b, c, d \mid cb, da, db - ba, dc, pa - ca, pb - ba, pc, pd, \text{ class } 2 \rangle \quad (6.47)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa, pb, pc, pd, \text{ class } 2 \rangle \quad (6.48)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa - ba, pb, pc, pd, \text{ class } 2 \rangle \quad (6.49)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa, pb, pc - ba, pd, \text{ class } 2 \rangle \quad (6.50)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa - ca, pb, pc, pd, \text{ class } 2 \rangle \quad (6.51)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa, pb, pc - ca, pd, \text{ class } 2 \rangle \quad (6.52)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa - ba, pb - ca, pc, pd, \text{ class } 2 \rangle \quad (6.53)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa - ca, pb, pc - ba, pd, \text{ class } 2 \rangle \quad (6.54)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa, pb - ca, pc - ba, pd, \text{ class } 2 \rangle \quad (6.55)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa, pb - \omega ca, pc - ba, pd, \text{ class } 2 \rangle \quad (6.56)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa - ba, pb, pc - ca, pd, \text{ class 2} \rangle \quad (6.57)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa, pb - xba, pc - ca, pd, \text{ class 2} \rangle (x \neq 0) \quad (6.58)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc, pa, pb, pc - ba, pd - ca, \text{ class 2} \rangle \quad (6.59)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc - \omega ba, pa, pb, pc, pd, \text{ class 2} \rangle \quad (6.60)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc - \omega ba, pa - ba, pb, pc, pd, \text{ class 2} \rangle \quad (6.60B)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc - \omega ba, pa, pb, pc - ba, pd - ca, \text{ class 2} \rangle \quad (6.61)$$

$$\langle a, b, c, d \mid cb, da, db - ca, dc - \omega ba, pa, pb - ba, pc - xba - yca, pd, \text{ class 2} \rangle \quad (6.62)$$

Algebra 6.62 has two parameters x, y , with $y \neq 0$. Parameter pairs (x, y) and (z, t) give isomorphic algebras if and only if

$$\begin{pmatrix} 1 & 0 \\ z & t \end{pmatrix} = \begin{pmatrix} \mu & \nu \\ \omega\nu & \mu \end{pmatrix} \begin{pmatrix} 1 & 0 \\ x & y \end{pmatrix} \begin{pmatrix} \mu + \nu x & \nu y \\ \omega\nu y & \mu + \nu x \end{pmatrix}^{-1}$$

for some non-singular matrix $\begin{pmatrix} \mu & \nu \\ \omega\nu & \mu \end{pmatrix}$. There are p distinct algebras.

7.3 Descendants of 4.3

$3p^2 + 13p + 37 + \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid ca, cb, pa, pb, pc, \text{ class 3} \rangle \quad (6.118)$$

$$\langle a, b, c \mid ca, cb, pa - bab, pb, pc, \text{ class 3} \rangle \quad (6.119)$$

$$\langle a, b, c \mid ca, cb, pa - \omega bab, pb, pc, \text{ class 3} \rangle \quad (6.120)$$

$$\langle a, b, c \mid ca, cb, pa - baa, pb, pc, \text{ class 3} \rangle \quad (6.121)$$

$$\langle a, b, c \mid ca, cb, pa - baa, pb - xbab, pc, \text{ class 3} \rangle (x \neq 0, x \sim x^{-1}) \quad (6.122)$$

$$\langle a, b, c \mid ca, cb, pa - baa - bab, pb - bab, pc, \text{ class 3} \rangle \quad (6.123)$$

$$\langle a, b, c \mid ca, cb, pa - baa - \omega bab, pb - bab, pc, \text{ class 3} \rangle \quad (6.124)$$

$$\langle a, b, c \mid ca, cb, pa - \omega bab, pb - baa, pc, \text{ class 3} \rangle \quad (6.125)$$

$$\langle a, b, c \mid ca, cb, pa - xbab, pb - baa - bab, pc, \text{ class 3} \rangle (1 + 4x \text{ not a square}) \quad (6.126)$$

$$\langle a, b, c \mid ca, cb, pa, pb, pc - bab, \text{ class 3} \rangle \quad (6.127)$$

$$\langle a, b, c \mid ca, cb, pa - baa, pb, pc - bab, \text{class } 3 \rangle \quad (6.128)$$

$$\langle a, b, c \mid ca, cb, pa, pb - baa, pc - bab, \text{class } 3 \rangle \quad (6.129)$$

$$\langle a, b, c \mid ca, cb, pa, pb - \omega baa, pc - bab, \text{class } 3 \rangle \quad (6.130)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb, pc, \text{class } 3 \rangle \quad (6.131)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb, pc, \text{class } 3 \rangle \quad (6.132)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb, pc, \text{class } 3 \rangle \quad (6.133)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa, pb, pc, \text{class } 3 \rangle \quad (6.134)$$

$$\langle a, b, c \mid ca - bab, cb, pa - xbaa, pb - bab, pc, \text{class } 3 \rangle \quad (6.135)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa - bab, pb - bab, pc, \text{class } 3 \rangle \quad (6.136)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa - \omega bab, pb - bab, pc, \text{class } 3 \rangle \quad (6.137)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa - xbab, pb - baa, pc, \text{class } 3 \rangle \quad (6.138)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - baa, pc, \text{class } 3 \rangle \quad (6.139)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - baa, pc, \text{class } 3 \rangle \quad (6.140)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - baa, pc, \text{class } 3 \rangle \quad (6.141)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa - xbab, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.142)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.143)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.144)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.145)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - xbaa, pc - bab, \text{class } 3 \rangle \text{ (all } x) \quad (6.146)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa, pb, pc - bab, \text{class } 3 \rangle \quad (6.147)$$

$$\langle a, b, c \mid ca - bab, cb, pa - xbab, pb, pc - baa, \text{class } 3 \rangle \text{ (} x = 0, 1, \omega, \omega^2, \omega^3, p = 1 \bmod 4) \quad (6.148)$$

$$\langle a, b, c \mid ca - bab, cb, pa - xbab, pb, pc - baa, \text{class } 3 \rangle \text{ (} x = 0, 1, \omega, p = 3 \bmod 4) \quad (6.148A)$$

$$\langle a, b, c \mid ca - bab, cb, pa - xbab, pb - ybab, pc - baa, \text{class } 3 \rangle \quad (6.149)$$

$$(y = 1, \omega, \omega^2, (x, y) \sim (ax, y) \text{ if } a^3 = 1, p = 1 \pmod{3})$$

$$\langle a, b, c \mid ca - bab, cb, pa - xbab, pb - bab, pc - baa, \text{class } 3 \rangle (p = 2 \pmod{3}) \quad (6.149A)$$

$$\langle a, b, c \mid ca - baa, cb, pa, pb, pc, \text{class } 3 \rangle \quad (6.150)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa, pb, pc, \text{class } 3 \rangle \quad (6.151)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb, pc, \text{class } 3 \rangle \quad (6.152)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb, pc, \text{class } 3 \rangle \quad (6.153)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb, pc, \text{class } 3 \rangle \quad (6.154)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb, pc, \text{class } 3 \rangle \quad (6.155)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb - baa, pc, \text{class } 3 \rangle (p = 1 \pmod{4}) \quad (6.156)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb - \omega baa, pc, \text{class } 3 \rangle (p = 1 \pmod{4}) \quad (6.157)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb - \omega baa, pc, \text{class } 3 \rangle (p = 1 \pmod{4}) \quad (6.158)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb - baa, pc, \text{class } 3 \rangle (p = 3 \pmod{4}) \quad (6.156A)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb - \omega baa, pc, \text{class } 3 \rangle (p = 3 \pmod{4}) \quad (6.157A)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb - baa, pc, \text{class } 3 \rangle (p = 3 \pmod{4}) \quad (6.158A)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - xbab, pb - baa, pc, \text{class } 3 \rangle \quad (6.159)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - xbab, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.160)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa, pb - xbab, pc, \text{class } 3 \rangle (x \neq 0, x \sim x^{-1}) \quad (6.160A)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb - xbab, pc, \text{class } 3 \rangle (x \neq 0) \quad (6.161)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb - xbab, pc, \text{class } 3 \rangle (x \neq 0) \quad (6.162)$$

In presentations 6.163 and 6.164 below we give a complete set of representatives for the isomorphism classes of algebras

$$\langle a, b, c \mid ca - baa, cb, pa - zbaa - tbab, pb - xbaa - ybab, pc, \text{class } 3 \rangle$$

where $x, y, z, t \neq 0$, $y \neq \pm z$. Here (x, y, z, t) and $(-t, z, y, -x)$ give isomorphic algebras, as do (x, y, z, t) and (a^2x, aby, abz, b^2t) for all $a, b \neq 0$. So we can take $z = 1$ and $y = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}$. With y, z fixed we can take $t = 1, \omega$.

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb - xbaa - ybab, pc, \text{ class } 3 \rangle (x \neq 0, y = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (6.163)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb - xbaa - ybab, pc, \text{ class } 3 \rangle (x \neq 0, y = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (6.164)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb - x^2baa + bab, pc, \text{ class } 3 \rangle (x \neq 0, x \sim -x) \quad (6.165)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb - x^2baa - bab, pc, \text{ class } 3 \rangle (x \neq 0, x \sim -x) \quad (6.165)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb - x^2\omega baa + bab, pc, \text{ class } 3 \rangle (x \neq 0, x \sim -x) \quad (6.166)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb - x^2\omega baa - bab, pc, \text{ class } 3 \rangle (x \neq 0, x \sim -x) \quad (6.166)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb - x^2\omega baa + bab, pc, \text{ class } 3 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (6.167)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb - x^2\omega baa - bab, pc, \text{ class } 3 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (6.167)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb - x^2baa + bab, pc, \text{ class } 3 \rangle (x \neq 0, x \sim -x, p = 3 \bmod 4) \quad (6.167A)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb - x^2baa - bab, pc, \text{ class } 3 \rangle (x \neq 0, x \sim -x, p = 3 \bmod 4) \quad (6.167A)$$

$$\langle a, b, c \mid ca - baa, cb, pa, pb, pc - baa, \text{ class } 3 \rangle \quad (6.168)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb, pc - baa, \text{ class } 3 \rangle \quad (6.169)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb, pc - baa, \text{ class } 3 \rangle \quad (6.170)$$

$$\langle a, b, c \mid ca - baa, cb, pa - xbab, pb - bab, pc - baa, \text{ class } 3 \rangle \quad (6.171)$$

$$\langle a, b, c \mid ca - baa, cb, pa - xbab, pb - ybab, pc - baa - bab, \text{ class } 3 \rangle ((x, y) \sim (y, x)) \quad (6.172)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa, pb, pc, \text{ class } 3 \rangle \quad (6.173)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - xbaa - bab, pb, pc, \text{ class } 3 \rangle (x \sim -x) \quad (6.174)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - xbaa - \omega bab, pb, pc, \text{ class } 3 \rangle (x \sim -x) \quad (6.175)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa, pb, pc, \text{ class } 3 \rangle \quad (6.176)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \omega baa, pb, pc, \text{ class } 3 \rangle (p = 1 \bmod 4) \quad (6.177)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - tbaa - xbab, pb - ybaa - zbab, pc, \text{class } 3 \rangle \quad (6.178)$$

Algebra 6.178 has four parameters x, y, z, t where $tz - xy \neq 0 \pmod{p}$. If we write the parameters in a matrix $A = \begin{pmatrix} t & x \\ y & z \end{pmatrix}$, then two such parameter matrices A and B define isomorphic algebras if and only if

$$B = \frac{1}{\det P} PAP^{-1} \pmod{p}$$

for some matrix P of the form

$$\begin{pmatrix} \alpha & \beta \\ \omega\beta & \alpha \end{pmatrix} \text{ or } \begin{pmatrix} \alpha & \beta \\ -\omega\beta & -\alpha \end{pmatrix}.$$

There are $p^2 + \frac{p-1}{2} - \frac{\gcd(p-1,4)}{2}$ orbits of non-singular matrices A under this action.

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - xbaa, pb - ybaa, pc - bab, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (6.179)$$

7.4 Descendants of 4.6

2 algebras

$$\langle a, b \mid ba, \text{class } 3 \rangle \quad (6.366)$$

$$\langle a, b \mid ba - p^2a, \text{class } 3 \rangle \quad (6.367)$$

7.5 Descendants of 4.7

$p + 15$ algebras

$$\langle a, b \mid p^2a, pb, \text{class } 3 \rangle \quad (6.368)$$

$$\langle a, b \mid p^2a, pb - bab, \text{class } 3 \rangle \quad (6.369)$$

$$\langle a, b \mid p^2a, pb - baa, \text{class } 3 \rangle \quad (6.370)$$

$$\langle a, b \mid p^2a, pb - \omega baa, \text{class } 3 \rangle \quad (6.371)$$

$$\langle a, b \mid p^2a - bab, pb, \text{class } 3 \rangle \quad (6.372)$$

$$\langle a, b \mid p^2a - bab, pb - baa, \text{class } 3 \rangle \quad (6.373)$$

$$\langle a, b \mid p^2a - bab, pb - \omega baa, \text{class } 3 \rangle \quad (6.374)$$

$$\langle a, b \mid p^2a - \omega bab, pb, \text{class } 3 \rangle \quad (6.375)$$

$$\langle a, b \mid p^2a - \omega bab, pb - baa, \text{class } 3 \rangle \quad (6.376)$$

$$\langle a, b \mid p^2a - \omega bab, pb - \omega baa, \text{class } 3 \rangle \quad (6.377)$$

$$\langle a, b \mid p^2a - baa, pb - xbab, \text{class } 3 \rangle \quad (6.378)$$

$$\langle a, b \mid baa, pb, \text{class } 3 \rangle \quad (6.379)$$

$$\langle a, b \mid baa, pb - bab, \text{class } 3 \rangle \quad (6.380)$$

$$\langle a, b \mid bab, pb, \text{class } 3 \rangle \quad (6.381)$$

$$\langle a, b \mid bab, pb - baa, \text{class } 3 \rangle \quad (6.382)$$

$$\langle a, b \mid bab, pb - \omega baa, \text{class } 3 \rangle \quad (6.383)$$

7.6 Descendants of 4.8

1 algebra

$$\langle a, b \mid pb - ba, \text{class } 3 \rangle \quad (6.384)$$

7.7 Descendants of 5.1

7 algebras

$$\langle a, b, c, d, e \mid ba, ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{class } 2 \rangle \quad (6.2)$$

$$\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pc, pd, pe, \text{class } 2 \rangle \quad (6.3)$$

$$\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pa - ba, pb, pc, pd, pe, \text{class } 2 \rangle \quad (6.4)$$

$$\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pc - ba, pd, pe, \text{class } 2 \rangle \quad (6.5)$$

$$\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pa, pb, pc, pd, pe, \text{class } 2 \rangle \quad (6.6)$$

$$\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pa - ba, pb, pc, pd, pe, \text{class } 2 \rangle \quad (6.7)$$

$$\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pa, pb, pc, pd, pe - ba, \text{class } 2 \rangle \quad (6.8)$$

7.8 Descendants of 5.2

4 algebras

$$\langle a, b, c, d \mid ba, ca, da, cb, db, dc, pb, pc, pd, \text{class } 3 \rangle \quad (6.63)$$

$$\langle a, b, c, d \mid ba - p^2a, ca, da, cb, db, dc, pb, pc, pd, \text{class } 3 \rangle \quad (6.64)$$

$$\langle a, b, c, d \mid ba, ca, da, cb - p^2a, db, dc, pb, pc, pd, \text{class } 3 \rangle \quad (6.65)$$

$$\langle a, b, c, d \mid ba, ca, da - p^2a, cb - p^2a, db, dc, pb, pc, pd, \text{class } 3 \rangle \quad (6.66)$$

7.9 Descendants of 5.3

18 algebras

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 3 \rangle \quad (6.67)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa - baa, pb, pc, pd, \text{ class } 3 \rangle \quad (6.68)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb - baa, pc, pd, \text{ class } 3 \rangle \quad (6.69)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3 \rangle \quad (6.70)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb, pc - baa, pd, \text{ class } 3 \rangle \quad (6.71)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb, pc, pd, \text{ class } 3 \rangle \quad (6.72)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa - baa, pb, pc, pd, \text{ class } 3 \rangle \quad (6.73)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb - baa, pc, pd, \text{ class } 3 \rangle \quad (6.74)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3 \rangle \quad (6.75)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb, pc - baa, pd, \text{ class } 3 \rangle \quad (6.76)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb, pc, pd - baa, \text{ class } 3 \rangle \quad (6.77)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb, pc, pd, \text{ class } 3 \rangle \quad (6.78)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa - baa, pb, pc, pd, \text{ class } 3 \rangle \quad (6.79)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb - baa, pc, pd, \text{ class } 3 \rangle \quad (6.80)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb - \omega baa, pc, pd, \text{ class } 3 \rangle \quad (6.81)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb, pc - baa, pd, \text{ class } 3 \rangle \quad (6.82)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb - baa, pc - baa, pd, \text{ class } 3 \rangle \quad (6.83)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb - \omega baa, pc - baa, pd, \text{ class } 3 \rangle \quad (6.84)$$

7.10 Descendants of 5.8

4 algebras

$$\langle a, b, c \mid ba, ca, cb - p^2a, p^2b, pc, \text{class } 3 \rangle \quad (6.180)$$

$$\langle a, b, c \mid ba, ca - p^2a, cb, p^2b, pc, \text{class } 3 \rangle \quad (6.181)$$

$$\langle a, b, c \mid ba, ca, cb, p^2b, pc, \text{class } 3 \rangle \quad (6.182)$$

$$\langle a, b, c \mid ba - p^2a, ca, cb, p^2b, pc, \text{class } 3 \rangle \quad (6.183)$$

7.11 Descendants of 5.9

23 algebras

$$\langle a, b, c \mid baa, bab, ca, cb, pb, pc, \text{class } 3 \rangle \quad (6.184)$$

$$\langle a, b, c \mid baa, bab, ca - p^2a, cb, pb, pc, \text{class } 3 \rangle \quad (6.185)$$

$$\langle a, b, c \mid baa, bab, ca, cb - p^2a, pb, pc, \text{class } 3 \rangle \quad (6.186)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb, pc, \text{class } 3 \rangle \quad (6.187)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb, pc, \text{class } 3 \rangle \quad (6.188)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - baa, pc, \text{class } 3 \rangle \quad (6.189)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - baa, pc, \text{class } 3 \rangle \quad (6.190)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.191)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.192)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb, pc - baa, \text{class } 3 \rangle \quad (6.193)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb, pc - baa, \text{class } 3 \rangle \quad (6.194)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a - baa, pb, pc, \text{class } 3 \rangle \quad (6.195)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a - baa, pb, pc, \text{class } 3 \rangle \quad (6.196)$$

$$\langle a, b, c \mid baa, ca, cb, p^2a, pb, pc, \text{class } 3 \rangle \quad (6.197)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a, pb, pc, \text{class } 3 \rangle \quad (6.198)$$

$$\langle a, b, c \mid baa, ca, cb, p^2a, pb - bab, pc, \text{class } 3 \rangle \quad (6.199)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a, pb - bab, pc, \text{class } 3 \rangle \quad (6.200)$$

$$\langle a, b, c \mid baa, ca, cb, p^2a, pb, pc - bab, \text{class } 3 \rangle \quad (6.201)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a, pb, pc - bab, \text{class } 3 \rangle \quad (6.202)$$

$$\langle a, b, c \mid baa, ca, cb, p^2a - bab, pb, pc, \text{class } 3 \rangle \quad (6.203)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a - bab, pb, pc, \text{class } 3 \rangle \quad (6.204)$$

$$\langle a, b, c \mid baa, ca, cb, p^2a - \omega bab, pb, pc, \text{class } 3 \rangle \quad (6.205)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a - \omega bab, pb, pc, \text{class } 3 \rangle \quad (6.206)$$

7.12 Descendants of 5.10

5 algebras

$$\langle a, b, c \mid baa, ca, cb, pb - ba, pc, \text{class } 3 \rangle \quad (6.207)$$

$$\langle a, b, c \mid baa, ca - p^2a, cb, pb - ba, pc, \text{class } 3 \rangle \quad (6.208)$$

$$\langle a, b, c \mid baa, ca, cb - p^2a, pb - ba, pc, \text{class } 3 \rangle \quad (6.209)$$

$$\langle a, b, c \mid ca, cb, p^2a, pb - ba, pc, \text{class } 3 \rangle \quad (6.210)$$

$$\langle a, b, c \mid ca, cb, p^2a, pb - ba, pc - baa, \text{class } 3 \rangle \quad (6.211)$$

7.13 Descendants of 5.11

4 algebras

$$\langle a, b, c \mid ca, cb, pb, pc - ba, \text{class } 3 \rangle \quad (6.212)$$

$$\langle a, b, c \mid ca, cb - p^2a, pb, pc - ba, \text{class } 3 \rangle \quad (6.213)$$

$$\langle a, b, c \mid ca, cb - \omega p^2a, pb, pc - ba, \text{class } 3 \rangle \quad (6.214)$$

$$\langle a, b, c \mid ca - p^2a, cb, pb, pc - ba, \text{class } 3 \rangle \quad (6.215)$$

7.14 Descendants of 5.12

12 algebras

$$\langle a, b, c \mid baa, bab, ca, cb, pa, pb, \text{class } 3 \rangle \quad (6.216)$$

$$\langle a, b, c \mid baa, bab, ca - p^2c, cb, pa, pb, \text{class } 3 \rangle \quad (6.217)$$

$$\langle a, b, c \mid bab, ca, cb, pa, pb, p^2c, \text{class } 3 \rangle \quad (6.218)$$

$$\langle a, b, c \mid bab, ca, cb, pa - baa, pb, p^2c, \text{class } 3 \rangle \quad (6.219)$$

$$\langle a, b, c \mid bab, ca, cb, pa, pb - baa, p^2c, \text{class } 3 \rangle \quad (6.220)$$

$$\langle a, b, c \mid bab, ca, cb, pa, pb - \omega baa, p^2c, \text{class } 3 \rangle \quad (6.221)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, p^2c, \text{class } 3 \rangle \quad (6.222)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa - baa, pb, p^2c, \text{class } 3 \rangle \quad (6.223)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - baa, p^2c, \text{class } 3 \rangle \quad (6.224)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega baa, p^2c, \text{class } 3 \rangle \quad (6.225)$$

$$\langle a, b, c \mid bab, ca, cb, pa, pb, p^2c - baa, \text{class } 3 \rangle \quad (6.226)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, p^2c - baa, \text{class } 3 \rangle \quad (6.227)$$

7.15 Descendants of 5.13

$p + 1$ algebras

$$\langle a, b, c \mid ca, cb, pa - ba, pb, \text{class } 3 \rangle \quad (6.228)$$

$$\langle a, b, c \mid ca - p^2c, cb, pa - ba, pb, \text{class } 3 \rangle \quad (6.229)$$

$$\langle a, b, c \mid ca, cb - xp^2c, pa - ba, pb, \text{class } 3 \rangle \quad (x \neq 0) \quad (6.230)$$

7.16 Descendants of 5.14

35 algebras

- $$\langle a, b, c \mid cb, baa, bac, caa, cac, pa, pb, pc, \text{class } 3 \rangle \quad (6.231)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac, pa, pb - bab, pc, \text{class } 3 \rangle \quad (6.232)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - bab, pb, pc, \text{class } 3 \rangle \quad (6.233)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - bab, pb - bab, pc, \text{class } 3 \rangle \quad (6.234)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - \omega bab, pb, pc, \text{class } 3 \rangle \quad (6.235)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - \omega bab, pb - bab, pc, \text{class } 3 \rangle \quad (6.236)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac, pa, pb, pc - bab, \text{class } 3 \rangle \quad (6.237)$$
- $$\langle a, b, c \mid cb, baa, bac, caa - bab, cac, pa, pb, pc, \text{class } 3 \rangle \quad (6.238)$$
- $$\langle a, b, c \mid cb, baa, bac, caa - bab, cac, pa - bab, pb, pc, \text{class } 3 \rangle \quad (6.239)$$
- $$\langle a, b, c \mid cb, baa, bac, caa - bab, cac, pa - \omega bab, pb, pc, \text{class } 3 \rangle \quad (6.240)$$
- $$\langle a, b, c \mid cb, baa, bac, caa - bab, cac, pa, pb - bab, pc, \text{class } 3 \rangle \quad (6.241)$$
- $$\langle a, b, c \mid cb, baa, bac, caa - bab, cac, pa, pb, pc - bab, \text{class } 3 \rangle \quad (6.242)$$
- $$\langle a, b, c \mid cb, baa, bac, caa - bab, cac, pa, pb, pc - \omega bab, \text{class } 3 \rangle \quad (6.243)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac + bab, pa, pb, pc, \text{class } 3 \rangle \quad (6.244)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac + bab, pa - bab, pb, pc, \text{class } 3 \rangle \quad (6.245)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac + bab, pa, pb - bab, pc, \text{class } 3 \rangle \quad (6.246)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac + bab, pa - bab, pb - bab, pc, \text{class } 3 \rangle \quad (6.247)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac + bab, pa - \omega bab, pb - bab, pc, \text{class } 3 \rangle \quad (6.248)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac + bab, pa, pb - bab, pc - bab, \text{class } 3 \rangle \quad (6.249)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac + bab, pa - bab, pb - bab, pc - bab, \text{class } 3 \rangle \quad (6.250)$$
- $$\langle a, b, c \mid cb, baa, bac, caa, cac + \omega bab, pa, pb, pc, \text{class } 3 \rangle \quad (6.251)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac + \omega bab, pa - bab, pb, pc, \text{class } 3 \rangle \quad (6.252)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac + \omega bab, pa, pb - bab, pc, \text{class } 3 \rangle \quad (6.253)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac + \omega bab, pa - bab, pb - bab, pc, \text{class } 3 \rangle \quad (6.254)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac + \omega bab, pa - \omega bab, pb - bab, pc, \text{class } 3 \rangle \quad (6.255)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb, pc, \text{class } 3 \rangle \quad (6.256)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac, pa - baa, pb, pc, \text{class } 3 \rangle \quad (6.257)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb - baa, pc, \text{class } 3 \rangle \quad (6.258)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.259)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb, pc - baa, \text{class } 3 \rangle \quad (6.260)$$

$$\langle a, b, c \mid cb - baa, bab, bac, caa, cac, pa, pb, pc, \text{class } 3 \rangle \quad (6.261)$$

$$\langle a, b, c \mid cb - baa, bab, bac, caa, cac, pa - baa, pb, pc, \text{class } 3 \rangle \quad (6.262)$$

$$\langle a, b, c \mid cb - baa, bab, bac, caa, cac, pa, pb - baa, pc, \text{class } 3 \rangle \quad (6.263)$$

$$\langle a, b, c \mid cb - baa, bab, bac, caa, cac, pa, pb - \omega baa, pc, \text{class } 3 \rangle \quad (6.264)$$

$$\langle a, b, c \mid cb - baa, bab, bac, caa, cac, pa, pb, pc - baa, \text{class } 3 \rangle \quad (6.265)$$

7.17 Descendants of 5.15

$2p + 13$ algebras

$$\langle a, b, c \mid cb, caa, pa - ba, pb - cac, pc, \text{class } 3 \rangle \quad (6.266)$$

$$\langle a, b, c \mid cb, caa, pa - ba, pb, pc, \text{class } 3 \rangle \quad (6.267)$$

$$\langle a, b, c \mid cb, caa, pa - ba, pb, pc - cac, \text{class } 3 \rangle \quad (6.268)$$

$$\langle a, b, c \mid cb, caa, pa - ba - cac, pb, pc, \text{class } 3 \rangle \quad (6.269)$$

$$\langle a, b, c \mid cb, caa, pa - ba - cac, pb, pc - cac, \text{class } 3 \rangle \quad (6.270)$$

$$\langle a, b, c \mid cb, caa, pa - ba - \omega cac, pb, pc, \text{class } 3 \rangle \quad (6.271)$$

$$\langle a, b, c \mid cb, caa, pa - ba - \omega cac, pb, pc - cac, \text{class } 3 \rangle \quad (6.272)$$

$$\langle a, b, c \mid cb, cac, pa - ba, pb - caa, pc, \text{class } 3 \rangle \quad (6.273)$$

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc, \text{class } 3 \rangle \quad (6.274)$$

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc - caa, \text{class } 3 \rangle \quad (6.275)$$

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc - \omega caa, \text{class } 3 \rangle \quad (6.276)$$

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb - caa, pc, \text{class } 3 \rangle \quad (6.277)$$

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb, pc - xcaa, \text{class } 3 \rangle \quad (6.278)$$

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb - caa, pc, \text{class } 3 \rangle \quad (6.279)$$

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb, pc - xcaa, \text{class } 3 \rangle \quad (6.280)$$

7.18 Descendants of 5.16

$4p + 8$ algebras

$$\langle a, b, c \mid cb, caa, pa, pb - ba, pc, \text{class } 3 \rangle \quad (6.281)$$

$$\langle a, b, c \mid cb, caa, pa, pb - ba - cac, pc, \text{class } 3 \rangle \quad (6.282)$$

$$\langle a, b, c \mid cb, caa, pa - cac, pb - ba, pc, \text{class } 3 \rangle \quad (6.283)$$

$$\langle a, b, c \mid cb, caa, pa - cac, pb - ba - cac, pc, \text{class } 3 \rangle \quad (6.284)$$

$$\langle a, b, c \mid cb, caa, pa - \omega cac, pb - ba, pc, \text{class } 3 \rangle \quad (6.285)$$

$$\langle a, b, c \mid cb, caa, pa - \omega cac, pb - ba - cac, pc, \text{class } 3 \rangle \quad (6.286)$$

$$\langle a, b, c \mid cb, caa, pa - xcac, pb - ba, pc - cac, \text{class } 3 \rangle \quad (6.287)$$

$$\langle a, b, c \mid cb, caa, pa - xcac, pb - ba - cac, pc - cac, \text{class } 3 \rangle \quad (6.288)$$

$$\langle a, b, c \mid cb, cac, pa, pb - ba, pc - xcaa, \text{class } 3 \rangle \quad (6.289)$$

$$\langle a, b, c \mid cb - caa, cac, pa, pb - ba, pc - xcaa, \text{class } 3 \rangle \quad (6.290)$$

$$\langle a, b, c \mid cb, cac, pa - caa, pb - ba, pc, \text{class } 3 \rangle \quad (6.291)$$

$$\langle a, b, c \mid cb - caa, cac, pa - caa, pb - ba, pc, \text{class } 3 \rangle \quad (6.292)$$

7.19 Descendants of 5.18

$2p + 13 + 3 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid cb, baa, pa, pb - ca, pc - bab, \text{class } 3 \rangle \quad (6.293)$$

$$\langle a, b, c \mid cb, baa, pa, pb - ca, pc, \text{class } 3 \rangle \quad (6.294)$$

$$\langle a, b, c \mid cb, baa, pa - bab, pb - ca, pc, \text{class } 3 \rangle \quad (6.295)$$

$$\langle a, b, c \mid cb, baa, pa - \omega bab, pb - ca, pc, \text{class } 3 \rangle \quad (6.296)$$

$$\langle a, b, c \mid cb, baa, pa, pb - ca - bab, pc, \text{class } 3 \rangle \quad (6.297)$$

$$\langle a, b, c \mid cb, baa, pa - bab, pb - ca - bab, pc, \text{class } 3 \rangle \quad (6.298)$$

$$\langle a, b, c \mid cb, baa, pa - \omega bab, pb - ca - bab, pc, \text{class } 3 \rangle \quad (6.299)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc - baa, \text{class } 3 \rangle \quad (6.300)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc - \omega baa, \text{class } 3 \rangle \ (p = 1 \bmod 3) \quad (6.301)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle \ (p = 1 \bmod 3) \quad (6.302)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc, \text{class } 3 \rangle \quad (6.303)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca - baa, pc, \text{class } 3 \rangle \quad (6.304)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca - \omega baa, pc, \text{class } 3 \rangle \quad (6.305)$$

$$\langle a, b, c \mid cb, bab - baa, pa - baa, pb - ca - xbaa, pc, \text{class } 3 \rangle \quad (6.305B)$$

$$\langle a, b, c \mid cb, bab - baa, pa - \omega baa, pb - ca - xbaa, pc, \text{class } 3 \rangle \quad (6.305C)$$

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc - baa, \text{class } 3 \rangle \quad (6.306)$$

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc - \omega baa, \text{class } 3 \rangle \ (p = 1 \bmod 3) \quad (6.307)$$

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle \ (p = 1 \bmod 3) \quad (6.308)$$

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc - baa, \text{class } 3 \rangle \quad (6.309)$$

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc - \omega baa, \text{class } 3 \rangle \ (p = 1 \bmod 3) \quad (6.310)$$

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle \ (p = 1 \bmod 3) \quad (6.311)$$

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc, \text{class } 3 \rangle \quad (6.312)$$

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc, \text{class } 3 \rangle \quad (6.313)$$

$$\langle a, b, c \mid cb, bab, pa - baa, pb - ca, pc, \text{class } 3 \rangle \quad (6.314)$$

$$\langle a, b, c \mid cb - baa, bab, pa - baa, pb - ca, pc, \text{class } 3 \rangle \quad (6.315)$$

$$\langle a, b, c \mid cb - \omega baa, bab, pa - baa, pb - ca, pc, \text{class } 3 \rangle \quad (6.316)$$

$$\langle a, b, c \mid cb - \omega^2 baa, bab, pa - baa, pb - ca, pc, \text{class } 3 \rangle \quad (p = 1 \pmod{4}) \quad (6.317)$$

$$\langle a, b, c \mid cb - \omega^3 baa, bab, pa - baa, pb - ca, pc, \text{class } 3 \rangle \quad (p = 1 \pmod{4}) \quad (6.318)$$

7.20 Descendants of 5.19

3 algebras

$$\langle a, b, c \mid cb, pa - ba, pb - ca, pc, \text{class } 3 \rangle \quad (6.319)$$

$$\langle a, b, c \mid cb - bab, pa - ba, pb - ca, pc, \text{class } 3 \rangle \quad (6.320)$$

$$\langle a, b, c \mid cb - \omega bab, pa - ba, pb - ca, pc, \text{class } 3 \rangle \quad (6.321)$$

7.21 Descendants of 5.24

3 algebras

$$\langle a, b, c \mid ba, ca, cb, pb, pc, \text{class } 4 \rangle \quad (6.322)$$

$$\langle a, b, c \mid ba - p^3 a, ca, cb, pb, pc, \text{class } 4 \rangle \quad (6.323)$$

$$\langle a, b, c \mid ba, ca, cb - p^3 a, pb, pc, \text{class } 4 \rangle \quad (6.324)$$

7.22 Descendants of 5.27

$11 + 4 \gcd(p - 1, 3) + 2 \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid bab, ca, cb, pa, pb, pc, \text{class } 4 \rangle \quad (6.325)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, pa, pb, pc, \text{class } 4 \rangle \quad (6.326)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, pa, pb, pc, \text{class } 4 \rangle \quad (6.327)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, pa, pb, pc, \text{class } 4 \rangle \quad (6.328)$$

$$\langle a, b, c \mid bab, ca, cb, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (6.329)$$

$$\langle a, b, c | bab - baaa, ca, cb, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (6.330)$$

$$\langle a, b, c | bab - \omega baaa, ca, cb, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (6.331)$$

$$\langle a, b, c | bab - \omega^2 baaa, ca, cb, pa - baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (6.332)$$

$$\langle a, b, c | bab - \omega^3 baaa, ca, cb, pa - baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (6.333)$$

$$\langle a, b, c | bab, ca, cb - baaa, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (6.334)$$

$$\langle a, b, c | bab - baaa, ca, cb - baaa, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (6.335)$$

$$\langle a, b, c | bab - \omega baaa, ca, cb - baaa, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (6.336)$$

$$\langle a, b, c | bab - \omega^2 baaa, ca, cb - baaa, pa - baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (6.337)$$

$$\langle a, b, c | bab - \omega^3 baaa, ca, cb - baaa, pa - baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (6.338)$$

$$\langle a, b, c | bab, ca, cb, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (6.339)$$

$$\langle a, b, c | bab - baaa, ca, cb, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (6.340)$$

$$\langle a, b, c | bab, ca, cb - baaa, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (6.341)$$

$$\langle a, b, c | bab - baaa, ca, cb - baaa, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (6.342)$$

$$\langle a, b, c | bab, ca, cb, pa, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.343)$$

$$\langle a, b, c | bab - baaa, ca, cb, pa, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.344)$$

$$\langle a, b, c | bab, ca, cb - baaa, pa, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.345)$$

$$\langle a, b, c | bab - baaa, ca, cb - baaa, pa, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.346)$$

$$\langle a, b, c | bab, ca, cb, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.347)$$

$$\langle a, b, c | bab - baaa, ca, cb, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.348)$$

$$\langle a, b, c | bab, ca, cb - baaa, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.349)$$

$$\langle a, b, c | bab - baaa, ca, cb - baaa, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.350)$$

$$\langle a, b, c | bab, ca, cb, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (6.351)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (6.352)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (6.353)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (6.354)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - \omega baaa, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (6.355)$$

7.23 Descendants of 5.32

$4 + 2 \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (6.356)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.357)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.358)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - baaa, pc - baaa, \text{class } 4 \rangle \quad (6.359)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega baaa, pc - baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.360)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega^2 baaa, pc - baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.361)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, pc, \text{class } 4 \rangle \quad (6.362)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (6.363)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (6.364)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa - baaa, pb, pc - baaa, \text{class } 4 \rangle \quad (6.365)$$

7.24 Descendants of 5.37

$p + 8$ algebras

$$\langle a, b \mid baa, bab, p^2a, p^2b, \text{class } 3 \rangle \quad (6.385)$$

$$\langle a, b \mid baa, bab, pba, p^2b, \text{class } 3 \rangle \quad (6.386)$$

$$\langle a, b \mid baa, bab, p^2a - pba, p^2b, \text{class } 3 \rangle \quad (6.387)$$

$$\langle a, b \mid bab, pba, p^2a, p^2b, \text{class } 3 \rangle \quad (6.388)$$

$$\langle a, b \mid bab, pba, p^2a - baa, p^2b, \text{class } 3 \rangle \quad (6.389)$$

$$\langle a, b \mid bab, pba, p^2a, p^2b - baa, \text{class } 3 \rangle \quad (6.390)$$

$$\langle a, b \mid bab, pba, p^2a, p^2b - \omega baa, \text{class } 3 \rangle \quad (6.391)$$

$$\langle a, b \mid bab, pba - baa, p^2a, p^2b - xbaa, \text{class } 3 \rangle \quad (6.392)$$

$$\langle a, b \mid bab, pba - baa, p^2a - baa, p^2b, \text{class } 3 \rangle \quad (6.393)$$

7.25 Descendants of 5.38

$5 + 3 \gcd(p - 1, 3)$ algebras

$$\langle a, b \mid baab, babb, pa, pb, \text{class } 4 \rangle \quad (6.394)$$

$$\langle a, b \mid baab, babb, pa - baaa, pb, \text{class } 4 \rangle \quad (6.395)$$

$$\langle a, b \mid baab, babb, pa, pb - baaa, \text{class } 4 \rangle \quad (6.396)$$

$$\langle a, b \mid baab, babb, pa, pb - \omega baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.397)$$

$$\langle a, b \mid baab, babb, pa, pb - \omega^2 baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.398)$$

$$\langle a, b \mid baab, babb + baaa, pa, pb, \text{class } 4 \rangle \quad (6.399)$$

$$\langle a, b \mid baab, babb + baaa, pa - baaa, pb, \text{class } 4 \rangle \quad (6.400)$$

$$\langle a, b \mid baab, babb + baaa, pa - \omega baaa, pb, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.401)$$

$$\langle a, b \mid baab, babb + baaa, pa - \omega^2 baaa, pb, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.402)$$

$$\langle a, b \mid baab, babb + baaa, pa - baaa, pb - baaa, \text{class } 4 \rangle \quad (6.403)$$

$$\langle a, b \mid baab, babb + \omega baaa, pa, pb, \text{class } 4 \rangle \quad (6.404)$$

$$\langle a, b \mid baab, babb + \omega baaa, pa - baaa, pb, \text{class } 4 \rangle \quad (6.405)$$

$$\langle a, b \mid baab, babb + \omega baaa, pa - \omega baaa, pb, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.406)$$

$$\langle a, b \mid baab, babb + \omega baaa, pa - \omega^2 baaa, pb, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.407)$$

7.26 Descendants of 5.39

$1 + \gcd(p-1, 3) + (1/2) \gcd(p-1, 4)$ algebras

$$\langle a, b \mid pa - bab, pb - baaa, \text{class } 4 \rangle \quad (6.408)$$

$$\langle a, b \mid pa - bab, pb - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (6.409)$$

$$\langle a, b \mid pa - bab, pb - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (6.410)$$

$$\langle a, b \mid pa - bab, pb, \text{class } 4 \rangle \quad (6.411)$$

$$\langle a, b \mid pa - bab - baaa, pb, \text{class } 4 \rangle \quad (6.412)$$

$$\langle a, b \mid pa - bab - \omega baaa, pb, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (6.413)$$

7.27 Descendants of 5.40

$1 + \gcd(p-1, 3) + (1/2) \gcd(p-1, 4)$ algebras

$$\langle a, b \mid pa - \omega bab, pb - baaa, \text{class } 4 \rangle \quad (6.414)$$

$$\langle a, b \mid pa - \omega bab, pb - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (6.415)$$

$$\langle a, b \mid pa - \omega bab, pb - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (6.416)$$

$$\langle a, b \mid pa - \omega bab, pb, \text{class } 4 \rangle \quad (6.417)$$

$$\langle a, b \mid pa - \omega bab - baaa, pb, \text{class } 4 \rangle \quad (6.418)$$

$$\langle a, b \mid pa - \omega bab - \omega baaa, pb, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (6.419)$$

7.28 Descendants of 5.41

$p + 1 + \gcd(p-1, 3)$ algebras

$$\langle a, b \mid pa - baa, pb, \text{class } 4 \rangle \quad (6.420)$$

$$\langle a, b \mid pa - baa - babb, pb, \text{class } 4 \rangle \quad (6.421)$$

$$\langle a, b \mid pa - baa - \omega babb, pb, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (6.422)$$

$$\langle a, b \mid pa - baa - \omega^2 babb, pb, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (6.423)$$

$$\langle a, b \mid pa - baa - xbabb, pb - babb, \text{class } 4 \rangle \quad (6.424)$$

7.29 Descendants of 5.42

$p + 1$ algebras

$$\langle a, b \mid pa - baa, pb + bab, \text{class } 4 \rangle \quad (6.425)$$

$$\langle a, b \mid pa - baa - baab, pb + bab - xbaab, \text{class } 4 \rangle \quad (6.426)$$

7.30 Descendants of 5.45

Here we have a two parameter family of p algebras, with the isomorphism type depending on the value of $y^2 - \omega x^2 \pmod{p}$.

$$\langle a, b \mid pa + bab - xbaaa, pb + \omega baa - ybaaa, \text{class } 4 \rangle \quad (6.427)$$

7.31 Descendants of 5.47

2 algebras

$$\langle a, b \mid ba, p^2b, \text{class } 4 \rangle \quad (6.428)$$

$$\langle a, b \mid ba - p^3a, p^2b, \text{class } 4 \rangle \quad (6.429)$$

7.32 Descendants of 5.48

1 algebra

$$\langle a, b \mid ba - p^2a, p^2b, \text{class } 4 \rangle \quad (6.430)$$

7.33 Descendants of 5.49

4 algebras

$$\langle a, b \mid baa, bab, pb, \text{class } 4 \rangle \quad (6.431)$$

$$\langle a, b \mid baa - p^3a, bab, pb, \text{class } 4 \rangle \quad (6.432)$$

$$\langle a, b \mid baa, bab - p^3a, pb, \text{class } 4 \rangle \quad (6.433)$$

$$\langle a, b \mid baa, bab - \omega p^3a, pb, \text{class } 4 \rangle \quad (6.434)$$

7.34 Descendants of 5.50

$3 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid bab, p^2a, pb, \text{class } 4 \rangle \quad (6.435)$$

$$\langle a, b \mid bab - baaa, p^2a, pb, \text{class } 4 \rangle \quad (6.436)$$

$$\langle a, b \mid bab, p^2a - baaa, pb, \text{class } 4 \rangle \quad (6.437)$$

$$\langle a, b \mid bab - baaa, p^2a - baaa, pb, \text{class } 4 \rangle \quad (6.438)$$

$$\langle a, b \mid bab - \omega baaa, p^2a - baaa, pb, \text{class } 4 \rangle \quad (6.439)$$

$$\langle a, b \mid bab - \omega^2 baaa, p^2a - baaa, pb, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (6.440)$$

$$\langle a, b \mid bab - \omega^3 baaa, p^2a - baaa, pb, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (6.441)$$

$$\langle a, b \mid bab, p^2a, pb - baaa, \text{class } 4 \rangle \quad (6.442)$$

$$\langle a, b \mid bab, p^2a, pb - \omega baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.443)$$

$$\langle a, b \mid bab, p^2a, pb - \omega^2 baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.444)$$

$$\langle a, b \mid bab - baaa, p^2a, pb - baaa, \text{class } 4 \rangle \quad (6.445)$$

$$\langle a, b \mid bab - baaa, p^2a, pb - \omega baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.446)$$

$$\langle a, b \mid bab - baaa, p^2a, pb - \omega^2 baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.447)$$

7.35 Descendants of 5.51

$3(p+1)/2$ algebras

$$\langle a, b \mid bab, p^2a, pb - baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (6.448)$$

$$\langle a, b \mid bab, p^2a - baaa, pb - baa, \text{class } 4 \rangle \quad (6.449)$$

$$\langle a, b \mid bab - baaa, p^2a - xbaaa, pb - baa, \text{class } 4 \rangle \quad (6.450)$$

7.36 Descendants of 5.52

$3(p+1)/2$ algebras

$$\langle a, b \mid bab, p^2a, pb - \omega baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (6.451)$$

$$\langle a, b \mid bab, p^2a - baaa, pb - \omega baa, \text{class } 4 \rangle \quad (6.452)$$

$$\langle a, b \mid bab - baaa, p^2a - xbaaa, pb - \omega baa, \text{class } 4 \rangle \quad (6.453)$$

7.37 Descendants of 5.54

$3 + 2 \gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b \mid baa, p^2a, pb, \text{class } 4 \rangle \quad (6.454)$$

$$\langle a, b \mid baa, p^2a, pb - babb, \text{class } 4 \rangle \quad (6.455)$$

$$\langle a, b \mid baa, p^2a - babb, pb, \text{class } 4 \rangle \quad (6.456)$$

$$\langle a, b \mid baa, p^2a - \omega babb, pb, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.457)$$

$$\langle a, b \mid baa, p^2a - \omega^2 babb, pb, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.458)$$

$$\langle a, b \mid baa - babb, p^2a, pb, \text{class } 4 \rangle \quad (6.459)$$

$$\langle a, b \mid baa - babb, p^2a, pb - babb, \text{class } 4 \rangle \quad (6.460)$$

$$\langle a, b \mid baa - babb, p^2a, pb - \omega babb, \text{class } 4 \rangle \quad (6.461)$$

$$\langle a, b \mid baa - babb, p^2a, pb - \omega^2 babb, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (6.462)$$

$$\langle a, b \mid baa - babb, p^2a, pb - \omega^3 babb, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (6.463)$$

$$\langle a, b \mid baa - babb, p^2a - babb, pb, \text{class } 4 \rangle \quad (6.464)$$

$$\langle a, b \mid baa - babb, p^2a - \omega babb, pb, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.465)$$

$$\langle a, b \mid baa - babb, p^2a - \omega^2 babb, pb, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (6.466)$$

7.38 Descendants of 5.58

2 algebras

$$\langle a, b \mid baa, pb - ba, \text{class } 4 \rangle \quad (6.467)$$

$$\langle a, b \mid baa - p^3a, pb - ba, \text{class } 4 \rangle \quad (6.468)$$

7.39 Descendants of 5.60

$7 + 2 \gcd(p - 1, 3) + 3 \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid bab, baaab, pa, pb, \text{class } 5 \rangle \quad (6.469)$$

$$\langle a, b \mid bab, baaab, pa - baaaa, pb, \text{class } 5 \rangle \quad (6.470)$$

$$\langle a, b \mid bab, baaab, pa, pb - baaaa, \text{class } 5 \rangle \quad (6.471)$$

$$\langle a, b \mid bab, baaab, pa, pb - \omega baaaa, \text{class } 5 \rangle \quad (6.472)$$

$$\langle a, b \mid bab, baaab, pa, pb - \omega^2 baaaa, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (6.473)$$

$$\langle a, b \mid bab, baaab, pa, pb - \omega^3 baaaa, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (6.474)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb, \text{class } 5 \rangle \quad (6.475)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - baaaa, pb, \text{class } 5 \rangle \quad (6.476)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - \omega baaaa, pb, \text{class } 5 \rangle \quad (6.477)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - \omega^2 baaaa, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{3}) \quad (6.478)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - \omega^3 baaaa, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{3}) \quad (6.479)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - \omega^4 baaaa, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{3}) \quad (6.480)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - \omega^5 baaaa, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{3}) \quad (6.481)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb - baaaa, \text{class } 5 \rangle \quad (6.482)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb - \omega baaaa, \text{class } 5 \rangle \quad (6.483)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb - \omega^2 baaaa, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (6.484)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb - \omega^3 baaaa, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (6.485)$$

$$\langle a, b \mid bab, baaaa, pa, pb, \text{class } 5 \rangle \quad (6.486)$$

$$\langle a, b \mid bab, baaaa, pa - baaab, pb, \text{class } 5 \rangle \quad (6.487)$$

$$\langle a, b \mid bab, baaaa, pa - \omega baaab, pb, \text{class } 5 \rangle \quad (6.488)$$

$$\langle a, b \mid bab, baaaa, pa, pb - baaab, \text{class } 5 \rangle \quad (6.489)$$

$$\langle a, b \mid bab, baaaa, pa - baaab, pb - baaab, \text{class } 5 \rangle \quad (6.490)$$

$$\langle a, b \mid bab, baaaa, pa - \omega baaab, pb - baaab, \text{class } 5 \rangle \quad (6.491)$$

$$\langle a, b \mid bab, baaaa, pa - \omega^2 baaab, pb - baaab, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (6.492)$$

$$\langle a, b \mid bab, baaaa, pa - \omega^3 baaab, pb - baaab, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (6.493)$$

7.40 Descendants of 5.65

$2 \gcd(p-1, 3) + \gcd(p-1, 4) + 2 \gcd(p-1, 5)$ algebras

$$\langle a, b \mid bab - baaa, baaaa, pa, pb, \text{class } 5 \rangle \quad (6.494)$$

$$\langle a, b \mid bab - baaa, baaaa, pa, pb - baaab, \text{class } 5 \rangle \quad (6.495)$$

$$\langle a, b \mid bab - baaa, baaaa, pa, pb - \omega baaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (6.496)$$

$$\langle a, b \mid bab - baaa, baaaa, pa, pb - \omega^2 baaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (6.497)$$

$$\langle a, b \mid bab - baaa, baaaa, pa, pb - \omega^3 baaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (6.498)$$

$$\langle a, b \mid bab - baaa, baaaa, pa, pb - \omega^4 baaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (6.499)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - baaab, pb, \text{class } 5 \rangle \quad (6.500)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - \omega baaab, pb, \text{class } 5 \rangle \quad (6.501)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - \omega^2 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (6.502)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - \omega^3 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (6.503)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - \omega^4 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (6.504)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - \omega^5 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (6.505)$$

For the next 5 algebras, we have $x \neq 0$, $x \sim ax$ if $a^5 = 1$:

$$\langle a, b \mid bab - baaa, baaaa, pa - xbaaab, pb - baaab, \text{class } 5 \rangle \quad (6.506)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - xbaaab, pb - \omega baaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (6.506A)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - xbaaab, pb - \omega^2 baaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (6.506A)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - xbaaab, pb - \omega^3 baaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (6.506A)$$

$$\langle a, b \mid bab - baaa, baaaa, pa - xbaaab, pb - \omega^4baaab, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.506A)$$

$$\langle a, b \mid bab - baaa, baaab, pa, pb, \text{ class } 5 \rangle \quad (6.507)$$

$$\langle a, b \mid bab - baaa, baaab, pa, pb - baaaa, \text{ class } 5 \rangle \quad (6.508)$$

$$\langle a, b \mid bab - baaa, baaab, pa, pb - \omega baaaa, \text{ class } 5 \rangle \quad (6.509)$$

$$\langle a, b \mid bab - baaa, baaab, pa, pb - \omega^2baaaa, \text{ class } 5 \rangle (p = 1 \bmod 4) \quad (6.510)$$

$$\langle a, b \mid bab - baaa, baaab, pa, pb - \omega^3baaaa, \text{ class } 5 \rangle (p = 1 \bmod 4) \quad (6.511)$$

$$\langle a, b \mid bab - baaa, baaab, pa - baaaa, pb, \text{ class } 5 \rangle \quad (6.512)$$

$$\langle a, b \mid bab - baaa, baaab, pa - \omega baaaa, pb, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.513)$$

$$\langle a, b \mid bab - baaa, baaab, pa - \omega^2baaaa, pb, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.514)$$

$$\langle a, b \mid bab - baaa, baaab, pa - \omega^3baaaa, pb, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.515)$$

$$\langle a, b \mid bab - baaa, baaab, pa - \omega^4baaaa, pb, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.516)$$

For the next 5 algebras, we have $x \neq 0$, $x \sim ax$ if $a^5 = 1$:

$$\langle a, b \mid bab - baaa, baaab, pa - baaaa, pb - xbaaaa, \text{ class } 5 \rangle \quad (6.517)$$

$$\langle a, b \mid bab - baaa, baaab, pa - \omega baaaa, pb - xbaaaa, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.517A)$$

$$\langle a, b \mid bab - baaa, baaab, pa - \omega^2baaaa, pb - xbaaaa, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.517A)$$

$$\langle a, b \mid bab - baaa, baaab, pa - \omega^3baaaa, pb - xbaaaa, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.517A)$$

$$\langle a, b \mid bab - baaa, baaab, pa - \omega^4baaaa, pb - xbaaaa, \text{ class } 5 \rangle (p = 1 \bmod 5) \quad (6.517A)$$

7.41 Descendants of 5.73

2 algebras

$$\langle a, b \mid ba, pb, \text{ class } 5 \rangle \quad (6.518)$$

$$\langle a, b \mid ba - p^4a, pb, \text{ class } 5 \rangle \quad (6.519)$$

8 Order p^7

For $p \geq 5$ the number of nilpotent Lie rings of order p^7 is

$$\begin{aligned} & 3p^5 + 12p^4 + 44p^3 + 170p^2 + 707p + 2455 \\ & + (4p^2 + 44p + 291) \gcd(p-1, 3) + (p^2 + 19p + 135) \gcd(p-1, 4) \\ & + (3p + 31) \gcd(p-1, 5) + 4 \gcd(p-1, 7) + 5 \gcd(p-1, 8) + \gcd(p-1, 9). \end{aligned}$$

The following table lists the nilpotent Lie rings of order at most p^5 which have immediate descendants of order p^7 , and gives the number of descendants.

3.1	$p + 14$
4.1	1361 if $p = 3$, otherwise $p^5 + 2p^4 + 7p^3 + 25p^2 + 88p + 270 + (p + 4) \gcd(p-1, 3) + \gcd(p-1, 4)$
4.7	4
5.1	178 if $p = 3$, otherwise $p^2 + 15p + 125$
5.3	$p^4 + 5p^3 + 19p^2 + 64p + 140 + (p + 6) \gcd(p-1, 3) + (p + 7) \gcd(p-1, 4) + \gcd(p-1, 5)$
5.8	$p + 8$
5.9	$4p^2 + 26p + 107 + 5 \gcd(p-1, 3) + (p + 4) \gcd(p-1, 4)$
5.10	$2p + 7$
5.12	$3p^2 + 17p + 53 + \gcd(p-1, 3) + \gcd(p-1, 4)$
5.14	$2p^5 + 7p^4 + 19p^3 + 49p^2 + 128p + 256 + (p^2 + 7p + 29) \gcd(p-1, 3) + (p^2 + 7p + 24) \gcd(p-1, 4) + (p + 3) \gcd(p-1, 5)$
5.15	$3p^2 + 12p + 14 + (p + 2) \gcd(p-1, 4)$
5.16	$p^4 + 2p^3 + 5p^2 + 14p$
5.18	$3p^3 + 6p^2 + 6p + 11 + (p + 7) \gcd(p-1, 3) + (p + 1) \gcd(p-1, 4) + \gcd(p-1, 5)$
5.37	$p^2 + 8p + 25$
5.38	$p + 6 + (p^2 + 3p + 10) \gcd(p-1, 3)$
5.60	$2p^2 + p + 3 + 2(p + 1) \gcd(p-1, 3) + (2p + 4) \gcd(p-1, 4) + \gcd(p-1, 8)$
5.65	$p^3 + p^2 + p - 2 + 2 \gcd(p-1, 3) + \gcd(p-1, 4) + (p + 1) \gcd(p-1, 5)$

In the two tables below we give the list of 42 nilpotent Lie rings of order at most p^5 which have immediate descendants of order p^6 . For each of these Lie rings we give the number of immediate descendants of order p^6 which are capable, and we give the number of “grandchildren” of order p^7 . Note that by “grandchildren” we mean Lie rings whose immediate ancestors have order p^6 .

	capable descendants of order p^6
3.1	$3p + 27$
4.1	24
4.3	$5p + 37 + \gcd(p - 1, 4)$
4.6	2
4.7	16
4.8	1
5.0	1
5.1	2
5.2	1
5.3	2
5.8	2
5.9	9
5.10	1
5.11	2
5.12	3
5.13	1
5.14	3
5.15	$2p + 9$
5.16	4
5.18	11
5.19	0
5.24	1
5.27	4
5.32	1
5.37	2
5.38	3
5.39	$1 + \gcd(p - 1, 3) + \gcd(p - 1, 4)/2$
5.40	$1 + \gcd(p - 1, 3) + \gcd(p - 1, 4)/2$
5.41	$p + 1 + \gcd(p - 1, 3)$
5.42	$p + 1$
5.45	p
5.47	2
5.48	0
5.49	1
5.50	$2 + 2 \gcd(p - 1, 3)$
5.51	$(p + 1)/2$
5.52	$(p + 1)/2$
5.54	$3 + \gcd(p - 1, 4)$
5.58	1
5.60	2
5.65	1
5.73	1

	grandchildren of order p^7
3.1	$2p^2 + 63p + 362 + (p + 19) \gcd(p - 1, 3) + 5 \gcd(p - 1, 4) + \gcd(p - 1, 5)$
4.1	$p^3 + 13p^2 + 96p + 595 + (3p + 21) \gcd(p - 1, 3) + (p + 11) \gcd(p - 1, 4) + \gcd(p - 1, 5)$
4.3	$p^4 + 4p^3 + 17p^2 + 39p + 72 + (p^2 + 9p + 47) \gcd(p - 1, 3)$ $+ (2p + 8) \gcd(p - 1, 4) + 2 \gcd(p - 1, 5) + \gcd(p - 1, 7)$
4.6	4
4.7	$15p + 41 + 16 \gcd(p - 1, 3) + 4 \gcd(p - 1, 4)$
4.8	2
5.0	1
5.1	30
5.2	4
5.3	$35 + (p + 15) \gcd(p - 1, 3) + 4 \gcd(p - 1, 4)$
5.8	6
5.9	$5p + 49 + 11 \gcd(p - 1, 3) + 4 \gcd(p - 1, 4)$
5.10	5
5.11	7
5.12	$2p + 20 + 7 \gcd(p - 1, 3) + 3 \gcd(p - 1, 4)$
5.13	$p + 1$
5.14	$p^2 + 9p + 36 + (p^2 + 5p + 29) \gcd(p - 1, 3) + (p + 7) \gcd(p - 1, 4)$ $+ \gcd(p - 1, 7) + \gcd(p - 1, 8)$
5.15	$10p + 16 + (2p + 7) \gcd(p - 1, 3) + 2 \gcd(p - 1, 4) + 2 \gcd(p - 1, 5)$
5.16	$p^3 + 5p^2 + 13p + 6 + 3 \gcd(p - 1, 3)$
5.18	$2p^2 + 14p + 10 + (2p + 8) \gcd(p - 1, 3) + 7 \gcd(p - 1, 4) + \gcd(p - 1, 5)$
5.19	0
5.24	3
5.27	$p^2 + 10p + 34 + (p + 14) \gcd(p - 1, 3) + 13 \gcd(p - 1, 4) + 6 \gcd(p - 1, 5) + \gcd(p - 1, 7)$
5.32	$p^2 + 7p + 3 + 2 \gcd(p - 1, 3) + 3 \gcd(p - 1, 4) + \gcd(p - 1, 5)$
5.37	$5p + 10 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$
5.38	$p^3 + 3p^2 + 8p + 18 + 5 \gcd(p - 1, 3) + (p + 5) \gcd(p - 1, 4)$ $+ 3 \gcd(p - 1, 5) + 2 \gcd(p - 1, 8) + \gcd(p - 1, 9)$
5.39	$\frac{1}{2}(p^2 + 2p + 1 + (p + 5) \gcd(p - 1, 3) + (p + 3) \gcd(p - 1, 4))$
5.40	$\frac{1}{2}(p^2 + 2p + 1 + (p + 5) \gcd(p - 1, 3) + (p + 3) \gcd(p - 1, 4))$
5.41	$3p^2 + 4p + (p + 1) \gcd(p - 1, 3) + \gcd(p - 1, 4)$
5.42	$p + 3$
5.45	$p + 1$
5.47	3
5.48	0
5.49	4
5.50	$4p + 5 + (p + 7) \gcd(p - 1, 3) + 3 \gcd(p - 1, 4) + 2 \gcd(p - 1, 5)$
5.51	$\frac{1}{2}(p + 1)$
5.52	$\frac{1}{2}(p + 1)$
5.54	$7p + 9 + 4 \gcd(p - 1, 3) + 6 \gcd(p - 1, 4) + 2 \gcd(p - 1, 5)$
5.58	2
5.60	$4p + 3 + 2 \gcd(p - 1, 3) + 4 \gcd(p - 1, 5) + \gcd(p - 1, 7) + \gcd(p - 1, 8)$
5.65	$2p^2 + p + 2p \gcd(p - 1, 3) + p \gcd(p - 1, 5)$
5.73	2

We now give the cyclic and seven generator nilpotent Lie rings of order p^7 , and then list the rest, grouped according to their immediate ancestors.

$$\langle a \mid \text{class } 7 \rangle \tag{7.0}$$

$$\langle a, b, c, d, e, f, g \mid \text{class } 1 \rangle. \tag{7.1}$$

8.1 Descendants of 4.7

4 algebras

$$\langle a, b \mid pb, \text{class } 3 \rangle \quad (7.2)$$

$$\langle a, b \mid pb - baa, \text{class } 3 \rangle \quad (7.3)$$

$$\langle a, b \mid pb - \omega baa, \text{class } 3 \rangle \quad (7.4)$$

$$\langle a, b \mid pb - bab, \text{class } 3 \rangle \quad (7.5)$$

8.2 Descendants of 5.37

$p^2 + 8p + 25$ algebras

$$\langle a, b \mid baa, bab, pba, \text{class } 3 \rangle \quad (7.6)$$

$$\langle a, b \mid baa, bab, p^2b, \text{class } 3 \rangle \quad (7.7)$$

$$\langle a, b \mid baa, bab, p^2b - pba, \text{class } 3 \rangle \quad (7.8)$$

$$\langle a, b \mid bab, p^2a, p^2b - xbaa - pba, \text{class } 3 \rangle \quad (7.9)$$

$$\langle a, b \mid bab, p^2a - baa, p^2b - xbaa - pba, \text{class } 3 \rangle \quad (7.10)$$

$$\langle a, b \mid bab, p^2a, p^2b - baa, \text{class } 3 \rangle \quad (7.11)$$

$$\langle a, b \mid bab, p^2a - pba, p^2b - baa, \text{class } 3 \rangle \quad (7.12)$$

$$\langle a, b \mid bab, p^2a, p^2b - \omega baa, \text{class } 3 \rangle \quad (7.13)$$

$$\langle a, b \mid bab, p^2a - pba, p^2b - \omega baa, \text{class } 3 \rangle \quad (7.14)$$

$$\langle a, b \mid bab, p^2a, p^2b, \text{class } 3 \rangle \quad (7.15)$$

$$\langle a, b \mid bab, p^2a - baa, p^2b, \text{class } 3 \rangle \quad (7.16)$$

$$\langle a, b \mid bab, p^2a - pba, p^2b, \text{class } 3 \rangle \quad (7.17)$$

$$\langle a, b \mid bab, p^2a - baa - pba, p^2b, \text{class } 3 \rangle \quad (7.18)$$

$$\langle a, b \mid bab, pba, p^2a, \text{class } 3 \rangle \quad (7.19)$$

$$\langle a, b \mid bab, pba, p^2a - baa, \text{class } 3 \rangle \quad (7.20)$$

$$\langle a, b \mid bab, pba - baa, p^2a, \text{class } 3 \rangle \quad (7.21)$$

$$\langle a, b | bab, pba - baa, p^2a - baa, \text{class } 3 \rangle \quad (7.22)$$

$$\langle a, b | bab, pba, p^2b, \text{class } 3 \rangle \quad (7.23)$$

$$\langle a, b | bab, pba, p^2b - baa, \text{class } 3 \rangle \quad (7.24)$$

$$\langle a, b | bab, pba, p^2b - \omega baa, \text{class } 3 \rangle \quad (7.25)$$

$$\langle a, b | bab, pba - baa, p^2b - xbaa, \text{class } 3 \rangle \quad (7.26)$$

$$\langle a, b | p^2a - baa, p^2b, pba, \text{class } 3 \rangle \quad (7.27)$$

$$\langle a, b | p^2a - baa, p^2b - xbab, pba, \text{class } 3 \rangle (x \neq 0, x \sim x^{-1}) \quad (7.28)$$

$$\langle a, b | p^2a - baa - bab, p^2b - bab, pba, \text{class } 3 \rangle \quad (7.29)$$

$$\langle a, b | p^2a - baa - \omega bab, p^2b - bab, pba, \text{class } 3 \rangle \quad (7.30)$$

$$\langle a, b | p^2a, p^2b, pba, \text{class } 3 \rangle \quad (7.31)$$

$$\langle a, b | p^2a - bab, p^2b, pba, \text{class } 3 \rangle \quad (7.32)$$

$$\langle a, b | p^2a - \omega bab, p^2b, pba, \text{class } 3 \rangle \quad (7.33)$$

$$\langle a, b | p^2a - \omega bab, p^2b - baa, pba, \text{class } 3 \rangle \quad (7.34)$$

$$\langle a, b | p^2a - xbab, p^2b - baa - bab, pba, \text{class } 3 \rangle (1 + 4x \text{ not a square}) \quad (7.35)$$

$$\langle a, b | p^2a - bab, p^2b - xbaa, pba - baa, \text{class } 3 \rangle \quad (7.36)$$

$$\langle a, b | p^2a - \omega bab, p^2b - xbaa, pba - baa, \text{class } 3 \rangle \quad (7.37)$$

$$\langle a, b | p^2a - ybab, p^2b - xbaa - bab, pba - baa, \text{class } 3 \rangle (y \neq 0) \quad (7.38)$$

$$\langle a, b | p^2a, p^2b - xbaa, pba - baa, \text{class } 3 \rangle \quad (7.39)$$

$$\langle a, b | p^2a - baa, p^2b - xbaa - bab, pba - baa, \text{class } 3 \rangle (x \neq 0) \quad (7.40)$$

$$\langle a, b | p^2a, p^2b - bab, pba - baa, \text{class } 3 \rangle \quad (7.41)$$

$$\langle a, b | p^2a - baa, p^2b - xbab, pba - baa, \text{class } 3 \rangle \quad (7.42)$$

8.3 Descendants of 5.38

$p + 6 + (p^2 + 3p + 10) \gcd(p - 1, 3)$ algebras

$$\langle a, b \mid babb, pa, pb, \text{class } 4 \rangle \quad (7.43)$$

$$\langle a, b \mid babb, pa, pb - baab, \text{class } 4 \rangle \quad (7.44)$$

$$\langle a, b \mid babb, pa - baaa, pb - xbaab, \text{class } 4 \rangle \quad (7.45)$$

$$\langle a, b \mid babb, pa - baab, pb, \text{class } 4 \rangle \quad (7.46)$$

$$\langle a, b \mid babb, pa - baaa - baab, pb - 2baab, \text{class } 4 \rangle \quad (7.47)$$

$$\langle a, b \mid babb, pa - baaa - \omega baab, pb - 2baab, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.48)$$

$$\langle a, b \mid babb, pa - baaa - \omega^2 baab, pb - 2baab, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.49)$$

$$\langle a, b \mid babb, pa, pb - baaa, \text{class } 4 \rangle \quad (7.50)$$

$$\langle a, b \mid babb, pa - baab, pb - baaa, \text{class } 4 \rangle \quad (7.51)$$

$$\langle a, b \mid babb, pa - \omega baab, pb - baaa, \text{class } 4 \rangle \quad (7.52)$$

$$\langle a, b \mid babb, pa - xbaab, pb - baaa - baab, \text{class } 4 \rangle \quad (7.53)$$

$$\langle a, b \mid babb, pa, pb - \omega baaa, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.54)$$

$$\langle a, b \mid babb, pa - baab, pb - \omega baaa, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.55)$$

$$\langle a, b \mid babb, pa - \omega baab, pb - \omega baaa, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.56)$$

$$\langle a, b \mid babb, pa - xbaab, pb - \omega baaa - baab, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.57)$$

$$\langle a, b \mid babb, pa, pb - \omega^2 baaa, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.58)$$

$$\langle a, b \mid babb, pa - baab, pb - \omega^2 baaa, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.59)$$

$$\langle a, b \mid babb, pa - \omega baab, pb - \omega^2 baaa, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.60)$$

$$\langle a, b \mid babb, pa - xbaab, pb - \omega^2 baaa - baab, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.61)$$

Algebra 5.38 has $\gcd(p - 1, 3)(p^2 + 3p + 10) + p + 6$ immediate descendants of order p^7 . Of these $\frac{1}{2}((p^2 + 3p + 11) \gcd(p - 1, 3) + 1)$ come from one 4 parameter family of algebras, and $\frac{1}{2}(\gcd(p - 1, 3)(p^2 + p + 1) + 5)$ come from another four parameter family. These two families have presentations:

$$\langle a, b \mid babb - baaa, pa - xbaaa - ybaab, pb - zbaaa - tbaab, \text{ class 4} \rangle \quad (7.62)$$

$$\langle a, b \mid babb - \omega baaa, pa - xbaaa - ybaab, pb - zbaaa - tbaab, \text{ class 4} \rangle \quad (7.63)$$

In both 7.62 and 7.63 we take the four parameters as entries in a 2×2 matrix

$$A = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

and in both cases we consider the orbits of matrices A of this form over $\text{GF}(p)$ under an action of the subgroup of $\text{GL}(2, p)$ consisting of non-singular matrices of the form

$$\begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} \text{ or } \begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix}.$$

In 7.62 two matrices A and B give isomorphic Lie rings if and only if

$$B = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} A \begin{pmatrix} (\alpha^4 - \beta^4) & 2\alpha\beta(\alpha^2 - \beta^2) \\ 2\alpha\beta(\alpha^2 - \beta^2) & \alpha^4 - \beta^4 \end{pmatrix}^{-1}$$

or

$$B = \begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix} A \begin{pmatrix} -(\alpha^4 - \beta^4) & -2\alpha\beta(\alpha^2 - \beta^2) \\ 2\alpha\beta(\alpha^2 - \beta^2) & \alpha^4 - \beta^4 \end{pmatrix}^{-1}$$

for some α, β .

In 7.63, two matrices A and B give isomorphic Lie rings if and only if

$$B = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} A \begin{pmatrix} \alpha^4 - \omega^2\beta^4 & 2\alpha\beta(\alpha^2 - \omega\beta^2) \\ 2\omega\alpha\beta(\alpha^2 - \omega\beta^2) & \alpha^4 - \omega^2\beta^4 \end{pmatrix}^{-1}$$

or

$$B = \begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix} A \begin{pmatrix} -(\alpha^4 - \omega^2\beta^4) & -2\alpha\beta(\alpha^2 - \omega\beta^2) \\ 2\omega\alpha\beta(\alpha^2 - \omega\beta^2) & \alpha^4 - \omega^2\beta^4 \end{pmatrix}^{-1}$$

for some α, β .

8.4 Descendants of 5.60

$2p^2 + p + 3 + 2(p + 1) \gcd(p - 1, 3) + (2p + 4) \gcd(p - 1, 4) + \gcd(p - 1, 8)$ algebras

$$\langle a, b \mid bab, pa, pb, \text{ class 5} \rangle \quad (7.64)$$

$$\langle a, b \mid bab, pa, pb - baaab, \text{ class 5} \rangle \quad (7.65)$$

$$\langle a, b \mid bab, pa - baaaa, pb - xbaaab, \text{ class 5} \rangle \quad (7.66)$$

$$\langle a, b \mid bab, pa - baaab, pb, \text{ class 5} \rangle \quad (7.67)$$

$$\langle a, b \mid bab, pa - \omega baaab, pb, \text{ class 5} \rangle \quad (7.68)$$

$$\langle a, b \mid bab, pa - baaaa - baaab, pb - baaab, \text{ class 5} \rangle \quad (7.69)$$

$$\langle a, b \mid bab, pa - baaaa - \omega baaab, pb - baaab, \text{ class 5} \rangle \quad (7.70)$$

$$\langle a, b \mid bab, pa - baaaa - \omega^2 baaab, pb - baaab, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.71)$$

$$\langle a, b \mid bab, pa - baaaa - \omega^3 baaab, pb - baaab, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.72)$$

$$\langle a, b \mid bab, pa, pb - baaaa, \text{class } 5 \rangle \quad (7.73)$$

$$\langle a, b \mid bab, pa - baaab, pb - baaaa, \text{class } 5 \rangle \quad (7.74)$$

$$\langle a, b \mid bab, pa - \omega baaab, pb - baaaa, \text{class } 5 \rangle \quad (7.75)$$

$$\langle a, b \mid bab, pa - xbaaab, pb - baaaa - baaab, \text{class } 5 \rangle \quad (7.76)$$

$$\langle a, b \mid bab, pa, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.77)$$

$$\langle a, b \mid bab, pa - baaab, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.78)$$

$$\langle a, b \mid bab, pa - \omega baaab, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.79)$$

$$\langle a, b \mid bab, pa - xbaaab, pb - \omega baaaa - baaab, \text{class } 5 \rangle \quad (7.80)$$

$$\langle a, b \mid bab, pa, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.81)$$

$$\langle a, b \mid bab, pa - baaab, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.82)$$

$$\langle a, b \mid bab, pa - \omega baaab, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.83)$$

$$\langle a, b \mid bab, pa - xbaaab, pb - \omega^2 baaaa - baaab, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.84)$$

$$\langle a, b \mid bab, pa, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.85)$$

$$\langle a, b \mid bab, pa - baaab, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.86)$$

$$\langle a, b \mid bab, pa - \omega baaab, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.87)$$

$$\langle a, b \mid bab, pa - xbaaab, pb - \omega^3 baaaa - baaab, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.88)$$

$$\langle a, b \mid bab - baaaa, pa, pb, \text{class } 5 \rangle \quad (7.89)$$

$$\langle a, b \mid bab - baaaa, pa, pb - baaab, \text{class } 5 \rangle \quad (7.90)$$

$$\langle a, b \mid bab - baaaa, pa, pb - \omega baaab, \text{class } 5 \rangle \quad (7.91)$$

$$\langle a, b \mid bab - baaaa, pa, pb - \omega^2 baaab, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.92)$$

$$\langle a, b \mid bab - baaaa, pa, pb - \omega^3 baaab, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.93)$$

$$\langle a, b \mid bab - baaaa, pa, pb - \omega^4 baaab, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.94)$$

$$\langle a, b \mid bab - baaaa, pa, pb - \omega^5 baaab, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.95)$$

$$\langle a, b \mid bab - baaaa, pa - baaaa, pb - xbaaab, \text{class } 5 \rangle \quad (7.96)$$

$$\langle a, b \mid bab - baaaa, pa - \omega baaaa, pb - xbaaab, \text{class } 5 \rangle \quad (7.97)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^2 baaaa, pb - xbaaab, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.98)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^3 baaaa, pb - xbaaab, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.99)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^4 baaaa, pb - xbaaab, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.100)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^5 baaaa, pb - xbaaab, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.101)$$

$$\langle a, b \mid bab - baaaa, pa - baaab, pb, \text{class } 5 \rangle \quad (7.102)$$

$$\langle a, b \mid bab - baaaa, pa - \omega baaab, pb, \text{class } 5 \rangle \quad (7.103)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^2 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.104)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^3 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.105)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^4 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 8) \quad (7.106)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^5 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 8) \quad (7.107)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^6 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 8) \quad (7.108)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^7 baaab, pb, \text{class } 5 \rangle (p = 1 \bmod 8) \quad (7.109)$$

$$\langle a, b \mid bab - baaaa, pa - baaaa - xbaaab, pb - baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.110)$$

$$\langle a, b \mid bab - baaaa, pa - \omega baaaa - xbaaab, pb - \omega baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.111)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^2 baaaa - xbaaab, pb - \omega^2 baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.112)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^3 baaaa - xbaaab, pb - \omega^3 baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.113)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^4 baaaa - xbaaab, pb - \omega^4 baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.114)$$

$$\langle a, b \mid bab - baaaa, pa - \omega^5 baaaa - xbaaab, pb - \omega^5 baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.115)$$

$$\langle a, b \mid bab - baaaa, pa - xbaaab, pb - baaaa - ybaaab, \text{class } 5 \rangle (p = 3 \bmod 4) \quad (7.116)$$

$$\langle a, b \mid bab - baaaa, pa - xbaaab, pb - \omega baaaa - ybaaab, \text{class } 5 \rangle (p = 3 \bmod 4) \quad (7.117)$$

$$\langle a, b \mid bab - baaaa, pa - xbaaab, pb - baaaa - ybaaab, \text{class } 5 \rangle ((x, y) \sim (x, -y), p = 1 \bmod 4) \quad (7.118)$$

$$\langle a, b \mid bab - baaaa, pa - xbaaab, pb - \omega baaaa - ybaaab, \text{class } 5 \rangle ((x, y) \sim (x, -y), p = 1 \bmod 4) \quad (7.119)$$

$$\langle a, b \mid bab - baaaa, pa - xbaaab, pb - \omega^2 baaaa - ybaaab, \text{class } 5 \rangle ((x, y) \sim (x, -y), p = 1 \bmod 4) \quad (7.120)$$

$$\langle a, b \mid bab - baaaa, pa - xbaaab, pb - \omega^3 baaaa - ybaaab, \text{class } 5 \rangle ((x, y) \sim (x, -y), p = 1 \bmod 4) \quad (7.121)$$

8.5 Descendants of 5.65

$p^3 + p^2 + p - 2 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4) + (p + 1) \gcd(p - 1, 5)$ algebras

$$\langle a, b \mid bab - baaa, pa - xbaaaa - ybaaab, pb - baaaa - zbaaab, \text{class } 5 \rangle (x \neq 0, (x, y, z) \sim (ax, a^2y, az) \text{ if } a^4 = 1) \quad (7.122)$$

$$\langle a, b \mid bab - baaa, pa - xbaaaa - ybaaab, pb - \omega baaaa - zbaaab, \text{class } 5 \rangle (x \neq 0, (x, y, z) \sim (ax, a^2y, az) \text{ if } a^4 = 1) \quad (7.123)$$

$$\langle a, b \mid bab - baaa, pa - xbaaaa - ybaaab, pb - \omega^2 baaaa - zbaaab, \text{class } 5 \rangle (x \neq 0, (x, y, z) \sim (ax, a^2y, az) \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.124)$$

$$\langle a, b \mid bab - baaa, pa - xbaaaa - ybaaab, pb - \omega^3 baaaa - zbaaab, \text{class } 5 \rangle (x \neq 0, (x, y, z) \sim (ax, a^2y, az) \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.125)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - baaaa - ybaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (a^2x, ay) \text{ if } a^4 = 1) \quad (7.126)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega baaaa - ybaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (a^2x, ay) \text{ if } a^4 = 1) \quad (7.127)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega^2 baaaa - ybaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (a^2x, ay) \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.128)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega^3 baaaa - ybaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (a^2x, ay) \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.129)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - baaaa, \text{class } 5 \rangle (x \neq 0, p = 3 \bmod 4) \quad (7.130)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega baaaa, \text{class } 5 \rangle (x \neq 0, p = 3 \bmod 4) \quad (7.131)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - baaaa, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (7.132)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega baaaa, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (7.133)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega^2 baaaa, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (7.134)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega^3 baaaa, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (7.135)$$

$$\langle a, b \mid bab - baaa, pa, pb - baaaa, \text{class } 5 \rangle \quad (7.136)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.137)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.138)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.139)$$

$$\langle a, b \mid bab - baaa, pa - baaaa, pb - xbaaab, \text{class } 5 \rangle \quad (7.140)$$

$$\langle a, b \mid bab - baaa, pa - \omega baaaa, pb - xbaaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (7.141)$$

$$\langle a, b \mid bab - baaa, pa - \omega^2 baaaa, pb - xbaaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (7.142)$$

$$\langle a, b \mid bab - baaa, pa - \omega^3 baaaa, pb - xbaaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (7.143)$$

$$\langle a, b \mid bab - baaa, pa - \omega^4 baaaa, pb - xbaaab, \text{class } 5 \rangle (p = 1 \bmod 5) \quad (7.144)$$

$$\langle a, b \mid bab - baaa, pa - baaaa - ybaaab, pb - xbaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^5 = 1) \quad (7.145)$$

$$\langle a, b \mid bab - baaa, pa - \omega baaaa - ybaaab, pb - xbaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.146)$$

$$\langle a, b \mid bab - baaa, pa - \omega^2 baaaa - ybaaab, pb - xbaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.147)$$

$$\langle a, b \mid bab - baaa, pa - \omega^3 baaaa - ybaaab, pb - xbaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.148)$$

$$\langle a, b \mid bab - baaa, pa - \omega^4 baaaa - ybaaab, pb - xbaaab, \text{class } 5 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.149)$$

$$\langle a, b \mid bab - baaa, pa, pb - baaab, \text{class } 5 \rangle \quad (7.150)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega baaab, \text{class } 5 \rangle (p = 1 \pmod{5}) \quad (7.151)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega^2 baaab, \text{class } 5 \rangle (p = 1 \pmod{5}) \quad (7.152)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega^3 baaab, \text{class } 5 \rangle (p = 1 \pmod{5}) \quad (7.153)$$

$$\langle a, b \mid bab - baaa, pa, pb - \omega^4 baaab, \text{class } 5 \rangle (p = 1 \pmod{5}) \quad (7.154)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.155)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.156)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega^2 baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.157)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega^3 baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.158)$$

$$\langle a, b \mid bab - baaa, pa - xbaaab, pb - \omega^4 baaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.159)$$

$$\langle a, b \mid bab - baaa, pa, pb, \text{class } 5 \rangle \quad (7.160)$$

$$\langle a, b \mid bab - baaa, pa - baaab, pb, \text{class } 5 \rangle \quad (7.161)$$

$$\langle a, b \mid bab - baaa, pa - \omega baaab, pb, \text{class } 5 \rangle \quad (7.162)$$

$$\langle a, b \mid bab - baaa, pa - \omega^2 baaab, pb, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.163)$$

$$\langle a, b \mid bab - baaa, pa - \omega^3 baaab, pb, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.164)$$

$$\langle a, b \mid bab - baaa, pa - \omega^4 baaab, pb, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.165)$$

$$\langle a, b \mid bab - baaa, pa - \omega^5 baaab, pb, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.166)$$

8.6 Descendants of 6.366

2 algebras

$$\langle a, b \mid ba, p^3b, \text{class } 4 \rangle \quad (7.167)$$

$$\langle a, b \mid ba - p^3a, p^3b, \text{class } 4 \rangle \quad (7.168)$$

8.7 Descendants of 6.367

2 algebras

$$\langle a, b \mid ba - p^2a, p^3a, \text{class } 4 \rangle \quad (7.169)$$

$$\langle a, b \mid ba - p^2a, p^3b, \text{class } 4 \rangle \quad (7.170)$$

8.8 Descendants of 6.368

$9 + 6 \gcd(p - 1, 3)$ algebras

$$\langle a, b \mid baaa, baab, p^2a, pb, \text{ class } 4 \rangle \quad (7.171)$$

$$\langle a, b \mid baaa, baab, p^2a, pb - babb, \text{ class } 4 \rangle \quad (7.172)$$

$$\langle a, b \mid baaa, baab, p^2a - babb, pb, \text{ class } 4 \rangle \quad (7.173)$$

$$\langle a, b \mid baaa, baab, p^2a - \omega babb, pb, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.174)$$

$$\langle a, b \mid baaa, baab, p^2a - \omega^2 babb, pb, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.175)$$

$$\langle a, b \mid baab, babb - baaa, p^2a, pb, \text{ class } 4 \rangle \quad (7.176)$$

$$\langle a, b \mid baab, babb - baaa, p^2a, pb - baaa, \text{ class } 4 \rangle \quad (7.177)$$

$$\langle a, b \mid baab, babb - baaa, p^2a, pb - \omega baaa, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.178)$$

$$\langle a, b \mid baab, babb - baaa, p^2a, pb - \omega^2 baaa, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.179)$$

$$\langle a, b \mid baab, babb - baaa, p^2a - baaa, pb, \text{ class } 4 \rangle \quad (7.180)$$

$$\langle a, b \mid baab, babb - baaa, p^2a - \omega baaa, pb, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.181)$$

$$\langle a, b \mid baab, babb - baaa, p^2a - \omega^2 baaa, pb, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.182)$$

$$\langle a, b \mid baab, babb - \omega baaa, p^2a, pb, \text{ class } 4 \rangle \quad (7.183)$$

$$\langle a, b \mid baab, babb - \omega baaa, p^2a, pb - baaa, \text{ class } 4 \rangle \quad (7.184)$$

$$\langle a, b \mid baab, babb - \omega baaa, p^2a, pb - \omega baaa, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.185)$$

$$\langle a, b \mid baab, babb - \omega baaa, p^2a, pb - \omega^2 baaa, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.186)$$

$$\langle a, b \mid baab, babb - \omega baaa, p^2a - baaa, pb, \text{ class } 4 \rangle \quad (7.187)$$

$$\langle a, b \mid baab, babb - \omega baaa, p^2a - \omega baaa, pb, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.188)$$

$$\langle a, b \mid baab, babb - \omega baaa, p^2a - \omega^2 baaa, pb, \text{ class } 4 \rangle \ (p = 1 \bmod 3) \quad (7.189)$$

$$\langle a, b \mid baaa, babb, p^2a, pb, \text{ class } 4 \rangle \quad (7.190)$$

$$\langle a, b \mid baaa, babb, p^2a, pb - baab, \text{ class } 4 \rangle \quad (7.191)$$

$$\langle a, b \mid baaa, babb, p^2a - baab, pb, \text{class } 4 \rangle \quad (7.192)$$

$$\langle a, b \mid baab, babb, p^2a, pb, \text{class } 4 \rangle \quad (7.193)$$

$$\langle a, b \mid baab, babb, p^2a, pb - baaa, \text{class } 4 \rangle \quad (7.194)$$

$$\langle a, b \mid baab, babb, p^2a, pb - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.195)$$

$$\langle a, b \mid baab, babb, p^2a, pb - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.196)$$

$$\langle a, b \mid baab, babb, p^2a - baaa, pb, \text{class } 4 \rangle \quad (7.197)$$

8.9 Descendants of 6.369

$5 + \gcd(p - 1, 3)$ algebras

$$\langle a, b \mid baaa, p^2a, pb - bab, \text{class } 4 \rangle \quad (7.198)$$

$$\langle a, b \mid baaa, p^2a, pb - bab - baab, \text{class } 4 \rangle \quad (7.199)$$

$$\langle a, b \mid baaa, p^2a - baab, pb - bab, \text{class } 4 \rangle \quad (7.200)$$

$$\langle a, b \mid baab, p^2a, pb - bab, \text{class } 4 \rangle \quad (7.201)$$

$$\langle a, b \mid baab, p^2a, pb - bab - baaa, \text{class } 4 \rangle \quad (7.202)$$

$$\langle a, b \mid baab, p^2a, pb - bab - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.203)$$

$$\langle a, b \mid baab, p^2a, pb - bab - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.204)$$

$$\langle a, b \mid baab, p^2a - baaa, pb - bab, \text{class } 4 \rangle \quad (7.205)$$

8.10 Descendants of 6.370

$2p + 2 + (p + 1)/2 + \gcd(p - 1, 3) + \gcd(p - 1, 4)/2$ algebras

$$\langle a, b \mid baaa, p^2a, pb - baa, \text{class } 4 \rangle \quad (7.206)$$

$$\langle a, b \mid baaa, p^2a, pb - baa - babb, \text{class } 4 \rangle \quad (7.207)$$

$$\langle a, b \mid baaa, p^2a, pb - baa - \omega babb, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.208)$$

$$\langle a, b \mid baaa, p^2a - babb, pb - baa, \text{class } 4 \rangle \quad (7.209)$$

$$\langle a, b \mid baaa, p^2a - \omega babb, pb - baa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.210)$$

$$\langle a, b \mid baaa, p^2a - \omega^2babb, pb - baa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.211)$$

$$\langle a, b \mid babb, p^2a, pb - baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.212)$$

$$\langle a, b \mid babb, p^2a - baaa, pb - baa, \text{class } 4 \rangle \quad (7.213)$$

$$\langle a, b \mid babb - baaa, p^2a, pb - baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.214)$$

$$\langle a, b \mid babb - baaa, p^2a - xbaaa, pb - baa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.215)$$

$$\langle a, b \mid babb - \omega baaa, p^2a, pb - baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.216)$$

$$\langle a, b \mid babb - \omega baaa, p^2a - xbaaa, pb - baa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.217)$$

8.11 Descendants of 6.371

$2p + 2 + (p + 1)/2 + \gcd(p - 1, 3) + \gcd(p - 1, 4)/2$ algebras

$$\langle a, b \mid baaa, p^2a, pb - \omega baa, \text{class } 4 \rangle \quad (7.218)$$

$$\langle a, b \mid baaa, p^2a, pb - \omega baa - babb, \text{class } 4 \rangle \quad (7.219)$$

$$\langle a, b \mid baaa, p^2a, pb - \omega baa - \omega babb, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.220)$$

$$\langle a, b \mid baaa, p^2a - babb, pb - \omega baa, \text{class } 4 \rangle \quad (7.221)$$

$$\langle a, b \mid baaa, p^2a - \omega babb, pb - \omega baa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.222)$$

$$\langle a, b \mid baaa, p^2a - \omega^2babb, pb - \omega baa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.223)$$

$$\langle a, b \mid babb, p^2a, pb - \omega baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.224)$$

$$\langle a, b \mid babb, p^2a - baaa, pb - \omega baa, \text{class } 4 \rangle \quad (7.225)$$

$$\langle a, b \mid babb - baaa, p^2a, pb - \omega baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.226)$$

$$\langle a, b \mid babb - baaa, p^2a - xbaaa, pb - \omega baa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.227)$$

$$\langle a, b \mid babb - \omega baaa, p^2a, pb - \omega baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.228)$$

$$\langle a, b \mid babb - \omega baaa, p^2a - xbaaa, pb - \omega baa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.229)$$

8.12 Descendants of 6.372

$(p+1)/2 + \gcd(p-1, 3) + \gcd(p-1, 4)/2$ algebras

$$\langle a, b \mid p^2a - bab, pb, \text{class } 4 \rangle \quad (7.230)$$

$$\langle a, b \mid p^2a - bab, pb - baaa, \text{class } 4 \rangle \quad (7.231)$$

$$\langle a, b \mid p^2a - bab, pb - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.232)$$

$$\langle a, b \mid p^2a - bab, pb - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.233)$$

$$\langle a, b \mid p^2a - bab - baaa, pb - xbaaa, \text{class } 4 \rangle \quad (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.234)$$

$$\langle a, b \mid p^2a - bab - \omega baaa, pb - xbaaa, \text{class } 4 \rangle \quad (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.235)$$

$$\langle a, b \mid p^2a - bab - baaa, pb - xbaaa, \text{class } 4 \rangle \quad (x \sim -x, p = 3 \pmod{4}) \quad (7.236)$$

8.13 Descendants of 6.373

p algebras

$$\langle a, b \mid p^2a - bab, pb - baa - xbaaa, \text{class } 4 \rangle \quad (x \sim -x) \quad (7.237)$$

$$\langle a, b \mid p^2a - bab - xbaaa, pb - baa, \text{class } 4 \rangle \quad (x \neq 0, x \sim -x) \quad (7.238)$$

8.14 Descendants of 6.374

p algebras

$$\langle a, b \mid p^2a - bab, pb - \omega baa - xbaaa, \text{class } 4 \rangle \quad (x \sim -x) \quad (7.239)$$

$$\langle a, b \mid p^2a - bab - xbaaa, pb - \omega baa, \text{class } 4 \rangle \quad (x \neq 0, x \sim -x) \quad (7.240)$$

8.15 Descendants of 6.375

$(p+1)/2 + \gcd(p-1, 3) + \gcd(p-1, 4)/2$ algebras

$$\langle a, b \mid p^2a - \omega bab, pb, \text{class } 4 \rangle \quad (7.241)$$

$$\langle a, b \mid p^2a - \omega bab, pb - baaa, \text{class } 4 \rangle \quad (7.242)$$

$$\langle a, b \mid p^2a - \omega bab, pb - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.243)$$

$$\langle a, b \mid p^2a - \omega bab, pb - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.244)$$

$$\langle a, b \mid p^2a - \omega bab - baaa, pb - xbaaa, \text{class } 4 \rangle \quad (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.245)$$

$$\langle a, b \mid p^2a - \omega bab - \omega baaa, pb - xbaaa, \text{class } 4 \rangle \quad (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.246)$$

$$\langle a, b \mid p^2a - \omega bab - baaa, pb - xbaaa, \text{class } 4 \rangle \quad (x \sim -x, p = 3 \pmod{4}) \quad (7.247)$$

8.16 Descendants of 6.376

p algebras

$$\langle a, b \mid p^2a - \omega bab, pb - baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.248)$$

$$\langle a, b \mid p^2a - \omega bab - xbaaa, pb - baa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.249)$$

8.17 Descendants of 6.377

p algebras

$$\langle a, b \mid p^2a - \omega bab, pb - \omega baa - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.250)$$

$$\langle a, b \mid p^2a - \omega bab - xbaaa, pb - \omega baa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.251)$$

8.18 Descendants of 6.378

$p + 1 + \gcd(p - 1, 3)$ algebras

$$\langle a, b \mid p^2a - baa, pb, \text{class } 4 \rangle \quad (7.252)$$

$$\langle a, b \mid p^2a - baa - babb, pb, \text{class } 4 \rangle \quad (7.253)$$

$$\langle a, b \mid p^2a - baa - \omega babb, pb, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.254)$$

$$\langle a, b \mid p^2a - baa - \omega^2 babb, pb, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.255)$$

$$\langle a, b \mid p^2a - baa - xbabb, pb - babb, \text{class } 4 \rangle \quad (7.256)$$

8.19 Descendants of 6.379

$5 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid p^3a, baa, pb, \text{class } 4 \rangle \quad (7.257)$$

$$\langle a, b \mid p^3a, baa, pb - babb, \text{class } 4 \rangle \quad (7.258)$$

$$\langle a, b \mid p^3a, baa - babb, pb, \text{class } 4 \rangle \quad (7.259)$$

$$\langle a, b \mid p^3a, baa - babb, pb - babb, \text{class } 4 \rangle \quad (7.260)$$

$$\langle a, b \mid p^3a, baa - babb, pb - \omega babb, \text{class } 4 \rangle \quad (7.261)$$

$$\langle a, b \mid p^3a, baa - babb, pb - \omega^2 babb, \text{class } 4 \rangle (p = 1 \pmod{4}) \quad (7.262)$$

$$\langle a, b \mid p^3a, baa - babb, pb - \omega^3 babb, \text{class } 4 \rangle (p = 1 \pmod{4}) \quad (7.263)$$

$$\langle a, b \mid babb, baa, pb, \text{class } 4 \rangle \quad (7.264)$$

$$\langle a, b \mid babb, baa - p^3a, pb, \text{class } 4 \rangle \quad (7.265)$$

$$\langle a, b \mid p^3a - babb, baa, pb, \text{class } 4 \rangle \quad (7.266)$$

$$\langle a, b \mid p^3a - babb, baa - babb, pb, \text{class } 4 \rangle \quad (7.267)$$

$$\langle a, b \mid p^3a - \omega babb, baa, pb, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.268)$$

$$\langle a, b \mid p^3a - \omega babb, baa - babb, pb, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.269)$$

$$\langle a, b \mid p^3a - \omega^2 babb, baa, pb, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.270)$$

$$\langle a, b \mid p^3a - \omega^2 babb, baa - babb, pb, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.271)$$

8.20 Descendants of 6.380

p algebras

$$\langle a, b \mid baa - xp^3a, pb - bab, \text{class } 4 \rangle \quad (7.272)$$

8.21 Descendants of 6.381

$6 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid baaa, bab, pb, \text{class } 4 \rangle \quad (7.273)$$

$$\langle a, b \mid baaa, bab - p^3a, pb, \text{class } 4 \rangle \quad (7.274)$$

$$\langle a, b \mid baaa, bab - \omega p^3a, pb, \text{class } 4 \rangle \quad (7.275)$$

$$\langle a, b \mid p^3a, bab, pb, \text{class } 4 \rangle \quad (7.276)$$

$$\langle a, b \mid p^3a, bab - baaa, pb, \text{class } 4 \rangle \quad (7.277)$$

$$\langle a, b \mid p^3a, bab, pb - baaa, \text{class } 4 \rangle \quad (7.278)$$

$$\langle a, b \mid p^3a, bab - baaa, pb - baaa, \text{class } 4 \rangle \quad (7.279)$$

$$\langle a, b \mid p^3a, bab, pb - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.280)$$

$$\langle a, b \mid p^3a, bab - baaa, pb - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.281)$$

$$\langle a, b \mid p^3a, bab, pb - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.282)$$

$$\langle a, b \mid p^3a, bab - baaa, pb - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.283)$$

$$\langle a, b \mid p^3 a - baaa, bab, pb, \text{class } 4 \rangle \quad (7.284)$$

$$\langle a, b \mid p^3 a - baaa, bab - baaa, pb, \text{class } 4 \rangle \quad (7.285)$$

$$\langle a, b \mid p^3 a - baaa, bab - \omega baaa, pb, \text{class } 4 \rangle \quad (7.286)$$

$$\langle a, b \mid p^3 a - baaa, bab - \omega^2 baaa, pb, \text{class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.287)$$

$$\langle a, b \mid p^3 a - baaa, bab - \omega^3 baaa, pb, \text{class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.288)$$

8.22 Descendants of 6.382

$p + 5 + (p - 1)/2$ algebras

$$\langle a, b \mid baaa, bab, pb - baa, \text{class } 4 \rangle \quad (7.289)$$

$$\langle a, b \mid baaa, bab - p^3 a, pb - baa, \text{class } 4 \rangle \quad (7.290)$$

$$\langle a, b \mid baaa, bab - \omega p^3 a, pb - baa, \text{class } 4 \rangle \quad (7.291)$$

$$\langle a, b \mid p^3 a, bab, pb - baa - xbaaa, \text{class } 4 \rangle \quad (x \sim -x) \quad (7.292)$$

$$\langle a, b \mid p^3 a, bab - baaa, pb - baa, \text{class } 4 \rangle \quad (7.293)$$

$$\langle a, b \mid p^3 a - baaa, bab - xbaaa, pb - baa, \text{class } 4 \rangle \quad (7.294)$$

8.23 Descendants of 6.383

$p + 5 + (p - 1)/2$ algebras

$$\langle a, b \mid baaa, bab, pb - \omega baa, \text{class } 4 \rangle \quad (7.295)$$

$$\langle a, b \mid baaa, bab - p^3 a, pb - \omega baa, \text{class } 4 \rangle \quad (7.296)$$

$$\langle a, b \mid baaa, bab - \omega p^3 a, pb - \omega baa, \text{class } 4 \rangle \quad (7.297)$$

$$\langle a, b \mid p^3 a, bab, pb - \omega baa - xbaaa, \text{class } 4 \rangle \quad (x \sim -x) \quad (7.298)$$

$$\langle a, b \mid p^3 a, bab - baaa, pb - \omega baa, \text{class } 4 \rangle \quad (7.299)$$

$$\langle a, b \mid p^3 a - baaa, bab - xbaaa, pb - \omega baa, \text{class } 4 \rangle \quad (7.300)$$

8.24 Descendants of 6.384

2 algebras

$$\langle a, b \mid baaa, pb - ba, \text{class } 4 \rangle \quad (7.301)$$

$$\langle a, b \mid p^3a, pb - ba, \text{class } 4 \rangle \quad (7.302)$$

8.25 Descendants of 6.386

$p + 5$ algebras

$$\langle a, b \mid baa, bab, pba, p^2b, \text{class } 4 \rangle \quad (7.303)$$

$$\langle a, b \mid baa - p^3a, bab, pba, p^2b, \text{class } 4 \rangle \quad (7.304)$$

$$\langle a, b \mid baa, bab - p^3a, pba, p^2b, \text{class } 4 \rangle \quad (7.305)$$

$$\langle a, b \mid baa, bab - \omega p^3a, pba, p^2b, \text{class } 4 \rangle \quad (7.306)$$

$$\langle a, b \mid baa, bab - xp^3a, pba - p^3a, p^2b, \text{class } 4 \rangle \quad (7.307)$$

$$\langle a, b \mid baa - p^3a, bab, pba - p^3a, p^2b, \text{class } 4 \rangle \quad (7.308)$$

8.26 Descendants of 6.388

$4p + 5 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid bab, pba, p^2a, p^2b, \text{class } 4 \rangle \quad (7.309)$$

$$\langle a, b \mid bab - baaa, pba, p^2a, p^2b, \text{class } 4 \rangle \quad (7.310)$$

$$\langle a, b \mid bab, pba - baaa, p^2a, p^2b, \text{class } 4 \rangle \quad (7.311)$$

$$\langle a, b \mid bab - baaa, pba - baaa, p^2a, p^2b, \text{class } 4 \rangle \quad (7.312)$$

$$\langle a, b \mid bab, pba - \omega baaa, p^2a, p^2b, \text{class } 4 \rangle \quad (7.313)$$

$$\langle a, b \mid bab - baaa, pba - \omega baaa, p^2a, p^2b, \text{class } 4 \rangle \quad (7.314)$$

$$\langle a, b \mid bab, pba, p^2a - baaa, p^2b, \text{class } 4 \rangle \quad (7.315)$$

$$\langle a, b \mid bab - baaa, pba, p^2a - baaa, p^2b, \text{class } 4 \rangle \quad (7.316)$$

$$\langle a, b \mid bab - \omega baaa, pba, p^2a - baaa, p^2b, \text{class } 4 \rangle \quad (7.317)$$

$$\langle a, b \mid bab - \omega^2 baaa, pba, p^2a - baaa, p^2b, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.318)$$

$$\langle a, b \mid bab - \omega^3baaa, pba, p^2a - baaa, p^2b, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.319)$$

$$\langle a, b \mid bab - xbaaa, pba - baaa, p^2a - baaa, p^2b, \text{class } 4 \rangle \quad (7.320)$$

$$\langle a, b \mid bab - xbaaa, pba - \omega baaa, p^2a - baaa, p^2b, \text{class } 4 \rangle \quad (7.321)$$

$$\langle a, b \mid bab, pba, p^2a, p^2b - baaa, \text{class } 4 \rangle \quad (7.322)$$

$$\langle a, b \mid bab - baaa, pba, p^2a, p^2b - baaa, \text{class } 4 \rangle \quad (7.323)$$

$$\langle a, b \mid bab, pba - xbaaa, p^2a, p^2b - baaa, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.324)$$

$$\langle a, b \mid bab - baaa, pba - xbaaa, p^2a, p^2b - baaa, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.325)$$

$$\langle a, b \mid bab, pba, p^2a, p^2b - \omega baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.326)$$

$$\langle a, b \mid bab - baaa, pba, p^2a, p^2b - \omega baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.327)$$

$$\langle a, b \mid bab, pba - xbaaa, p^2a, p^2b - \omega baaa, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.328)$$

$$\langle a, b \mid bab - baaa, pba - xbaaa, p^2a, p^2b - \omega baaa, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.329)$$

$$\langle a, b \mid bab, pba, p^2a, p^2b - \omega^2baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.330)$$

$$\langle a, b \mid bab - baaa, pba, p^2a, p^2b - \omega^2baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.331)$$

$$\langle a, b \mid bab, pba - xbaaa, p^2a, p^2b - \omega^2baaa, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.332)$$

$$\langle a, b \mid bab - baaa, pba - xbaaa, p^2a, p^2b - \omega^2baaa, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.333)$$

8.27 Descendants of 6.394

$p^2 + 3p + 10 + 2 \gcd(p - 1, 3) + 3 \gcd(p - 1, 4) + 2 \gcd(p - 1, 5) + \gcd(p - 1, 8) + \gcd(p - 1, 9)$ algebras

$$\langle a, b \mid baaaa, baab, babb, pa, pb, \text{class } 5 \rangle \quad (7.334)$$

$$\langle a, b \mid baaaa, baab, babb, pa - baaab, pb, \text{class } 5 \rangle \quad (7.335)$$

$$\langle a, b \mid baaaa, baab, babb, pa - \omega baaab, pb, \text{class } 5 \rangle \quad (7.336)$$

$$\langle a, b \mid baaaa, baab, babb, pa, pb - baaab, \text{class } 5 \rangle \quad (7.337)$$

$$\langle a, b \mid baaaa, baab, babb, pa - baaab, pb - baaab, \text{class } 5 \rangle \quad (7.338)$$

$$\langle a, b \mid baaaa, baab, babb, pa - \omega baaab, pb - baaab, \text{class } 5 \rangle \quad (7.339)$$

$$\langle a, b \mid baaaa, baab, babb, pa - \omega^2 baaab, pb - baaab, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (7.340)$$

$$\langle a, b \mid baaaa, baab, babb, pa - \omega^3 baaab, pb - baaab, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (7.341)$$

$$\langle a, b \mid baaaa, baab, babb - baaab, pa, pb, \text{class } 5 \rangle \quad (7.342)$$

$$\langle a, b \mid baaaa, baab, babb - baaab, pa, pb - baaab, \text{class } 5 \rangle \quad (7.343)$$

$$\langle a, b \mid baaaa, baab, babb - baaab, pa, pb - xbaaab, \text{class } 5 \rangle \quad (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \pmod{5}) \quad (7.344)$$

In the following presentation we have $y = 1, \omega, \omega^2, \omega^3, \omega^4, \omega^5$, $(x, y) \sim (ax, y)$ if $a^6 = 1$, $p = 1 \pmod{3}$:

$$\langle a, b \mid baaaa, baab, babb - baaab, pa - ybaaab, pb - xbaaab, \text{class } 5 \rangle \quad (7.345)$$

$$\langle a, b \mid baaaa, baab, babb - baaab, pa - ybaaab, pb - xbaaab, \text{class } 5 \rangle \quad (y = 1, \omega, (x, y) \sim (-x, y), p = 2 \pmod{3}) \quad (7.346)$$

$$\langle a, b \mid baaaa, baab - baaab, babb, pa, pb, \text{class } 5 \rangle \quad (7.347)$$

$$\langle a, b \mid baaaa, baab - baaab, babb, pa - baaab, pb, \text{class } 5 \rangle \quad (7.348)$$

$$\langle a, b \mid baaaa, baab - baaab, babb, pa - \omega baaab, pb, \text{class } 5 \rangle \quad (7.349)$$

$$\langle a, b \mid baaaa, baab - baaab, babb, pa - xbaaab, pb - baaab, \text{class } 5 \rangle \quad (7.350)$$

$$\langle a, b \mid baaaa, baab - baaab, babb - baaab, pa - xbaaab, pb - ybaaab, \text{class } 5 \rangle \quad (7.351)$$

$$\langle a, b \mid baaab, baab, babb, pa, pb, \text{class } 5 \rangle \quad (7.352)$$

$$\langle a, b \mid baaab, baab, babb, pa, pb - baaaa, \text{class } 5 \rangle \quad (7.353)$$

$$\langle a, b \mid baaab, baab, babb, pa, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.354)$$

$$\langle a, b \mid baaab, baab, babb, pa, pb - \omega^2 baaaa, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (7.355)$$

$$\langle a, b \mid baaab, baab, babb, pa, pb - \omega^3 baaaa, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (7.356)$$

$$\langle a, b \mid baaab, baab, babb, pa - baaaa, pb, \text{class } 5 \rangle \quad (7.357)$$

$$\langle a, b \mid baaab, baab - baaaa, babb, pa, pb, \text{class } 5 \rangle \quad (7.358)$$

$$\langle a, b \mid baaab, baab - baaaa, babb, pa - baaaa, pb, \text{class } 5 \rangle \quad (7.359)$$

$$\langle a, b \mid baaab, baab - baaaa, babb, pa - xbaaaa, pb, \text{class } 5 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.360)$$

$$\langle a, b \mid baaab, baab - baaaa, babb, pa, pb - baaaa, \text{class } 5 \rangle \quad (7.361)$$

$$\langle a, b \mid baaab, baab - baaaa, babb, pa, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.362)$$

$$\langle a, b \mid baaab, baab - baaaa, babb, pa, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.363)$$

$$\langle a, b \mid baaab, baab - baaaa, babb, pa, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.364)$$

$$\langle a, b \mid baaab, baab, babb - baaaa, pa, pb, \text{class } 5 \rangle \quad (7.365)$$

$$\langle a, b \mid baaab, baab, babb - baaaa, pa - baaaa, pb, \text{class } 5 \rangle \quad (7.366)$$

$$\langle a, b \mid baaab, baab, babb - baaaa, pa - xbaaaa, pb, \text{class } 5 \rangle (x = \omega, \omega^2, p = 1 \bmod 3) \quad (7.367)$$

$$\langle a, b \mid baaab, baab, babb - baaaa, pa - xbaaaa, pb, \text{class } 5 \rangle (x = \omega^3, \omega^4, \omega^5, \omega^6, \omega^7, \omega^8, p = 1 \bmod 9) \quad (7.368)$$

$$\langle a, b \mid baaab, baab, babb - baaaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle (y = 1, \omega, (x, y) \sim (ax, y) \text{ if } a^8 = 1) \quad (7.369)$$

$$\langle a, b \mid baaab, baab, babb - baaaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle (y = \omega^2, \omega^3, (x, y) \sim (ax, y) \text{ if } a^8 = 1, p = 1 \bmod 4) \quad (7.370)$$

$$\langle a, b \mid baaab, baab, babb - baaaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle (y = \omega^4, \omega^5, \omega^6, \omega^7, (x, y) \sim (ax, y) \text{ if } a^8 = 1, p = 1 \bmod 8) \quad (7.371)$$

8.28 Descendants of 6.399

$(1/2)p^3 + (3/2)p^2 + 4p + 7 + 3 \gcd(p-1, 3) + (p+3) \gcd(p-1, 4)/2 + \gcd(p-1, 5) + \gcd(p-1, 8)/2$ algebras

$$\langle a, b \mid baaab, baab, babb + baaa, pa, pb, \text{class } 5 \rangle \quad (7.372)$$

$$\langle a, b \mid baaab, baab, babb + baaa, pa - baaaa, pb, \text{class } 5 \rangle \quad (7.373)$$

$$\langle a, b \mid baaab, baab, babb + baaa, pa - \omega baaaa, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.374)$$

$$\langle a, b \mid baaab, baab, babb + baaa, pa - \omega^2 baaaa, pb, \text{class } 5 \rangle (p = 1 \bmod 8) \quad (7.375)$$

$$\langle a, b \mid baaab, baab, babb + baaa, pa - \omega^3 baaaa, pb, \text{class } 5 \rangle (p = 1 \bmod 8) \quad (7.376)$$

$$\langle a, b \mid baaab, baab, babb + baaa, pa - xbaaaa, pb - baaaa, \text{class } 5 \rangle (x \sim -x) \quad (7.377)$$

$$\langle a, b \mid baaab, baab, babb + baaa, pa - xbaaaa, pb - \omega baaaa, \text{class } 5 \rangle (x \sim -x) \quad (7.378)$$

$$\langle a, b \mid baaab, baab, babb + baaa, pa - xbaaaa, pb - \omega^2baaaa, \text{class } 5 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.379)$$

$$\langle a, b \mid baaab, baab, babb + baaa, pa - xbaaaa, pb - \omega^3baaaa, \text{class } 5 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.380)$$

$$\langle a, b \mid baaab, baab - baaaa, babb + baaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (-x, y)) \quad (7.381)$$

$$\langle a, b \mid baaab, baab, babb + baaa - baaaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (-x, y)) \quad (7.382)$$

$$\langle a, b \mid baaab, baab - xbaaaa, babb + baaa - baaaa, pa - ybaaaa, pb - zbaaaa, \text{class } 5 \rangle (x \neq 0, (x, y, z) \sim (-x, -y, z)) \quad (7.383)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa, pa, pb, \text{class } 5 \rangle \quad (7.384)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa, pa, pb - baaaa, \text{class } 5 \rangle \quad (7.385)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa, pa, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.386)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa, pa, pb - \omega^2baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.387)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa, pa, pb - \omega^3baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.388)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa, pa - baaaa, pb - baaaa, \text{class } 5 \rangle \quad (7.389)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa, pa - \omega baaaa, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.390)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa, pa - baaaa, pb + baaaa, \text{class } 5 \rangle \quad (7.391)$$

$$\langle a, b \mid baaab - baaaa, baab, babb + baaa - baaaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (y, x)) \quad (7.392)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa, pb - xbaaaa, \text{class } 5 \rangle \quad (7.393)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - baaaa, pb - baaaa, \text{class } 5 \rangle \quad (7.394)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - \omega baaaa, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.395)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - baaaa, pb + baaaa, \text{class } 5 \rangle \quad (7.396)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - \omega baaaa, pb + \omega baaaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.397)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - \omega^2baaaa, pb + \omega^2baaaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.398)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa, pb - xbaaaa, \text{class } 5 \rangle \quad (7.399)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa - baaaa, pb - baaaa, \text{class } 5 \rangle \quad (7.400)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa - \omega baaaa, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.401)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa - xbaaaa, pb - xbaaaa, \text{class } 5 \rangle (x = \omega^2, \omega^3, \omega^4, \omega^5, p = 1 \bmod 3) \quad (7.402)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa - baaaa, pb + baaaa, \text{class } 5 \rangle \quad (7.403)$$

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa - xbaaaa, pb + xbaaaa, \text{class } 5 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.404)$$

In the following presentation, z is chosen so as to be any *one* integer such that $z^2 - 4$ is not a square mod p :

$$\langle a, b \mid baaab - baaaa, baab - baaaa, babb + baaa - zbaaaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle ((x, y, z) \sim (y, x, z)) \quad (7.405)$$

8.29 Descendants of 6.404

$(p^3 + p^2 + 2p + 2 + (p + 1) \gcd(p - 1, 4) + \gcd(p - 1, 8))/2$ algebras

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa, pb, \text{class } 5 \rangle \quad (7.406)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa - baaaa, pb, \text{class } 5 \rangle \quad (7.407)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa - \omega baaaa, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.408)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa - \omega^2 baaaa, pb, \text{class } 5 \rangle (p = 1 \bmod 8) \quad (7.409)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa - \omega^3 baaaa, pb, \text{class } 5 \rangle (p = 1 \bmod 8) \quad (7.410)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa - xbaaaa, pb - baaaa, \text{class } 5 \rangle (x \sim -x) \quad (7.411)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa - xbaaaa, pb - \omega baaaa, \text{class } 5 \rangle (x \sim -x) \quad (7.412)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa - xbaaaa, pb - \omega^2 baaaa, \text{class } 5 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.413)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa, pa - xbaaaa, pb - \omega^3 baaaa, \text{class } 5 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.414)$$

$$\langle a, b \mid baaab, baab - baaaa, babb + \omega baaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (-x, y)) \quad (7.415)$$

$$\langle a, b \mid baaab, baab, babb + \omega baaa - baaaa, pa - xbaaaa, pb - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (-x, y)) \quad (7.416)$$

$$\langle a, b \mid baaab, baab - xbaaaa, babb + \omega baaa - baaaa, pa - ybaaaa, pb - zbaaaa, \text{class } 5 \rangle (x \neq 0, (x, y, z) \sim (-x, -y, z)) \quad (7.417)$$

8.30 Descendants of 6.408

$(1/2)(p+1)(p-1+\gcd(p-1,3))$ algebras

$$\langle a, b \mid pa - bab - xbaaaa, pb - baaa - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (\pm x, ay) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.418)$$

$$\langle a, b \mid pa - bab - xbaaaa, pb - \omega baaa - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (\pm x, ay) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.419)$$

$$\langle a, b \mid pa - bab - xbaaaa, pb - \omega^2 baaa - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (\pm x, ay) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.420)$$

$$\langle a, b \mid pa - bab - xbaaaa, pb - baaa - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (-x, y), p = 2 \pmod{3}) \quad (7.421)$$

8.31 Descendants of 6.411

$2 + 2\gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a, b \mid baaaa, pa - bab, pb, \text{class } 5 \rangle \quad (7.422)$$

$$\langle a, b \mid baaaa, pa - bab, pb - baaab, \text{class } 5 \rangle \quad (7.423)$$

$$\langle a, b \mid baaaa, pa - bab, pb - \omega baaab, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.424)$$

$$\langle a, b \mid baaaa, pa - bab, pb - \omega^2 baaab, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.425)$$

$$\langle a, b \mid baaab, pa - bab, pb, \text{class } 5 \rangle \quad (7.426)$$

$$\langle a, b \mid baaab, pa - bab - baaaa, pb, \text{class } 5 \rangle \quad (7.427)$$

$$\langle a, b \mid baaab, pa - bab - \omega baaaa, pb, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.428)$$

$$\langle a, b \mid baaab, pa - bab - \omega^2 baaaa, pb, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.429)$$

$$\langle a, b \mid baaab, pa - bab, pb - baaaa, \text{class } 5 \rangle \quad (7.430)$$

$$\langle a, b \mid baaab, pa - bab, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.431)$$

$$\langle a, b \mid baaab, pa - bab, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \pmod{4}) \quad (7.432)$$

$$\langle a, b \mid baaab, pa - bab, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \pmod{4}) \quad (7.433)$$

8.32 Descendants of 6.412

$p - 1 + (1/2)(p + 1) \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid baaaa, pa - bab - baaa, pb - xbaaab, \text{class } 5 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.434)$$

$$\langle a, b \mid baaaa, pa - bab - \omega baaa, pb - xbaaab, \text{class } 5 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.435)$$

$$\langle a, b \mid baaaa, pa - bab - baaa, pb - xbaaab, \text{class } 5 \rangle (x \sim -x, p = 3 \pmod{4}) \quad (7.436)$$

$$\langle a, b \mid baaab, pa - bab - baaa, pb - xbaaaa, \text{class } 5 \rangle \quad (7.437)$$

$$\langle a, b \mid baaab, pa - bab - \omega baaa, pb - xbaaaa, \text{class } 5 \rangle (p = 1 \pmod{4}) \quad (7.438)$$

$$\langle a, b \mid baaab, pa - bab - baaa - xbaaaa, pb + 2baaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.439)$$

$$\langle a, b \mid baaab, pa - bab - \omega baaa - xbaaaa, pb + 2\omega^2baaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.440)$$

$$\langle a, b \mid baaab, pa - bab - baaa - xbaaaa, pb + 2baaaa, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 3 \pmod{4}) \quad (7.441)$$

8.33 Descendants of 6.414

$(1/2)(p + 1)(p - 1 + \gcd(p - 1, 3))$ algebras

$$\langle a, b \mid pa - \omega bab - xbaaaa, pb - baaa - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (\pm x, ay) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.442)$$

$$\langle a, b \mid pa - \omega bab - xbaaaa, pb - \omega baaa - ybaaaa, babb + baaaa, \text{class } 5 \rangle ((x, y) \sim (\pm x, ay) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.443)$$

$$\langle a, b \mid pa - \omega bab - xbaaaa, pb - \omega^2baaa - ybaaaa, babb + \omega baaaa, \text{class } 5 \rangle ((x, y) \sim (\pm x, ay) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.444)$$

$$\langle a, b \mid pa - \omega bab - xbaaaa, pb - baaa - ybaaaa, \text{class } 5 \rangle ((x, y) \sim (-x, y), p = 2 \pmod{3}) \quad (7.445)$$

8.34 Descendants of 6.417

$2 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid baaaa, pa - \omega bab, pb, \text{class } 5 \rangle \quad (7.446)$$

$$\langle a, b \mid baaaa, pa - \omega bab, pb - baaab, \text{class } 5 \rangle \quad (7.447)$$

$$\langle a, b \mid baaaa, pa - \omega bab, pb - \omega baaab, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.448)$$

$$\langle a, b \mid baaaa, pa - \omega bab, pb - \omega^2baaab, \text{class } 5 \rangle (p = 1 \pmod{3}) \quad (7.449)$$

$$\langle a, b \mid baaab, pa - \omega bab, pb, \text{class } 5 \rangle \quad (7.450)$$

$$\langle a, b \mid baaab, pa - \omega bab - baaaa, pb, \text{class } 5 \rangle \quad (7.451)$$

$$\langle a, b \mid baaab, pa - \omega bab - \omega baaaa, pb, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.452)$$

$$\langle a, b \mid baaab, pa - \omega bab - \omega^2 baaaa, pb, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.453)$$

$$\langle a, b \mid baaab, pa - \omega bab, pb - baaaa, \text{class } 5 \rangle \quad (7.454)$$

$$\langle a, b \mid baaab, pa - \omega bab, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.455)$$

$$\langle a, b \mid baaab, pa - \omega bab, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.456)$$

$$\langle a, b \mid baaab, pa - \omega bab, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.457)$$

8.35 Descendants of 6.418

$p - 1 + (1/2)(p + 1) \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid baaaa, pa - \omega bab - baaa, pb - xbaaab, \text{class } 5 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.458)$$

$$\langle a, b \mid baaaa, pa - \omega bab - \omega baaa, pb - xbaaab, babb + baaab, \text{class } 5 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.459)$$

$$\langle a, b \mid baaaa, pa - \omega bab - baaa, pb - xbaaab, \text{class } 5 \rangle (x \sim -x, p = 3 \bmod 4) \quad (7.460)$$

$$\langle a, b \mid baaab, pa - \omega bab - baaa, pb - xbaaaa, \text{class } 5 \rangle \quad (7.461)$$

$$\langle a, b \mid baaab, pa - \omega bab - \omega baaa, pb - xbaaaa, baab + baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.462)$$

$$\langle a, b \mid baaab, pa - \omega bab - baaa - xbaaaa, pb + 2\omega^{-1}baaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.463)$$

$$\langle a, b \mid baaab, pa - \omega bab - \omega baaa - xbaaaa, pb + 2\omega baaaa, baab + baaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.464)$$

$$\langle a, b \mid baaab, pa - \omega bab - baaa - xbaaaa, pb + 2\omega^{-1}baaaa, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 3 \bmod 4) \quad (7.465)$$

8.36 Descendants of 6.420

$5p + 1 + \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid babba, pa - baa - xbabbb, pb - babbb, \text{class } 5 \rangle \quad (7.466)$$

$$\langle a, b \mid babba, pa - baa - xbabbb, pb - \omega babbb, \text{class } 5 \rangle \quad (7.467)$$

$$\langle a, b \mid babba, pa - baa, pb, \text{class } 5 \rangle \quad (7.468)$$

$$\langle a, b \mid babba, pa - baa - babbb, pb, \text{class } 5 \rangle \quad (7.469)$$

$$\langle a, b \mid babba, pa - baa - \omega babbb, pb, \text{class } 5 \rangle \quad (7.470)$$

$$\langle a, b \mid babba, pa - baa - \omega^2 babbb, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (7.471)$$

$$\langle a, b \mid babba, pa - baa - \omega^3 babbb, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (7.472)$$

$$\langle a, b \mid babbb, pa - baa, pb - xabba, \text{class } 5 \rangle \quad (7.473)$$

$$\langle a, b \mid babbb - babba, pa - baa, pb - xabba, \text{class } 5 \rangle \quad (7.474)$$

$$\langle a, b \mid babbb - \omega babba, pa - baa, pb - xabba, \text{class } 5 \rangle \quad (7.475)$$

8.37 Descendants of 6.421

$2p^2 - p - 1 + (p + 1)\gcd(p - 1, 3)$ algebras

$$\langle a, b \mid babba, pa - baa - babb - xbabbb, pb - ybabbb, \text{class } 5 \rangle \quad (x \neq 0, (x, y) \sim (ax, a^2y) \text{ if } a^3 = 1) \quad (7.476)$$

$$\langle a, b \mid babba, pa - baa - \omega babb - xbabbb, pb - ybabbb, \text{class } 5 \rangle \quad (x \neq 0, (x, y) \sim (ax, a^2y) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.477)$$

$$\langle a, b \mid babba, pa - baa - \omega^2 babb - xbabbb, pb - ybabbb, \text{class } 5 \rangle \quad (x \neq 0, (x, y) \sim (ax, a^2y) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.478)$$

$$\langle a, b \mid babba, pa - baa - babb, pb - xbabbb, \text{class } 5 \rangle \quad (x \sim ax \text{ if } a^3 = 1) \quad (7.479)$$

$$\langle a, b \mid babba, pa - baa - \omega babb, pb - xbabbb, \text{class } 5 \rangle \quad (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.480)$$

$$\langle a, b \mid babba, pa - baa - \omega^2 babb, pb - xbabbb, \text{class } 5 \rangle \quad (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.481)$$

$$\langle a, b \mid babbb - xabba, pa - baa - babb, pb - yabba, \text{class } 5 \rangle \quad ((x, y) \sim (ax, y) \text{ if } a^3 = 1) \quad (7.482)$$

$$\langle a, b \mid babbb - xabba, pa - baa - \omega babb, pb - yabba, \text{class } 5 \rangle \quad ((x, y) \sim (ax, y) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.483)$$

$$\langle a, b \mid babbb - xabba, pa - baa - \omega^2 babb, pb - yabba, \text{class } 5 \rangle \quad ((x, y) \sim (ax, y) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.484)$$

8.38 Descendants of 6.424

p^2 algebras

$$\langle a, b \mid pa - baa - xbabb, pb - babb - ybabba, \text{class } 5 \rangle \quad (7.485)$$

8.39 Descendants of 6.425

3 algebras

$$\langle a, b \mid baabb, pa - baa, pb + bab, \text{class } 5 \rangle \quad (7.486)$$

$$\langle a, b \mid baabb - baaba, pa - baa, pb + bab, \text{class } 5 \rangle \quad (7.487)$$

$$\langle a, b \mid baabb - \omega baaba, pa - baa, pb + bab, \text{class } 5 \rangle \quad (7.488)$$

8.40 Descendants of 6.426

p algebras

$$\langle a, b \mid pa - baa - baab, pb + bab - xbaab, \text{class } 5 \rangle \quad (7.489)$$

8.41 Descendants of 6.427

$p + 1$ algebras

$$\langle a, b \mid pa + bab, pb + \omega baa - xbaaa, baaab, \text{class } 5 \rangle (x \sim -x) \quad (7.490)$$

In the following two presentation, y is chosen so as to be any *one* integer such that $1 - \omega y^2$ is not a square mod p :

$$\langle a, b \mid pa + bab - xybaaa, pb + \omega baa - xbaaa, baaab - \omega ybaaaa, \text{class } 5 \rangle (x \neq 0, (x, y) \sim (-x, y)) \quad (7.491)$$

$$\langle a, b \mid pa + bab, pb + \omega baa, baaab + \omega ybaaaa, \text{class } 5 \rangle \quad (7.492)$$

8.42 Descendants of 6.428

2 algebras

$$\langle a, b \mid ba, p^2b, \text{class } 5 \rangle \quad (7.493)$$

$$\langle a, b \mid ba - p^4a, p^2b, \text{class } 5 \rangle \quad (7.494)$$

8.43 Descendants of 6.429

1 algebra

$$\langle a, b \mid ba - p^3a, p^2b, \text{class } 5 \rangle \quad (7.495)$$

8.44 Descendants of 6.431

4 algebras

$$\langle a, b \mid baa, bab, pb, \text{class } 5 \rangle \quad (7.496)$$

$$\langle a, b \mid baa, bab - p^4a, pb, \text{class } 5 \rangle \quad (7.497)$$

$$\langle a, b \mid baa, bab - \omega p^4a, pb, \text{class } 5 \rangle \quad (7.498)$$

$$\langle a, b \mid baa - p^4a, bab, pb, \text{class } 5 \rangle \quad (7.499)$$

8.45 Descendants of 6.435

$7 + 2 \gcd(p - 1, 3) + 2 \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid baaaa, bab, p^2a, pb, \text{class } 5 \rangle \quad (7.500)$$

$$\langle a, b \mid baaaa, bab, p^2a - baaab, pb, \text{class } 5 \rangle \quad (7.501)$$

$$\langle a, b \mid baaaa, bab, p^2a - \omega baaab, pb, \text{class } 5 \rangle \quad (7.502)$$

$$\langle a, b \mid baaaa, bab, p^2a, pb - baaab, \text{class } 5 \rangle \quad (7.503)$$

$$\langle a, b \mid baaab, bab, p^2a - baaaa, pb, \text{class } 5 \rangle \quad (7.504)$$

$$\langle a, b \mid baaab, bab - baaaa, p^2a - baaaa, pb, \text{class } 5 \rangle \quad (7.505)$$

$$\langle a, b \mid baaab, bab - \omega baaaa, p^2a - baaaa, pb, \text{class } 5 \rangle \quad (7.506)$$

$$\langle a, b \mid baaab, bab - \omega^2 baaaa, p^2a - baaaa, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{3}) \quad (7.507)$$

$$\langle a, b \mid baaab, bab - \omega^3 baaaa, p^2a - baaaa, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{3}) \quad (7.508)$$

$$\langle a, b \mid baaab, bab - \omega^4 baaaa, p^2a - baaaa, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{3}) \quad (7.509)$$

$$\langle a, b \mid baaab, bab - \omega^5 baaaa, p^2a - baaaa, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{3}) \quad (7.510)$$

$$\langle a, b \mid baaab, bab, p^2a, pb, \text{class } 5 \rangle \quad (7.511)$$

$$\langle a, b \mid baaab, bab - baaaa, p^2a, pb, \text{class } 5 \rangle \quad (7.512)$$

$$\langle a, b \mid baaab, bab, p^2a, pb - baaaa, \text{class } 5 \rangle \quad (7.513)$$

$$\langle a, b \mid baaab, bab - baaaa, p^2a, pb - baaaa, \text{class } 5 \rangle \quad (7.514)$$

$$\langle a, b \mid baaab, bab, p^2a, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.515)$$

$$\langle a, b \mid baaab, bab - baaaa, p^2a, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.516)$$

$$\langle a, b \mid baaab, bab, p^2a, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.517)$$

$$\langle a, b \mid baaab, bab - baaaa, p^2a, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.518)$$

$$\langle a, b \mid baaab, bab, p^2a, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.519)$$

$$\langle a, b \mid baaab, bab - baaaa, p^2a, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.520)$$

8.46 Descendants of 6.436

$2 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4) + 2 \gcd(p - 1, 5)$ algebras

$$\langle a, b \mid baaaa, bab - baaa, p^2a, pb, \text{class } 5 \rangle \quad (7.521)$$

$$\langle a, b \mid baaaa, bab - baaa, p^2a - baaab, pb, \text{class } 5 \rangle \quad (7.522)$$

$$\langle a, b \mid baaaa, bab - baaa, p^2a - \omega baaab, pb, \text{class } 5 \rangle \quad (7.523)$$

$$\langle a, b \mid baaaa, bab - baaa, p^2a - xbaaab, pb, \text{class } 5 \rangle (x = \omega^2, \omega^3, \omega^4, \omega^5, p = 1 \bmod 3) \quad (7.524)$$

$$\langle a, b \mid baaaa, bab - baaa, p^2a, pb - baaab, \text{class } 5 \rangle \quad (7.525)$$

$$\langle a, b \mid baaaa, bab - baaa, p^2a, pb - xbaaab, \text{class } 5 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.526)$$

$$\langle a, b \mid baaab, bab - baaa, p^2a, pb, \text{class } 5 \rangle \quad (7.527)$$

$$\langle a, b \mid baaab, bab - baaa, p^2a - baaaa, pb, \text{class } 5 \rangle \quad (7.528)$$

$$\langle a, b \mid baaab, bab - baaa, p^2a - xbaaaa, pb, \text{class } 5 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.529)$$

$$\langle a, b \mid baaab, bab - baaa, p^2a, pb - baaaa, \text{class } 5 \rangle \quad (7.530)$$

$$\langle a, b \mid baaab, bab - baaa, p^2a, pb - \omega baaaa, \text{class } 5 \rangle \quad (7.531)$$

$$\langle a, b \mid baaab, bab - baaa, p^2a, pb - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.532)$$

$$\langle a, b \mid baaab, bab - baaa, p^2a, pb - \omega^3 baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.533)$$

8.47 Descendants of 6.442

$2p - 2 + (p + 2) \gcd(p - 1, 3)$ algebras

$$\langle a, b \mid bab, p^2a, pb - baaa, \text{class } 5 \rangle \quad (7.534)$$

$$\langle a, b \mid bab, p^2a - baaaa, pb - baaa, \text{class } 5 \rangle \quad (7.535)$$

$$\langle a, b \mid bab - baaaa, p^2a - xbaaaa, pb - baaa, \text{class } 5 \rangle \quad (7.536)$$

$$\langle a, b \mid bab, p^2a, pb - \omega baaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.537)$$

$$\langle a, b \mid bab, p^2a - baaaa, pb - \omega baaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.538)$$

$$\langle a, b \mid bab - baaaa, p^2a - xbaaaa, pb - \omega baaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.539)$$

$$\langle a, b \mid bab, p^2a, pb - \omega^2 baaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.540)$$

$$\langle a, b \mid bab, p^2a - baaaa, pb - \omega^2 baaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.541)$$

$$\langle a, b \mid bab - baaaa, p^2a - xbaaaa, pb - \omega^2 baaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.542)$$

$$\langle a, b \mid bab, p^2a, pb - baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, p = 2 \bmod 3) \quad (7.543)$$

$$\langle a, b \mid bab - baaaa, p^2a, pb - baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, p = 2 \bmod 3) \quad (7.544)$$

$$\langle a, b \mid bab, p^2a, pb - baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.545)$$

$$\langle a, b \mid bab - baaaa, p^2a, pb - baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.546)$$

$$\langle a, b \mid bab, p^2a, pb - \omega baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.547)$$

$$\langle a, b \mid bab - baaaa, p^2a, pb - \omega baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.548)$$

$$\langle a, b \mid bab, p^2a, pb - \omega^2 baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.549)$$

$$\langle a, b \mid bab - baaaa, p^2a, pb - \omega^2 baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.550)$$

8.48 Descendants of 6.445

$2p - 2 + \gcd(p - 1, 3)$ algebras

$$\langle a, b \mid bab - baaa, p^2a - xbaaaa, pb - baaa, \text{class } 5 \rangle (p = 2 \bmod 3) \quad (7.551)$$

$$\langle a, b \mid bab - baaa, p^2a, pb - baaa - xbaaaa, \text{class } 5 \rangle (p = 2 \bmod 3) \quad (7.552)$$

$$\langle a, b \mid bab - baaa, p^2a - xbaaaa, pb - baaa, \text{class } 5 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.553)$$

$$\langle a, b \mid bab - baaa, p^2a, pb - baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.554)$$

$$\langle a, b \mid bab - baaa, p^2a - xbaaaa, pb - \omega baaa, \text{class } 5 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.555)$$

$$\langle a, b \mid bab - baaa, p^2a, pb - \omega baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.556)$$

$$\langle a, b \mid bab - baaa, p^2a - xbaaaa, pb - \omega^2 baaa, \text{class } 5 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.557)$$

$$\langle a, b \mid bab - baaa, p^2a, pb - \omega^2 baaa - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.558)$$

8.49 Descendants of 6.448

$(p + 1)/2$ algebras

$$\langle a, b \mid bab, p^2a, pb - baa - xbaaa, \text{class } 5 \rangle (x \sim -x) \quad (7.559)$$

8.50 Descendants of 6.451

$(p + 1)/2$ algebras

$$\langle a, b \mid bab, p^2a, pb - \omega baa - xbaaa, \text{class } 5 \rangle (x \sim -x) \quad (7.560)$$

8.51 Descendants of 6.454

$8 + 2\gcd(p - 1, 3) + 4\gcd(p - 1, 4)$ algebras

$$\langle a, b \mid babba, baa, p^2a, pb, \text{class } 5 \rangle \quad (7.561)$$

$$\langle a, b \mid babba, baa, p^2a, pb - babbb, \text{class } 5 \rangle \quad (7.562)$$

$$\langle a, b \mid babba, baa - babbb, p^2a, pb, \text{class } 5 \rangle \quad (7.563)$$

$$\langle a, b \mid babba, baa - babbb, p^2a, pb - babbb, \text{class } 5 \rangle \quad (7.564)$$

$$\langle a, b \mid babba, baa - babbb, p^2a, pb - \omega babbb, \text{class } 5 \rangle \quad (7.565)$$

$$\langle a, b \mid babba, baa - babbb, p^2a, pb - xbabbb, \text{class } 5 \rangle (x = \omega^2, \omega^3, \omega^4, \omega^5, p = 1 \bmod 3) \quad (7.566)$$

$$\langle a, b \mid babba, baa, p^2a - babbb, pb, \text{class } 5 \rangle \quad (7.567)$$

$$\langle a, b \mid babba, baa, p^2a - \omega babb, pb, \text{class } 5 \rangle \quad (7.568)$$

$$\langle a, b \mid babba, baa, p^2a - \omega^2 babb, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.569)$$

$$\langle a, b \mid babba, baa, p^2a - \omega^3 babb, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.570)$$

$$\langle a, b \mid babba, baa - babb, p^2a - babb, pb, \text{class } 5 \rangle \quad (7.571)$$

$$\langle a, b \mid babba, baa - babb, p^2a - \omega babb, pb, \text{class } 5 \rangle \quad (7.572)$$

$$\langle a, b \mid babba, baa - babb, p^2a - \omega^2 babb, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.573)$$

$$\langle a, b \mid babba, baa - babb, p^2a - \omega^3 babb, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.574)$$

$$\langle a, b \mid babb, baa, p^2a, pb, \text{class } 5 \rangle \quad (7.575)$$

$$\langle a, b \mid babb, baa, p^2a - babba, pb, \text{class } 5 \rangle \quad (7.576)$$

$$\langle a, b \mid babb, baa, p^2a, pb - babba, \text{class } 5 \rangle \quad (7.577)$$

$$\langle a, b \mid babb, baa, p^2a, pb - \omega babba, \text{class } 5 \rangle \quad (7.578)$$

$$\langle a, b \mid babb - babba, baa, p^2a, pb, \text{class } 5 \rangle \quad (7.579)$$

$$\langle a, b \mid babb - babba, baa, p^2a - babba, pb, \text{class } 5 \rangle \quad (7.580)$$

$$\langle a, b \mid babb - babba, baa, p^2a - \omega babba, pb, \text{class } 5 \rangle \quad (7.581)$$

$$\langle a, b \mid babb - babba, baa, p^2a - \omega^2 babba, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.582)$$

$$\langle a, b \mid babb - babba, baa, p^2a - \omega^3 babba, pb, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.583)$$

$$\langle a, b \mid babb - babba, baa, p^2a, pb - babba, \text{class } 5 \rangle \quad (7.584)$$

$$\langle a, b \mid babb - babba, baa, p^2a, pb - \omega babba, \text{class } 5 \rangle \quad (7.585)$$

$$\langle a, b \mid babb - babba, baa, p^2a, pb - \omega^2 babba, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.586)$$

$$\langle a, b \mid babb - babba, baa, p^2a, pb - \omega^3 babba, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.587)$$

8.52 Descendants of 6.455

4 algebras

$$\langle a, b \mid baa, p^2a, pb - babb, \text{class } 5 \rangle \quad (7.588)$$

$$\langle a, b \mid baa, p^2a - babba, pb - babb, \text{class } 5 \rangle \quad (7.589)$$

$$\langle a, b \mid baa, p^2a, pb - babb - babba, \text{class } 5 \rangle \quad (7.590)$$

$$\langle a, b \mid baa, p^2a, pb - babb - \omega babba, \text{class } 5 \rangle \quad (7.591)$$

8.53 Descendants of 6.459

$4p + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4) + 2 \gcd(p - 1, 5)$ algebras

$$\langle a, b \mid babba, baa - babb, p^2a, pb, \text{class } 5 \rangle \quad (7.592)$$

$$\langle a, b \mid babba, baa - babb, p^2a - babbb, pb, \text{class } 5 \rangle \quad (7.593)$$

$$\langle a, b \mid babba, baa - babb, p^2a - \omega babbb, pb, \text{class } 5 \rangle \quad (7.594)$$

$$\langle a, b \mid babba, baa - babb, p^2a - \omega^2 babbb, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (7.595)$$

$$\langle a, b \mid babba, baa - babb, p^2a - \omega^3 babbb, pb, \text{class } 5 \rangle \quad (p = 1 \pmod{4}) \quad (7.596)$$

$$\langle a, b \mid babba, baa - babb, p^2a, pb - babbb, \text{class } 5 \rangle \quad (7.597)$$

$$\langle a, b \mid babba, baa - babb, p^2a, pb - xbabb, \text{class } 5 \rangle \quad (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \pmod{5}) \quad (7.598)$$

$$\langle a, b \mid babba, baa - babb - babbb, p^2a - xbabb, pb, \text{class } 5 \rangle \quad (7.599)$$

$$\langle a, b \mid babba, baa - babb - babbb, p^2a, pb - xbabb, \text{class } 5 \rangle \quad (x \neq 0) \quad (7.600)$$

$$\langle a, b \mid babbb, baa - babb, p^2a, pb, \text{class } 5 \rangle \quad (7.601)$$

$$\langle a, b \mid babbb, baa - babb, p^2a - babba, pb, \text{class } 5 \rangle \quad (7.602)$$

$$\langle a, b \mid babbb, baa - babb, p^2a - xbabba, pb, \text{class } 5 \rangle \quad (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \pmod{5}) \quad (7.603)$$

$$\langle a, b \mid babbb, baa - babb, p^2a, pb - babba, \text{class } 5 \rangle \quad (7.604)$$

$$\langle a, b \mid babbb, baa - babb, p^2a, pb - \omega babba, \text{class } 5 \rangle \quad (7.605)$$

$$\langle a, b \mid babbb, baa - babb, p^2a, pb - xbabba, \text{class } 5 \rangle \quad (x = \omega^2, \omega^3, \omega^4, \omega^5, p = 1 \pmod{3}) \quad (7.606)$$

$$\langle a, b \mid babbb - babba, baa - babb, p^2a - xbabba, pb, \text{class } 5 \rangle \quad (7.607)$$

$$\langle a, b \mid babbb - babba, baa - babb, p^2a, pb - xbabba, \text{class } 5 \rangle \quad (x \neq 0) \quad (7.608)$$

8.54 Descendants of 6.460

$3p - 3 + \gcd(p - 1, 4)$ algebras

$$\langle a, b \mid baa - babb, p^2a - xbabba, pb - babb, \text{class } 5 \rangle (x \sim -x, p = 3 \bmod 4) \quad (7.609)$$

$$\langle a, b \mid baa - babb, p^2a - xbabba, pb - \omega babb, \text{class } 5 \rangle (x \sim -x, p = 3 \bmod 4) \quad (7.610)$$

$$\langle a, b \mid baa - babb, p^2a, pb - babb - xbabba, \text{class } 5 \rangle (x \neq 0, p = 3 \bmod 4) \quad (7.611)$$

$$\langle a, b \mid baa - babb, p^2a, pb - \omega babb - xbabba, \text{class } 5 \rangle (x \neq 0, p = 3 \bmod 4) \quad (7.612)$$

$$\langle a, b \mid baa - babb, p^2a - xbabba, pb - babb, \text{class } 5 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.613)$$

$$\langle a, b \mid baa - babb, p^2a, pb - babb - xbabba, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (7.614)$$

$$\langle a, b \mid baa - babb, p^2a - xbabba, pb - \omega babb, \text{class } 5 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.615)$$

$$\langle a, b \mid baa - babb, p^2a, pb - \omega babb - xbabba, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (7.616)$$

$$\langle a, b \mid baa - babb, p^2a - xbabba, pb - \omega^2 babb, \text{class } 5 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.617)$$

$$\langle a, b \mid baa - babb, p^2a, pb - \omega^2 babb - xbabba, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (7.618)$$

$$\langle a, b \mid baa - babb, p^2a - xbabba, pb - \omega^3 babb, \text{class } 5 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.619)$$

$$\langle a, b \mid baa - babb, p^2a, pb - \omega^3 babb - xbabba, \text{class } 5 \rangle (x \neq 0, x \sim -x, p = 1 \bmod 4) \quad (7.620)$$

8.55 Descendants of 6.467

2 algebras

$$\langle a, b \mid baa, pb - ba, \text{class } 5 \rangle \quad (7.621)$$

$$\langle a, b \mid baa - p^4a, pb - ba, \text{class } 5 \rangle \quad (7.622)$$

8.56 Descendants of 6.469

$4 + 2\gcd(p - 1, 3) + 3\gcd(p - 1, 5) + \gcd(p - 1, 8)$ algebras

$$\langle a, b \mid bab, baaab, pa, pb, \text{class } 6 \rangle \quad (7.623)$$

$$\langle a, b \mid bab, baaab, pa, pb - baaaaa, \text{class } 6 \rangle \quad (7.624)$$

$$\langle a, b \mid bab, baaab, pa, pb - xbaaaaa, \text{class } 6 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.625)$$

$$\langle a, b \mid bab, baaab, pa - baaaaa, pb, \text{class } 6 \rangle \quad (7.626)$$

$$\langle a, b \mid bab, baaab - baaaa, pa, pb, \text{ class } 6 \rangle \quad (7.627)$$

$$\langle a, b \mid bab, baaab - baaaa, pa, pb - baaaa, \text{ class } 6 \rangle \quad (7.628)$$

$$\langle a, b \mid bab, baaab - baaaa, pa, pb - xbaaaaa, \text{ class } 6 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.629)$$

$$\langle a, b \mid bab, baaab - baaaa, pa - baaaa, pb, \text{ class } 6 \rangle \quad (7.630)$$

$$\langle a, b \mid bab, baaab - baaaa, pa - \omega baaaa, pb, \text{ class } 6 \rangle \quad (7.631)$$

$$\langle a, b \mid bab, baaab - baaaa, pa - xbaaaaa, pb, \text{ class } 6 \rangle (x = \omega^2, \omega^3, \omega^4, \omega^5, p = 1 \bmod 3) \quad (7.632)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb, \text{ class } 6 \rangle \quad (7.633)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb - baaaa, \text{ class } 6 \rangle \quad (7.634)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb - xbaaaaa, \text{ class } 6 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.635)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - baaaa, pb, \text{ class } 6 \rangle \quad (7.636)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - \omega baaaa, pb, \text{ class } 6 \rangle \quad (7.637)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - \omega^2 baaaa, pb, \text{ class } 6 \rangle (p = 1 \bmod 4) \quad (7.638)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - \omega^3 baaaa, pb, \text{ class } 6 \rangle (p = 1 \bmod 4) \quad (7.639)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - xbaaaaa, pb, \text{ class } 6 \rangle (x = \omega^4, \omega^5, \omega^6, \omega^7, p = 1 \bmod 8) \quad (7.640)$$

8.57 Descendants of 6.475

$4p - 1 + \gcd(p - 1, 5) + \gcd(p - 1, 7)$ algebras

$$\langle a, b \mid bab - baaaa, baaab, pa, pb, \text{ class } 6 \rangle \quad (7.641)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb - baaaa, \text{ class } 6 \rangle \quad (7.642)$$

$$\langle a, b \mid bab - baaaa, baaab, pa, pb - xbaaaaa, \text{ class } 6 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.643)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - baaaa, pb, \text{ class } 6 \rangle \quad (7.644)$$

$$\langle a, b \mid bab - baaaa, baaab, pa - xbaaaaa, pb, \text{ class } 6 \rangle (x = \omega, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6, p = 1 \bmod 7) \quad (7.645)$$

$$\langle a, b \mid bab - baaaa, baaab - baaaa, pa, pb - xbaaaaa, \text{ class } 6 \rangle \quad (7.646)$$

$$\langle a, b \mid bab - baaaa, baaab - baaaa, pa - xbaaaaa, pb, \text{class } 6 \rangle (x \neq 0) \quad (7.647)$$

$$\langle a, b \mid bab - baaaa - baaaa, baaab, pa, pb - xbaaaaa, \text{class } 6 \rangle \quad (7.648)$$

$$\langle a, b \mid bab - baaaa - baaaa, baaab, pa - xbaaaaa, pb, \text{class } 6 \rangle (x \neq 0) \quad (7.649)$$

8.58 Descendants of 6.507

$2p^2 + p + 2p \gcd(p-1, 3) + p \gcd(p-1, 5)$ algebras

$$\langle a, b \mid bab - baaa, baaab - xbaaaaa, pa, pb, \text{class } 6 \rangle \quad (7.650)$$

$$\langle a, b \mid bab - baaa, baaab - xbaaaaa, pa - baaaa, pb, \text{class } 6 \rangle \quad (7.651)$$

$$\langle a, b \mid bab - baaa, baaab - xbaaaaa, pa - \omega baaaa, pb, \text{class } 6 \rangle \quad (7.652)$$

$$\langle a, b \mid bab - baaa, baaab - xbaaaaa, pa - ybaaaaa, pb, \text{class } 6 \rangle (y = \omega^2, \omega^3, \omega^4, \omega^5, p = 1 \pmod{3}) \quad (7.653)$$

$$\langle a, b \mid bab - baaa, baaab - xbaaaaa, pa - ybaaaaa, pb - baaaa, \text{class } 6 \rangle (p \neq 1 \pmod{5}) \quad (7.654)$$

In the presentation below we have $z = 1, \omega, \omega^2, \omega^3, \omega^4$, $(x, y, z) \sim (x, ay, z)$ if $a^5 = 1$, $p = 1 \pmod{5}$:

$$\langle a, b \mid bab - baaa, baaab - xbaaaaa, pa - ybaaaaa, pb - zbaaaaa, \text{class } 6 \rangle \quad (7.655)$$

$$\langle a, b \mid bab - baaa - baaaa, baaab - baaaa, pa - xbaaaaa, pb - ybaaaaa, \text{class } 6 \rangle ((x, y) \sim (x, -y)) \quad (7.656)$$

$$\langle a, b \mid bab - baaa - \omega baaaa, baaab - baaaa, pa - xbaaaaa, pb - ybaaaaa, \text{class } 6 \rangle ((x, y) \sim (x, -y)) \quad (7.657)$$

8.59 Descendants of 6.518

2 algebras

$$\langle a, b \mid ba, pb, \text{class } 6 \rangle \quad (7.658)$$

$$\langle a, b \mid ba - p^5 a, pb, \text{class } 6 \rangle \quad (7.659)$$

8.60 Descendants of 3.1

$p + 14$ algebras

$$\langle a, b, c \mid ca, cb, \text{class } 2 \rangle \quad (7.660)$$

$$\langle a, b, c \mid cb, pa, \text{class } 2 \rangle \quad (7.661)$$

$$\langle a, b, c \mid cb, pa - ba, \text{class } 2 \rangle \quad (7.662)$$

$$\langle a, b, c \mid cb, pb, \text{class } 2 \rangle \quad (7.663)$$

$$\langle a, b, c \mid cb, pb - ba, \text{class } 2 \rangle \quad (7.664)$$

$$\langle a, b, c \mid cb, pb - ca, \text{class } 2 \rangle \quad (7.665)$$

$$\langle a, b, c \mid pb, pc, \text{class } 2 \rangle \quad (7.666)$$

$$\langle a, b, c \mid pb - ba, pc, \text{class } 2 \rangle \quad (7.667)$$

$$\langle a, b, c \mid pb - ca, pc, \text{class } 2 \rangle \quad (7.668)$$

$$\langle a, b, c \mid pb - cb, pc, \text{class } 2 \rangle \quad (7.669)$$

$$\langle a, b, c \mid pb - ba, pc - cb, \text{class } 2 \rangle \quad (7.670)$$

$$\langle a, b, c \mid pb - ca, pc - cb, \text{class } 2 \rangle \quad (7.671)$$

$$\langle a, b, c \mid pb - ba, pc - ca, \text{class } 2 \rangle \quad (7.672)$$

$$\langle a, b, c \mid pb - ca, pc - ba, \text{class } 2 \rangle \quad (7.673)$$

$$\langle a, b, c \mid pb - \omega ca, pc - ba, \text{class } 2 \rangle \quad (7.674)$$

$$\langle a, b, c \mid pb - xca, pc - ba - ca, \text{class } 2 \rangle \quad (x \neq 0) \quad (7.675)$$

8.61 Descendants of 5.10

$2p + 7$ algebras

$$\langle a, b, c \mid ca, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.676)$$

$$\langle a, b, c \mid ca - baa, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.677)$$

$$\langle a, b, c \mid ca - p^2a, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.678)$$

$$\langle a, b, c \mid ca - xp^2a, cb - baa, pb - ba, pc, \text{class } 3 \rangle \quad (7.679)$$

$$\langle a, b, c \mid ca - baa - p^2a, cb - baa, pb - ba, pc, \text{class } 3 \rangle \quad (7.680)$$

$$\langle a, b, c \mid ca - xbaa - p^2a, cb - p^2a, pb - ba, pc, \text{class } 3 \rangle \quad (7.681)$$

$$\langle a, b, c \mid ca, cb - p^2a, pb - ba, pc, \text{class } 3 \rangle \quad (7.682)$$

$$\langle a, b, c \mid ca - baa, cb - p^2a, pb - ba, pc, \text{class } 3 \rangle \quad (7.683)$$

$$\langle a, b, c \mid ca - \omega baa, cb - p^2a, pb - ba, pc, \text{class } 3 \rangle \quad (7.684)$$

8.62 Descendants of 5.12

$3p^2 + 17p + 53 + \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid bab, ca, cb, pa, pb, \text{class } 3 \rangle \quad (7.685)$$

$$\langle a, b, c \mid bab, ca, cb, pa - baa, pb, \text{class } 3 \rangle \quad (7.686)$$

$$\langle a, b, c \mid bab, ca, cb, pa, pb - baa, \text{class } 3 \rangle \quad (7.687)$$

$$\langle a, b, c \mid bab, ca, cb, pa, pb - \omega baa, \text{class } 3 \rangle \quad (7.688)$$

$$\langle a, b, c \mid bab, ca - p^2c, cb, pa, pb - xbaa, \text{class } 3 \rangle \quad (7.689)$$

$$\langle a, b, c \mid bab, ca - p^2c, cb, pa - baa, pb, \text{class } 3 \rangle \quad (7.690)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, \text{class } 3 \rangle \quad (7.691)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa - baa, pb, \text{class } 3 \rangle \quad (7.692)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - baa, \text{class } 3 \rangle \quad (7.693)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega baa, \text{class } 3 \rangle \quad (7.694)$$

$$\langle a, b, c \mid bab, ca - p^2c, cb - baa, pa, pb - xbaa, \text{class } 3 \rangle \quad (7.695)$$

$$\langle a, b, c \mid bab, ca - p^2c, cb - baa, pa - baa, pb, \text{class } 3 \rangle \quad (7.696)$$

$$\langle a, b, c \mid bab, ca, cb - p^2c, pa, pb, \text{class } 3 \rangle \quad (7.697)$$

$$\langle a, b, c \mid bab, ca, cb - p^2c, pa, pb - baa, \text{class } 3 \rangle \quad (7.698)$$

$$\langle a, b, c \mid bab, ca, cb - p^2c, pa, pb - \omega baa, \text{class } 3 \rangle \quad (7.699)$$

$$\langle a, b, c \mid bab, ca, cb - p^2c, pa - baa, pb - xbaa, \text{class } 3 \rangle \quad (7.700)$$

$$\langle a, b, c \mid bab, ca, cb - baa - p^2c, pa, pb, \text{class } 3 \rangle \quad (7.701)$$

$$\langle a, b, c \mid bab, ca, cb - baa - p^2c, pa, pb - baa, \text{class } 3 \rangle \quad (7.702)$$

$$\langle a, b, c \mid bab, ca, cb - baa - p^2c, pa, pb - \omega baa, \text{class } 3 \rangle \quad (7.703)$$

$$\langle a, b, c \mid bab, ca, cb - baa - p^2c, pa - baa, pb - xbaa, \text{class } 3 \rangle \quad (7.704)$$

$$\langle a, b, c \mid p^2c, ca, cb, pa - baa, pb, \text{class } 3 \rangle \quad (7.705)$$

$$\langle a, b, c | p^2c, ca, cb, pa - baa, pb - xbab, \text{class } 3 \rangle (x \neq 0, x \sim x^{-1}) \quad (7.706)$$

$$\langle a, b, c | p^2c, ca, cb, pa - baa - bab, pb - bab, \text{class } 3 \rangle \quad (7.707)$$

$$\langle a, b, c | p^2c, ca, cb, pa - baa - \omega bab, pb - bab, \text{class } 3 \rangle \quad (7.708)$$

$$\langle a, b, c | p^2c, ca, cb, pa, pb, \text{class } 3 \rangle \quad (7.709)$$

$$\langle a, b, c | p^2c, ca, cb, pa - bab, pb, \text{class } 3 \rangle \quad (7.710)$$

$$\langle a, b, c | p^2c, ca, cb, pa - \omega bab, pb, \text{class } 3 \rangle \quad (7.711)$$

$$\langle a, b, c | p^2c, ca, cb, pa - \omega bab, pb - baa, \text{class } 3 \rangle \quad (7.712)$$

$$\langle a, b, c | p^2c, ca, cb, pa - xbab, pb - baa - bab, \text{class } 3 \rangle (1 + 4x \text{ not a square}) \quad (7.713)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa, pb, \text{class } 3 \rangle \quad (7.714)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa, pb - baa, \text{class } 3 \rangle \quad (7.715)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa, pb - \omega baa, \text{class } 3 \rangle \quad (7.716)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa, pb - bab, \text{class } 3 \rangle \quad (7.717)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - baa, pb - xbab, \text{class } 3 \rangle \quad (7.718)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - baa, pb - baa - bab, \text{class } 3 \rangle \quad (7.719)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - baa, pb - \omega baa - bab, \text{class } 3 \rangle \quad (7.720)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - bab, pb - xbaa - bab, \text{class } 3 \rangle \quad (7.721)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - bab, pb, \text{class } 3 \rangle \quad (7.722)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - bab, pb - baa, \text{class } 3 \rangle \quad (7.723)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - bab, pb - \omega baa, \text{class } 3 \rangle \quad (7.724)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - \omega bab, pb - xbaa - bab, \text{class } 3 \rangle \quad (7.725)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - \omega bab, pb, \text{class } 3 \rangle \quad (7.726)$$

$$\langle a, b, c | p^2c, ca, cb - baa, pa - \omega bab, pb - baa, \text{class } 3 \rangle \quad (7.727)$$

$$\langle a, b, c \mid p^2c, ca, cb - baa, pa - \omega bab, pb - \omega baa, \text{ class } 3 \rangle \quad (7.728)$$

In the following presentation we have a four parameter family of descendants of algebra 5.12 of order p^7 . The parameters are x, y, z, t .

We put the parameters x, y, z, t in a matrix $\begin{pmatrix} x & y \\ z & t \end{pmatrix}$, and the distinct algebras correspond to orbits of matrices $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$ with entries in $\text{GF}(p)$ under the action

$$A \rightarrow \frac{1}{\det P} PAP^{-1}$$

where P is the subgroup of $\text{GL}(2, p)$ consisting of non-singular matrices $\begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix}$ or $\begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix}$.

$$\langle a, b, c \mid p^2c, ca - bab, cb - baa, pa - xbaa - ybab, pb - zbaa - tbab, \text{ class } 3 \rangle \quad (7.729)$$

In the following presentation another four parameter family of descendants of algebra 5.12 of order p^7 . The parameters are x, y, z, t .

We put the parameters x, y, z, t in a matrix $\begin{pmatrix} x & y \\ z & t \end{pmatrix}$, and the distinct algebras correspond to orbits of matrices $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$ with entries in $\text{GF}(p)$ under the action

$$A \rightarrow \frac{1}{\det P} PAP^{-1}$$

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$$\langle a, b, c \mid p^2c, ca - \omega bab, cb - baa, pa - xbaa - ybab, pb - zbaa - tbab, \text{ class } 3 \rangle \quad (7.730)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb, pa, pb, \text{ class } 3 \rangle \quad (7.731)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb - baa, pa, pb, \text{ class } 3 \rangle \quad (7.732)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb - bab, pa, pb, \text{ class } 3 \rangle \quad (7.733)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb, pa, pb, \text{ class } 3 \rangle \quad (7.734)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - baa, pa, pb, \text{ class } 3 \rangle \quad (7.735)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - \omega baa, pa, pb, \text{ class } 3 \rangle \quad (7.736)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb, pa, pb - bab, \text{ class } 3 \rangle \quad (7.737)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb - baa, pa, pb - bab, \text{ class } 3 \rangle \quad (7.738)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb - bab, pa, pb - bab, \text{ class } 3 \rangle \quad (7.739)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - xbaa, pa, pb - bab, \text{ class } 3 \rangle (x \sim ax \text{ if } a^3 = 1) \quad (7.740)$$

$$\langle a, b, c \mid p^2c - baa, ca - \omega bab, cb - xbaa, pa, pb - bab, \text{class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.741)$$

$$\langle a, b, c \mid p^2c - baa, ca - \omega^2 bab, cb - xbaa, pa, pb - bab, \text{class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.742)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb - xbaa, pa - bab, pb, \text{class } 3 \rangle (x \sim -x) \quad (7.743)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb - xbaa - bab, pa - bab, pb, \text{class } 3 \rangle (x \sim -x) \quad (7.744)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - xbaa - ybab, pa - bab, pb, \text{class } 3 \rangle ((x, y) \sim (x, -y), p = 3 \bmod 4) \quad (7.745)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - xbaa, pa - bab, pb, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.746)$$

$$\langle a, b, c \mid p^2c - baa, ca - \omega bab, cb - xbaa, pa - bab, pb, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.747)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - xbaa - ybab, pa - bab, pb, \text{class } 3 \rangle (y \neq 0, (x, y) \sim (a^2x, ay) \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.748)$$

$$\langle a, b, c \mid p^2c - baa, ca - \omega bab, cb - xbaa - ybab, pa - bab, pb, \text{class } 3 \rangle (y \neq 0, (x, y) \sim (a^2x, ay) \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.749)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb - xbaa, pa - \omega bab, pb, \text{class } 3 \rangle (x \sim -x) \quad (7.750)$$

$$\langle a, b, c \mid p^2c - baa, ca, cb - xbaa - bab, pa - \omega bab, pb, \text{class } 3 \rangle (x \sim -x) \quad (7.751)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - xbaa - ybab, pa - \omega bab, pb, \text{class } 3 \rangle ((x, y) \sim (x, -y), p = 3 \bmod 4) \quad (7.752)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - xbaa, pa - \omega bab, pb, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.753)$$

$$\langle a, b, c \mid p^2c - baa, ca - \omega bab, cb - xbaa, pa - \omega bab, pb, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.754)$$

$$\langle a, b, c \mid p^2c - baa, ca - bab, cb - xbaa - ybab, pa - \omega bab, pb, \text{class } 3 \rangle (y \neq 0, (x, y) \sim (a^2x, ay) \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.755)$$

$$\langle a, b, c \mid p^2c - baa, ca - \omega bab, cb - xbaa - ybab, pa - \omega bab, pb, \text{class } 3 \rangle (y \neq 0, (x, y) \sim (a^2x, ay) \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.756)$$

8.63 Descendants of 5.14

Algebra 5.14 has

$$2p^5 + 7p^4 + 19p^3 + 49p^2 + 128p + 256 + (p^2 + 7p + 29) \gcd(p-1, 3) \\ + (p^2 + 7p + 24) \gcd(p-1, 4) + (p+3) \gcd(p-1, 5)$$

immediate descendants of order p^7 and p -class 3.

Algebra 5.14 has presentation

$$\langle a, b, c \mid cb, pa, pb, pc, \text{class } 2 \rangle,$$

and so if L is an immediate descendant of 5.14 of order p^7 then L_2 is generated by ba, ca modulo L_3 , and L_3 has order p^2 and is generated by baa, bab, bac, caa, cab . And $cb, pa, pb, pc \in L_3$. The commutator structure is given by the classification of nilpotent Lie algebras over \mathbb{Z}_p . So we may assume that one of the following holds:

$$\begin{aligned} & cb = caa = cab = cac = 0, \\ & caa = cab = cac = 0, cb = baa, \\ & cb = bab = bac = cab = cac = 0, \\ & cb = baa, bab = bac = cab = cac = 0, \\ & cb = bac = cac = 0, caa = bab, \\ & cb = baa, bac = cac = 0, caa = bab, \\ & cb = baa = bac = cac = 0, \\ & baa = bac = cac = 0, cb = caa, \\ & cb = bac = caa = 0, cac = bab, \\ & cb = bac = caa = 0, cac = \omega bab, \\ & bac = caa = 0, cb = baa, cac = bab, \\ & bac = caa = 0, cb = baa, cac = \omega bab, \\ & cb = bac = 0, caa = baa, cac = -bab, \\ & bac = 0, cb = caa = baa, cac = -bab, \\ & cb = baa = bac = caa = 0, \\ & cb = bac = caa = 0, baa = cac, \\ & cb = bac = 0, baa = cac, caa = bab, \\ & cb = bac = 0, baa = cac, caa = \omega bab, (p = 1 \bmod 3) \\ & cb = baa = caa = cac = 0, \\ & cb = baa = cac = 0, caa = bab, \\ & cb = caa = cac = 0, baa = bab, \\ & cb = baa = caa = 0, cac = \omega bab, \\ & cb = baa = 0, caa = bac, cac = \omega bab, \\ & cb = baa = 0, caa = kbab + bac, cac = \omega bab, (p = 2 \bmod 3), \end{aligned}$$

where k is any (one) integer which is not a value of

$$\frac{\lambda(\lambda^2 + 3\omega\mu^2)}{\mu(3\lambda^2 + \omega\mu^2)} \bmod p.$$

Since the total number of descendants of 5.14 of order p^7 is of order $2p^5$, we need presentations with at least 5 parameters in some of these cases. In each case the commutator structure is determined, and so to give a presentation for the Lie rings we only need to specify pa, pb, pc . These powers take values in L_3 , which has order p^2 , so we need 2 coefficients for each of pa, pb, pc . For the sake of simplicity I give a single presentation with 6 parameters for each of the 24 different commutator structures defined above, and I give the conditions for two sets of parameters to define isomorphic Lie rings.

8.63.1 Case 1

$$\langle a, b, c \mid cb, caa, cab, cac, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.757)$$

Here L_3 is generated by baa and bab , and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & \mu \\ 0 & 0 & \xi \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha\beta\lambda \\ 0 & \alpha\lambda^2 \end{pmatrix}^{-1}.$$

There are $3p + 22$ algebras in all in this case.

8.63.2 Case 2

$$\langle a, b, c \mid cb - baa, caa, cab, cac, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.758)$$

Here L_3 is generated by baa and bab , and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & \mu \\ 0 & 0 & \alpha^2 \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha\beta\lambda \\ 0 & \alpha\lambda^2 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $5p + 13 + \gcd(p - 1, 3) + \gcd(p - 1, 4)$.

8.63.3 Case 3

$$\langle a, b, c \mid cb, bab, bac, cab, cac, pa - x_1baa - x_2caa, pb - x_3baa - x_4caa, pc - x_5baa - x_6caa \rangle. \quad (7.759)$$

L_3 is generated by baa and caa and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ caa \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & \mu \\ 0 & \nu & \xi \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha^2\mu \\ \alpha^2\nu & \alpha^2\xi \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $2p + 8 + \gcd(p - 1, 4)$.

8.63.4 Case 4

$$\langle a, b, c \mid cb - baa, bab, bac, cab, cac, pa - x_1baa - x_2caa, pb - x_3baa - x_4caa, pc - x_5baa - x_6caa \rangle. \quad (7.760)$$

L_3 is generated by baa and caa and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ caa \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & 0 \\ 0 & \nu & \alpha^2 \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & 0 \\ \alpha^2\nu & \alpha^4 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $6p + 8 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4) + \gcd(p - 1, 5)$.

8.63.5 Case 5

$$\langle a, b, c \mid cb, bac, caa - bab, cac, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.761)$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & \mu \\ 0 & 0 & \alpha^{-1}\lambda^2 \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha^2\mu + \alpha\beta\lambda \\ 0 & \alpha\lambda^2 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $5p + 13 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$.

8.63.6 Case 6

$$\langle a, b, c \mid cb - baa, bac, caa - bab, cac, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.762)$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha^2 & \beta & \gamma \\ 0 & \pm\alpha^3 & \mu \\ 0 & 0 & \alpha^4 \end{pmatrix} A \begin{pmatrix} \pm\alpha^7 & \alpha^4\mu \pm \alpha^5\beta \\ 0 & \alpha^8 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is

$$p^2 + 3p - 3 + (p + 2) \gcd(p - 1, 3) + (p + 1) \gcd(p - 1, 4) + (p + 1) \gcd(p - 1, 5).$$

8.63.7 Case 7

$$\langle a, b, c \mid cb, baa, bac, cac, pa - x_1bab - x_2caa, pb - x_3bab - x_4caa, pc - x_5bab - x_6caa \rangle. \quad (7.763)$$

L_3 is generated by bab and caa and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ caa \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \lambda & 0 \\ 0 & 0 & \xi \end{pmatrix} A \begin{pmatrix} \alpha\lambda^2 & 0 \\ 0 & \alpha^2\xi \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $2p^2 + 11p + 43 + \gcd(p-1, 4)$.

8.63.8 Case 8

$$\langle a, b, c \mid cb - caa, baa, bac, cac, pa - x_1bab - x_2caa, pb - x_3bab - x_4caa, pc - x_5bab - x_6caa \rangle. \quad (7.764)$$

L_3 is generated by bab and caa and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ caa \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \alpha^2 & 0 \\ 0 & 0 & \xi \end{pmatrix} A \begin{pmatrix} \alpha^5 & 0 \\ 0 & \alpha^2\xi \end{pmatrix}^{-1}.$$

The total number of algebras in this case is

$$p^3 + 4p^2 + 6p + (p+5)\gcd(p-1, 3) + 3\gcd(p-1, 4) + \gcd(p-1, 5).$$

8.63.9 Cases 9 and 10

$$\langle a, b, c \mid cb, bac, caa, cac - bab, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.765)$$

$$\langle a, b, c \mid cb, bac, caa, cac - \omega bab, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.766)$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \pm\lambda \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha\beta\lambda \\ 0 & \alpha\lambda^2 \end{pmatrix}^{-1}.$$

The total number of algebras in both cases is

$$p^3 + \frac{5}{2}p^2 + 7p + \frac{19}{2} + \frac{p+4}{2}\gcd(p-1, 4).$$

8.63.10 Cases 11 and 12

$$\langle a, b, c \mid cb - baa, bac, caa, cac - bab, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.767)$$

$$\langle a, b, c \mid cb - baa, bac, caa, cac - \omega bab, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.768)$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & 0 \\ 0 & \pm\alpha^2 & 0 \\ 0 & 0 & \alpha^2 \end{pmatrix} A \begin{pmatrix} \pm\alpha^4 & \pm\alpha^3\beta \\ 0 & \alpha^5 \end{pmatrix}^{-1}.$$

The total number of algebras in both cases is

$$(p^4 + p^3 + 4p^2 + p - 1 + (p^2 + 2p + 3) \gcd(p - 1, 3) + (p + 2) \gcd(p - 1, 4))/2$$

8.63.11 Case 13

$$\langle a, b, c \mid cb, bac, caa - baa, cac + bab, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.769)$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & -\beta \\ 0 & \lambda & \mu \\ 0 & \mu & \lambda \end{pmatrix} A \begin{pmatrix} \alpha^2(\lambda + \mu) & \alpha\beta(\lambda + \mu) \\ 0 & \alpha(\lambda^2 - \mu^2) \end{pmatrix}^{-1}.$$

In this case there are $2p^2 + 11p + 27 + \gcd(p - 1, 4)$ immediate descendants of order p^7 and p -class 3.

8.63.12 Case 14

$$\langle a, b, c \mid cb - baa, bac, caa - baa, cac + bab, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.770)$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & \beta & -\beta \\ 0 & \lambda & \lambda - \alpha^2 \\ 0 & \lambda - \alpha^2 & \lambda \end{pmatrix} A \begin{pmatrix} 2\alpha^2\lambda - \alpha^4 & 2\alpha\beta\lambda - \alpha^3\beta \\ 0 & 2\alpha^3\lambda - \alpha^5 \end{pmatrix}^{-1}.$$

In this case there are $p^3 + 2p^2 + 6p + 10 + (p + 4) \gcd(p - 1, 3)$ algebras.

8.63.13 Case 15

$$\langle a, b, c \mid cb, baa, bac, caa, pa - x_1bab - x_2cac, pb - x_3bab - x_4cac, pc - x_5bab - x_6cac \rangle. \quad (7.771)$$

L_3 is generated by bab and cac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ cac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha\beta^2 & 0 \\ 0 & \alpha\gamma^2 \end{pmatrix}^{-1}$$

and

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & \beta \\ 0 & \gamma & 0 \end{pmatrix} A \begin{pmatrix} 0 & \alpha\beta^2 \\ \alpha\gamma^2 & 0 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is

$$p^3 + \frac{7}{2}p^2 + \frac{17}{2}p + \frac{59}{2} + \frac{5}{2} \gcd(p-1, 3) + \frac{p+1}{2} \gcd(p-1, 4).$$

8.63.14 Case 16

$$\langle a, b, c \mid cb, bac, caa, cac - baa, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.772)$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-1}\gamma^2 & 0 \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha\gamma^2 & 0 \\ 0 & \alpha^{-1}\gamma^4 \end{pmatrix}^{-1}.$$

The total number of algebras here is

$$2p^4 + 4p^3 + 8p^2 + 14p + 11 + 4 \gcd(p-1, 3) + 3 \gcd(p-1, 4).$$

8.63.15 Case 17

$$\langle a, b, c \mid cb, bac, caa - bab, cac - baa, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab \rangle. \quad (7.773)$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-1}\gamma^2 & 0 \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha\gamma^2 & 0 \\ 0 & \alpha^2\gamma \end{pmatrix}^{-1}$$

or

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & \alpha^{-1}\gamma^2 \\ 0 & \gamma & 0 \end{pmatrix} A \begin{pmatrix} 0 & \alpha\gamma^2 \\ \alpha^2\gamma & 0 \end{pmatrix}^{-1}$$

with $\alpha^3 = \gamma^3$.

If $p \not\equiv 1 \pmod{3}$ then $\alpha = \gamma$ and the number of orbits is

$$p^5 + p^4 + p^3 + p^2 + p + 2 + (p^2 + p + 1) \gcd(p - 1, 4)/2.$$

If $p \equiv 1 \pmod{3}$ then $\alpha = \gamma$ or $\xi\gamma$ or $\xi^2\gamma$ where $\xi^3 = 1$. The number of orbits is then

$$(p^5 + p^4 + p^3 + p^2 + 7p + 10)/3 + (p^2 + p + 1) \gcd(p - 1, 4)/2.$$

So in general the number of orbits is

$$(p^4 + 2p^3 + 3p^2 + 4p + 2) \frac{p - 1}{\gcd(p - 1, 3)} + 3p + 4 + (p^2 + p + 1) \gcd(p - 1, 4)/2.$$

8.63.16 Case 18

$$\langle a, b, c \mid cb, bac, caa - \omega bab, cac - baa, pa - x_1 baa - x_2 bab, pb - x_3 baa - x_4 bab, pc - x_5 baa - x_6 bab \rangle \quad (p \equiv 1 \pmod{3}). \quad (7.774)$$

This case is very similar to Case 17, though we do not have as many automorphisms. L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-1}\gamma^2 & 0 \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha\gamma^2 & 0 \\ 0 & \alpha^2\gamma \end{pmatrix}^{-1}$$

with $\alpha^3 = \gamma^3$.

The number of algebras is

$$(2p^5 + 2p^4 + 2p^3 + 2p^2 + 14p + 17)/3.$$

Combining Case 17 and Case 18, the total number of algebras in the two cases is

$$p^5 + p^4 + p^3 + p^2 - 2p - \frac{3}{2} + (3p + \frac{7}{2}) \gcd(p - 1, 3) + (p^2 + p + 1) \gcd(p - 1, 4)/2.$$

8.63.17 Case 19

$$\langle a, b, c \mid cb, baa, caa, cac, pa - x_1 bab - x_2 bac, pb - x_3 bab - x_4 bac, pc - x_5 bab - x_6 bac \rangle. \quad (7.775)$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & \gamma \\ 0 & 0 & \delta \end{pmatrix} A \begin{pmatrix} \alpha\beta^2 & 2\alpha\beta\gamma \\ 0 & \alpha\beta\delta \end{pmatrix}^{-1}$$

The total number of algebras in this case is $2p^2 + 11p + 27 + \gcd(p - 1, 4)$.

8.63.18 Case 20

$$\langle a, b, c \mid cb, baa, caa - bab, cac, pa - x_1bab - x_2bac, pb - x_3bab - x_4bac, pc - x_5bab - x_6bac \rangle. \quad (7.776)$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \alpha^{-1}\beta^2 \end{pmatrix} A \begin{pmatrix} \alpha\beta^2 & 0 \\ 0 & \beta^3 \end{pmatrix}^{-1}$$

The total number of algebras here is

$$2p^4 + 4p^3 + 6p^2 + 11p + 11 + 2 \gcd(p-1, 3) + (p+1) \gcd(p-1, 4).$$

8.63.19 Case 21

$$\langle a, b, c \mid cb, bab - baa, caa, cac, pa - x_1baa - x_2bac, pb - x_3baa - x_4bac, pc - x_5baa - x_6bac \rangle. \quad (7.777)$$

L_3 is generated by baa and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 2\beta \\ 0 & \alpha & \beta \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha^3 & 2\alpha^2\beta \\ 0 & \alpha^2\gamma \end{pmatrix}^{-1}.$$

The total number of algebras in this case is

$$2p^3 + 6p^2 + 7p + 7 + (p+1) \gcd(p-1, 4).$$

8.63.20 Case 22

$$\langle a, b, c \mid cb, baa, caa, cac - \omega bab, pa - x_1bab - x_2bac, pb - x_3bab - x_4bac, pc - x_5bab - x_6bac \rangle. \quad (7.778)$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \omega\beta & \pm\gamma \\ 0 & \omega\gamma & \pm\omega\beta \end{pmatrix} A \begin{pmatrix} \omega\alpha(\omega\beta^2 + \gamma^2) & \pm 2\omega\alpha\beta\gamma \\ 2\omega^2\alpha\beta\gamma & \pm\omega\alpha(\omega\beta^2 + \gamma^2) \end{pmatrix}^{-1}.$$

The total number of algebras in Case 22 is

$$(2p^3 + 3p^2 + 3p + 13 - \gcd(p-1, 3) + (p+1) \gcd(p-1, 4))/2.$$

8.63.21 Case 23

$$\langle a, b, c \mid cb, baa, caa - bac, cac - \omega bab, pa - x_1 bab - x_2 bac, pb - x_3 bab - x_4 bac, pc - x_5 bab - x_6 bac \rangle. \quad (7.779)$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \pm\alpha \end{pmatrix} A \begin{pmatrix} \alpha^3 & 0 \\ 0 & \pm\alpha^3 \end{pmatrix}^{-1}$$

or when $p = 2 \pmod 3$ and $12\omega\beta^2 = -1$,

$$A \rightarrow \begin{pmatrix} 4\omega\alpha\beta & -3\omega\alpha\beta & \frac{\alpha}{2} \\ 0 & -2\omega\alpha\beta & \alpha \\ 0 & \pm\omega\alpha & \mp 2\omega\alpha\beta \end{pmatrix} A \begin{pmatrix} \frac{8}{3}\omega^2\alpha^3\beta & \frac{4}{3}\omega\alpha^3 \\ \pm\frac{4}{3}\omega^2\alpha^3 & \pm\frac{8}{3}\omega^2\alpha^3\beta \end{pmatrix}^{-1}.$$

If $p = 1 \pmod 3$ there are $p^5 + p^4 + p^3 + p^2 + p + 2 + (p^2 + p + 1) \gcd(p - 1, 4)/2$ algebras. When $p = 2 \pmod 3$ the number of algebras here is

$$\frac{1}{3}p^5 + \frac{1}{3}p^4 + \frac{1}{3}p^3 + \frac{1}{3}p^2 + p + 2 + (p^2 + p + 1) \gcd(p - 1, 4)/2.$$

8.63.22 Case 24

$$\langle a, b, c \mid cb, baa, caa - xbab - bac, cac - \omega bab, pa - x_1 bab - x_2 bac, pb - x_3 bab - x_4 bac, pc - x_5 bab - x_6 bac \rangle \quad (p = 2 \pmod 3). \quad (7.780)$$

where x is any (fixed) integer which is not a value of

$$\frac{\lambda(\lambda^2 + 3\omega\mu^2)}{\mu(3\lambda^2 + \omega\mu^2)} \pmod p.$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \rightarrow \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix} A \begin{pmatrix} \alpha^3 & 0 \\ 0 & \alpha^3 \end{pmatrix}^{-1}$$

and

$$A \rightarrow \begin{pmatrix} -4\alpha & k\alpha\beta + 3\alpha & 3k\omega^{-1}\alpha + \alpha\beta \\ 0 & 2\alpha & 2\alpha\beta \\ 0 & 2\omega\alpha\beta & 2\alpha \end{pmatrix} A \begin{pmatrix} 32\alpha^3 & -32\alpha^3\beta \\ -32\omega\alpha^3\beta & 32\alpha^3 \end{pmatrix}^{-1}$$

where $\omega\beta^2 = -3$.

The number of orbits is

$$\frac{2}{3}p^5 + \frac{2}{3}p^4 + \frac{2}{3}p^3 + \frac{2}{3}p^2 + 2p + 3.$$

The total number of algebras from Case 23 and Case 24 is

$$p^5 + p^4 + p^3 + p^2 + 4p + \frac{13}{2} - (p + \frac{3}{2}) \gcd(p - 1, 3) + (p^2 + p + 1) \gcd(p - 1, 4)/2.$$

The total number of algebras from cases 17, 18, 23 and 24 is

$$2p^5 + 2p^4 + 2p^3 + 2p^2 + 2p + 5 + (2p + 2) \gcd(p - 1, 3) + (p^2 + p + 1) \gcd(p - 1, 4).$$

8.64 Descendants of 5.15

$3p^2 + 12p + 14 + (p + 2) \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid cb, pa - ba, pb, pc, \text{class } 3 \rangle \quad (7.781)$$

$$\langle a, b, c \mid cb, pa - ba - caa, pb, pc, \text{class } 3 \rangle \quad (7.782)$$

$$\langle a, b, c \mid cb, pa - ba - cac, pb, pc, \text{class } 3 \rangle \quad (7.783)$$

$$\langle a, b, c \mid cb, pa - ba - \omega cac, pb, pc, \text{class } 3 \rangle \quad (7.784)$$

$$\langle a, b, c \mid cb, pa - ba, pb, pc - caa, \text{class } 3 \rangle \quad (7.785)$$

$$\langle a, b, c \mid cb, pa - ba - cac, pb, pc - caa, \text{class } 3 \rangle \quad (7.786)$$

$$\langle a, b, c \mid cb, pa - ba - \omega cac, pb, pc - caa, \text{class } 3 \rangle \quad (7.787)$$

$$\langle a, b, c \mid cb, pa - ba - caa - xcac, pb, pc - caa, \text{class } 3 \rangle \quad (7.788)$$

$$\langle a, b, c \mid cb, pa - ba, pb, pc - \omega caa, \text{class } 3 \rangle \quad (7.789)$$

$$\langle a, b, c \mid cb, pa - ba - cac, pb, pc - \omega caa, \text{class } 3 \rangle \quad (7.790)$$

$$\langle a, b, c \mid cb, pa - ba - \omega cac, pb, pc - \omega caa, \text{class } 3 \rangle \quad (7.791)$$

$$\langle a, b, c \mid cb, pa - ba - caa - xcac, pb, pc - \omega caa, \text{class } 3 \rangle \quad (7.792)$$

$$\langle a, b, c \mid cb, pa - ba - xcaa, pb, pc - cac, \text{class } 3 \rangle \quad (7.793)$$

$$\langle a, b, c \mid cb, pa - ba - caa - cac, pb, pc - cac, \text{class } 3 \rangle \quad (7.794)$$

$$\langle a, b, c \mid cb, pa - ba - caa - \omega cac, pb, pc - cac, \text{class } 3 \rangle \quad (7.795)$$

$$\langle a, b, c \mid cb, pa - ba, pb - caa, pc, \text{class } 3 \rangle \quad (7.796)$$

$$\langle a, b, c \mid cb, pa - ba - cac, pb - caa, pc, \text{class } 3 \rangle \quad (7.797)$$

$$\langle a, b, c \mid cb, pa - ba - \omega cac, pb - caa, pc, \text{class } 3 \rangle \quad (7.798)$$

$$\langle a, b, c \mid cb, pa - ba - \omega^2 cac, pb - caa, pc, \text{class } 3 \rangle \quad (p = 1 \pmod{4}) \quad (7.799)$$

$$\langle a, b, c \mid cb, pa - ba - \omega^3 cac, pb - caa, pc, \text{class } 3 \rangle \quad (p = 1 \pmod{4}) \quad (7.800)$$

$$\langle a, b, c \mid cb, pa - ba - xcac, pb - caa, pc - cac, \text{class } 3 \rangle \quad (7.801)$$

$$\langle a, b, c \mid cb, pa - ba, pb - cac, pc, \text{class } 3 \rangle \quad (7.802)$$

$$\langle a, b, c \mid cb, pa - ba - caa, pb - cac, pc, \text{class } 3 \rangle \quad (7.803)$$

$$\langle a, b, c \mid cb, pa - ba, pb - cac, pc - caa, \text{class } 3 \rangle \quad (7.804)$$

$$\langle a, b, c \mid cb, pa - ba, pb - cac, pc - \omega caa, \text{class } 3 \rangle \quad (7.805)$$

$$\langle a, b, c \mid cb, pa - ba, pb - cac, pc - \omega^2 caa, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.806)$$

$$\langle a, b, c \mid cb, pa - ba, pb - cac, pc - \omega^3 caa, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.807)$$

$$\langle a, b, c \mid cb - caa, pa - ba, pb, pc, \text{class } 3 \rangle \quad (7.808)$$

$$\langle a, b, c \mid cb - caa, pa - ba - cac, pb, pc, \text{class } 3 \rangle \quad (7.809)$$

$$\langle a, b, c \mid cb - caa, pa - ba - \omega cac, pb, pc, \text{class } 3 \rangle \quad (7.810)$$

$$\langle a, b, c \mid cb - caa, pa - ba, pb, pc - xcaa, \text{class } 3 \rangle \quad (x \neq 0) \quad (7.811)$$

$$\langle a, b, c \mid cb - caa, pa - ba - cac, pb, pc - xcaa, \text{class } 3 \rangle \quad (x \neq 0) \quad (7.812)$$

$$\langle a, b, c \mid cb - caa, pa - ba - \omega cac, pb, pc - xcaa, \text{class } 3 \rangle \quad (x \neq 0) \quad (7.813)$$

$$\langle a, b, c \mid cb - caa, pa - ba - caa - ycac, pb, pc - xcaa, \text{class } 3 \rangle \quad (x \neq 0) \quad (7.814)$$

$$\langle a, b, c \mid cb - caa, pa - ba - xcac, pb, pc - cac, \text{class } 3 \rangle \quad (7.815)$$

$$\langle a, b, c \mid cb - caa, pa - ba - ycac, pb - caa, pc - xcac, \text{class } 3 \rangle \quad ((x, y) \sim (-x, y)) \quad (7.816)$$

$$\langle a, b, c \mid cb - caa, pa - ba, pb - cac, pc - xcaa, \text{class } 3 \rangle \quad (7.817)$$

$$\langle a, b, c \mid cb - caa, pa - ba - xcaa, pb - cac, pc + 2caa, \text{class } 3 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.818)$$

$$\langle a, b, c \mid cb - caa, pa - ba, pb - \omega cac, pc - xcaa, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.819)$$

$$\langle a, b, c \mid cb - caa, pa - ba - xcaa, pb - \omega cac, pc + 2caa, \text{class } 3 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.820)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba, pb, pc, \text{class } 3 \rangle \quad (7.821)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - cac, pb, pc, \text{class } 3 \rangle \quad (7.822)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - \omega cac, pb, pc, \text{class } 3 \rangle \quad (7.823)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba, pb, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.824)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - cac, pb, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.825)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - \omega cac, pb, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.826)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - caa - ycac, pb, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.827)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - xcac, pb, pc - cac, \text{class } 3 \rangle \quad (7.828)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - ycac, pb - caa, pc - xcac, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.829)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba, pb - cac, pc - xcaa, \text{class } 3 \rangle \quad (7.830)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - xcaa, pb - cac, pc + 2\omega caa, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.831)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba, pb - \omega cac, pc - xcaa, \text{class } 3 \rangle (p = 1 \pmod{4}) \quad (7.832)$$

$$\langle a, b, c \mid cb - \omega caa, pa - ba - xcaa, pb - \omega cac, pc + 2\omega caa, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.833)$$

8.65 Descendants of 5.16

$p^4 + 2p^3 + 5p^2 + 14p$ algebras

$$\langle a, b, c \mid cb, pa, pb - ba, pc, \text{class } 3 \rangle \quad (7.834)$$

$$\langle a, b, c \mid cb, pa, pb - ba - caa, pc, \text{class } 3 \rangle \quad (7.835)$$

$$\langle a, b, c \mid cb, pa, pb - ba - cac, pc, \text{class } 3 \rangle \quad (7.836)$$

$$\langle a, b, c \mid cb, pa - caa, pb - ba, pc, \text{class } 3 \rangle \quad (7.837)$$

$$\langle a, b, c \mid cb, pa - caa, pb - ba - cac, pc, \text{class } 3 \rangle \quad (7.838)$$

$$\langle a, b, c \mid cb, pa - caa, pb - ba - caa - xcac, pc, \text{class } 3 \rangle \quad (7.839)$$

$$\langle a, b, c \mid cb, pa - cac, pb - ba, pc, \text{class } 3 \rangle \quad (7.840)$$

$$\langle a, b, c \mid cb, pa - cac, pb - ba - caa, pc, \text{class } 3 \rangle \quad (7.841)$$

$$\langle a, b, c \mid cb, pa - cac, pb - ba - cac, pc, \text{class } 3 \rangle \quad (7.842)$$

$$\langle a, b, c \mid cb, pa - \omega cac, pb - ba, pc, \text{class } 3 \rangle \quad (7.843)$$

$$\langle a, b, c \mid cb, pa - \omega cac, pb - ba - caa, pc, \text{class } 3 \rangle \quad (7.844)$$

$$\langle a, b, c \mid cb, pa - \omega cac, pb - ba - cac, pc, \text{class } 3 \rangle \quad (7.845)$$

$$\langle a, b, c \mid cb, pa, pb - ba, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.846)$$

$$\langle a, b, c \mid cb, pa, pb - ba - cac, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.847)$$

$$\langle a, b, c \mid cb, pa, pb - ba - caa, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.848)$$

$$\langle a, b, c \mid cb, pa, pb - ba - caa - cac, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.849)$$

$$\langle a, b, c \mid cb, pa - cac, pb - ba, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.850)$$

$$\langle a, b, c \mid cb, pa - cac, pb - ba - caa, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.851)$$

$$\langle a, b, c \mid cb, pa - cac, pb - ba - ycaa - cac, pc - xcaa, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (x, -y)) \quad (7.852)$$

$$\langle a, b, c \mid cb, pa - \omega cac, pb - ba, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.853)$$

$$\langle a, b, c \mid cb, pa - \omega cac, pb - ba - caa, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.854)$$

$$\langle a, b, c \mid cb, pa - \omega cac, pb - ba - ycaa - cac, pc - xcaa, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (x, -y)) \quad (7.855)$$

$$\langle a, b, c \mid cb, pa - caa - ycac, pb - ba, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.856)$$

$$\langle a, b, c \mid cb, pa - caa - ycac, pb - ba - cac, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.857)$$

$$\langle a, b, c \mid cb, pa - caa - zcac, pb - ba - caa - ycac, pc - xcaa, \text{class } 3 \rangle (x \neq 0) \quad (7.858)$$

$$\langle a, b, c \mid cb, pa - xcaa, pb - ba, pc - cac, \text{class } 3 \rangle (x \neq 1) \quad (7.859)$$

$$\langle a, b, c \mid cb, pa - xcaa, pb - ba - cac, pc - cac, \text{class } 3 \rangle (x \neq 1) \quad (7.860)$$

$$\langle a, b, c \mid cb, pa - xcaa, pb - ba - caa - ycac, pc - cac, \text{class } 3 \rangle (x \neq 1) \quad (7.861)$$

$$\langle a, b, c \mid cb, pa - caa - xcac, pb - ba, pc - cac, \text{class } 3 \rangle \quad (7.862)$$

$$\langle a, b, c \mid cb, pa - caa - xcac, pb - ba - cac, pc - cac, \text{class } 3 \rangle \quad (7.863)$$

$$\langle a, b, c \mid cb, pa - caa - xcac, pb - ba - caa, pc - cac, \text{class } 3 \rangle \quad (7.864)$$

$$\langle a, b, c \mid cb - caa, pa, pb - ba, pc - xcaa, \text{class } 3 \rangle \quad (7.865)$$

$$\langle a, b, c \mid cb - caa, pa - cac, pb - ba, pc - xcaa, \text{class } 3 \rangle \quad (7.866)$$

$$\langle a, b, c \mid cb - caa, pa - \omega cac, pb - ba, pc - xcaa, \text{class } 3 \rangle \quad (7.867)$$

$$\langle a, b, c \mid cb - caa, pa - ycac, pb - ba, pc - xcaa - cac, \text{class } 3 \rangle \quad (7.868)$$

$$\langle a, b, c \mid cb - caa, pa - caa - ycac, pb - ba, pc - xcaa - zcac, \text{class } 3 \rangle \quad (7.869)$$

$$\langle a, b, c \mid cb - caa, pa - ycac, pb - ba - cac, pc - zcaa - xcac, \text{class } 3 \rangle ((x, y, z) \sim (-x, y, z)) \quad (7.870)$$

$$\langle a, b, c \mid cb - caa, pa - xcaa - ycac, pb - ba - cac, pc - zcaa - tcac, \text{class } 3 \rangle (x \neq 0, (x, y, z, t) \sim (-x, y, z, -t)) \quad (7.871)$$

$$\langle a, b, c \mid cb - caa, pa - ycac, pb - ba - \omega cac, pc - zcaa - xcac, \text{class } 3 \rangle ((x, y, z) \sim (-x, y, z)) \quad (7.872)$$

$$\langle a, b, c \mid cb - caa, pa - xcaa - ycac, pb - ba - \omega cac, pc - zcaa - tcac, \text{class } 3 \rangle (x \neq 0, (x, y, z, t) \sim (-x, y, z, -t)) \quad (7.873)$$

8.66 Descendants of 5.18

$3p^3 + 6p^2 + 6p + 11 + (p + 7) \gcd(p - 1, 3) + (p + 1) \gcd(p - 1, 4) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid cb, pa, pb - ca, pc, \text{class } 3 \rangle \quad (7.874)$$

$$\langle a, b, c \mid cb, pa, pb - ca - baa, pc, \text{class } 3 \rangle \quad (7.875)$$

$$\langle a, b, c \mid cb, pa, pb - ca - \omega baa, pc, \text{class } 3 \rangle \quad (7.876)$$

$$\langle a, b, c \mid cb, pa - bab, pb - ca, pc, \text{class } 3 \rangle \quad (7.877)$$

$$\langle a, b, c \mid cb, pa - bab, pb - ca - baa, pc, \text{class } 3 \rangle \quad (7.878)$$

$$\langle a, b, c \mid cb, pa - bab, pb - ca - \omega baa, pc, \text{class } 3 \rangle \quad (7.879)$$

$$\langle a, b, c \mid cb, pa - \omega bab, pb - ca, pc, \text{class } 3 \rangle \quad (7.880)$$

$$\langle a, b, c \mid cb, pa - \omega bab, pb - ca - baa, pc, \text{class } 3 \rangle \quad (7.881)$$

$$\langle a, b, c \mid cb, pa - \omega bab, pb - ca - \omega baa, pc, \text{class } 3 \rangle \quad (7.882)$$

$$\langle a, b, c \mid cb, pa, pb - ca - bab, pc, \text{class } 3 \rangle \quad (7.883)$$

$$\langle a, b, c \mid cb, pa, pb - ca - baa - bab, pc, \text{class } 3 \rangle \quad (7.884)$$

$$\langle a, b, c \mid cb, pa, pb - ca - \omega baa - bab, pc, \text{class } 3 \rangle \quad (7.885)$$

$$\langle a, b, c \mid cb, pa - bab, pb - ca - xbaa - bab, pc, \text{class } 3 \rangle \quad (7.886)$$

$$\langle a, b, c \mid cb, pa - \omega bab, pb - ca - xbaa - bab, pc, \text{class } 3 \rangle \quad (7.887)$$

$$\langle a, b, c \mid cb, pa - baa, pb - ca - xbab, pc, \text{class } 3 \rangle \quad (7.888)$$

$$\langle a, b, c \mid cb, pa - baa, pb - ca - baa - xbab, pc, \text{class } 3 \rangle \quad (7.889)$$

$$\langle a, b, c \mid cb, pa - baa, pb - ca - \omega baa - xbab, pc, \text{class } 3 \rangle \quad (7.890)$$

$$\langle a, b, c \mid cb, pa - baa - bab, pb - ca - xbaa - ybab, pc, \text{class } 3 \rangle \quad (7.891)$$

$$\langle a, b, c \mid cb, pa - baa - \omega bab, pb - ca - xbaa - ybab, pc, \text{class } 3 \rangle \quad (7.892)$$

$$\langle a, b, c \mid cb, pa, pb - ca, pc - baa, \text{class } 3 \rangle \quad (7.893)$$

$$\langle a, b, c \mid cb, pa - bab, pb - ca, pc - baa, \text{class } 3 \rangle \quad (7.894)$$

$$\langle a, b, c \mid cb, pa - \omega bab, pb - ca, pc - baa, \text{class } 3 \rangle \quad (7.895)$$

$$\langle a, b, c \mid cb, pa, pb - ca - bab, pc - baa, \text{class } 3 \rangle \quad (7.896)$$

$$\langle a, b, c \mid cb, pa - xbab, pb - ca - bab, pc - baa, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.897)$$

$$\langle a, b, c \mid cb, pa, pb - ca, pc - \omega baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.898)$$

$$\langle a, b, c \mid cb, pa - bab, pb - ca, pc - \omega baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.899)$$

$$\langle a, b, c \mid cb, pa - \omega bab, pb - ca, pc - \omega baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.900)$$

$$\langle a, b, c \mid cb, pa, pb - ca - bab, pc - \omega baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.901)$$

$$\langle a, b, c \mid cb, pa - xbab, pb - ca - bab, pc - \omega baa, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.902)$$

$$\langle a, b, c \mid cb, pa, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.903)$$

$$\langle a, b, c \mid cb, pa - bab, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.904)$$

$$\langle a, b, c \mid cb, pa - \omega bab, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.905)$$

$$\langle a, b, c \mid cb, pa, pb - ca - bab, pc - \omega^2 baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.906)$$

$$\langle a, b, c \mid cb, pa - xbab, pb - ca - bab, pc - \omega^2baa, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.907)$$

$$\langle a, b, c \mid cb, pa, pb - ca, pc - bab, \text{class } 3 \rangle \quad (7.908)$$

$$\langle a, b, c \mid cb, pa, pb - ca - baa, pc - bab, \text{class } 3 \rangle \quad (7.909)$$

$$\langle a, b, c \mid cb, pa, pb - ca - \omega baa, pc - bab, \text{class } 3 \rangle \quad (7.910)$$

$$\langle a, b, c \mid cb, pa - baa, pb - ca - xbaa, pc - bab, \text{class } 3 \rangle \quad (7.911)$$

$$\langle a, b, c \mid cb, pa - xbaa, pb - ca - ybaa, pc - baa - bab, \text{class } 3 \rangle (p = 2 \bmod 3) \quad (7.912)$$

$$\langle a, b, c \mid cb, pa, pb - ca, pc - baa - bab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.913)$$

$$\langle a, b, c \mid cb, pa, pb - ca, pc - \omega baa - bab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.914)$$

$$\langle a, b, c \mid cb, pa, pb - ca, pc - \omega^2baa - bab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.915)$$

$$\langle a, b, c \mid cb, pa, pb - ca - xbaa, pc - baa - bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.916)$$

$$\langle a, b, c \mid cb, pa, pb - ca - xbaa, pc - \omega baa - bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.917)$$

$$\langle a, b, c \mid cb, pa, pb - ca - xbaa, pc - \omega^2baa - bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.918)$$

$$\langle a, b, c \mid cb, pa - xbaa, pb - ca - ybaa, pc - baa - bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (ax, ay) \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.919)$$

$$\langle a, b, c \mid cb, pa - xbaa, pb - ca - ybaa, pc - \omega baa - bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (ax, ay) \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.920)$$

$$\langle a, b, c \mid cb, pa - xbaa, pb - ca - ybaa, pc - \omega^2baa - bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (ax, ay) \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.921)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca, pc, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.922)$$

$$\langle a, b, c \mid cb - baa, pa - bab, pb - ca, pc, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.923)$$

$$\langle a, b, c \mid cb - baa, pa - \omega bab, pb - ca, pc, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.924)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^2bab, pb - ca, pc, \text{class } 3 \rangle (p = 1 \bmod 4, p = 1 \bmod 3) \quad (7.925)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^3bab, pb - ca, pc, \text{class } 3 \rangle (p = 1 \bmod 4, p = 1 \bmod 3) \quad (7.926)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^4 bab, pb - ca, pc, \text{ class } 3 \rangle (p = 1 \bmod 4, p = 1 \bmod 3) \quad (7.927)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^5 bab, pb - ca, pc, \text{ class } 3 \rangle (p = 1 \bmod 4, p = 1 \bmod 3) \quad (7.928)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - bab, pc, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.929)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - \omega bab, pc, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.930)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - \omega^2 bab, pc, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.931)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - \omega^3 bab, pc, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.932)$$

$$\langle a, b, c \mid cb - baa, pa - baa - xbab, pb - ca - ybab, pc, \text{ class } 3 \rangle ((x, y) \sim (-x, y), p = 1 \bmod 4) \quad (7.933)$$

$$\langle a, b, c \mid cb - baa, pa - \omega baa - xbab, pb - ca - ybab, pc, \text{ class } 3 \rangle ((x, y) \sim (-x, y), p = 1 \bmod 4) \quad (7.934)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^2 baa - xbab, pb - ca - ybab, pc, \text{ class } 3 \rangle ((x, y) \sim (-x, y), p = 1 \bmod 4) \quad (7.935)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^3 baa - xbab, pb - ca - ybab, pc, \text{ class } 3 \rangle ((x, y) \sim (-x, y), p = 1 \bmod 4) \quad (7.936)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa - ybab, pb - ca - baa - zbab, pc, \text{ class } 3 \rangle (p = 1 \bmod 4) \quad (7.937)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa - ybab, pb - ca - \omega baa - zbab, pc, \text{ class } 3 \rangle (p = 1 \bmod 4) \quad (7.938)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca, pc, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.939)$$

$$\langle a, b, c \mid cb - baa, pa - bab, pb - ca, pc, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.940)$$

$$\langle a, b, c \mid cb - baa, pa - \omega bab, pb - ca, pc, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.941)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^2 bab, pb - ca, pc, \text{ class } 3 \rangle (p = 3 \bmod 4, p = 1 \bmod 3) \quad (7.942)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^3 bab, pb - ca, pc, \text{ class } 3 \rangle (p = 3 \bmod 4, p = 1 \bmod 3) \quad (7.943)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^4 bab, pb - ca, pc, \text{ class } 3 \rangle (p = 3 \bmod 4, p = 1 \bmod 3) \quad (7.944)$$

$$\langle a, b, c \mid cb - baa, pa - \omega^5 bab, pb - ca, pc, \text{ class } 3 \rangle (p = 3 \bmod 4, p = 1 \bmod 3) \quad (7.945)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - bab, pc, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.946)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - \omega bab, pc, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.947)$$

$$\langle a, b, c \mid cb - baa, pa - baa - xbab, pb - ca - ybab, pc, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.948)$$

$$\langle a, b, c \mid cb - baa, pa - \omega baa - xbab, pb - ca - ybab, pc, \text{class } 3 \rangle (p = 3 \bmod 4) \quad (7.949)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa - ybab, pb - ca - baa - zbab, pc, \text{class } 3 \rangle (p = 3 \bmod 4) \quad (7.950)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa - ybab, pb - ca - \omega baa - zbab, pc, \text{class } 3 \rangle (p = 3 \bmod 4) \quad (7.951)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca, pc - bab, \text{class } 3 \rangle \quad (7.952)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca, pc - \omega bab, \text{class } 3 \rangle (p = 1 \bmod 5) \quad (7.953)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca, pc - \omega^2 bab, \text{class } 3 \rangle (p = 1 \bmod 5) \quad (7.954)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca, pc - \omega^3 bab, \text{class } 3 \rangle (p = 1 \bmod 5) \quad (7.955)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca, pc - \omega^4 bab, \text{class } 3 \rangle (p = 1 \bmod 5) \quad (7.956)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca - xbaa, pc - bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.957)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca - xbaa, pc - \omega bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.958)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca - xbaa, pc - \omega^2 bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.959)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca - xbaa, pc - \omega^3 bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.960)$$

$$\langle a, b, c \mid cb - baa, pa, pb - ca - xbaa, pc - \omega^4 bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.961)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa, pb - ca - ybaa, pc - bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^5 = 1) \quad (7.962)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa, pb - ca - ybaa, pc - \omega bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.963)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa, pb - ca - ybaa, pc - \omega^2 bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.964)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa, pb - ca - ybaa, pc - \omega^3 bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.965)$$

$$\langle a, b, c \mid cb - baa, pa - xbaa, pb - ca - ybaa, pc - \omega^4 bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.966)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca, pc - baa, \text{class } 3 \rangle \quad (7.967)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - ybab, pc - baa, \text{class } 3 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^3 = 1) \quad (7.968)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca, pc - \omega baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.969)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - ybab, pc - \omega baa, \text{class } 3 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.970)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.971)$$

$$\langle a, b, c \mid cb - baa, pa - xbab, pb - ca - ybab, pc - \omega^2 baa, \text{class } 3 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.972)$$

$$\langle a, b, c \mid cb - baa, pa - ybab, pb - ca - zbab, pc - baa - xbab, \text{class } 3 \rangle (x \neq 0, (x, y, z) \sim (ax, y, a^2z) \text{ if } a^3 = 1) \quad (7.973)$$

$$\langle a, b, c \mid cb - baa, pa - ybab, pb - ca - zbab, pc - \omega baa - xbab, \text{class } 3 \rangle (x \neq 0, (x, y, z) \sim (ax, y, a^2z) \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.974)$$

$$\langle a, b, c \mid cb - baa, pa - ybab, pb - ca - zbab, pc - \omega^2 baa - xbab, \text{class } 3 \rangle (x \neq 0, (x, y, z) \sim (ax, y, a^2z) \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.975)$$

8.67 Descendants of 5.8

$p + 8$ algebras

$$\langle a, b, c \mid ba, ca - p^2a, cb - p^2b, pc, \text{class } 3 \rangle \quad (7.976)$$

$$\langle a, b, c \mid ba, ca - p^2b, cb - p^2a, pc, \text{class } 3 \rangle \quad (7.977)$$

$$\langle a, b, c \mid ba, ca - \omega p^2b, cb - p^2a, pc, \text{class } 3 \rangle \quad (7.978)$$

$$\langle a, b, c \mid ba, ca - xp^2b, cb - p^2a - p^2b, pc, \text{class } 3 \rangle (x \neq 0) \quad (7.979)$$

$$\langle a, b, c \mid ba, ca, cb - p^2a, pc, \text{class } 3 \rangle \quad (7.980)$$

$$\langle a, b, c \mid ba - p^2b, ca, cb - p^2a, pc, \text{class } 3 \rangle \quad (7.981)$$

$$\langle a, b, c \mid ba, ca, cb - p^2b, pc, \text{class } 3 \rangle \quad (7.982)$$

$$\langle a, b, c \mid ba - p^2a, ca, cb - p^2b, pc, \text{class } 3 \rangle \quad (7.983)$$

$$\langle a, b, c \mid ba, ca, cb, pc, \text{class } 3 \rangle \quad (7.984)$$

$$\langle a, b, c \mid ba - p^2a, ca, cb, pc, \text{class } 3 \rangle \quad (7.985)$$

8.68 Descendants of 5.9

$4p^2 + 26p + 107 + 5 \gcd(p - 1, 3) + (p + 4) \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid baa, ca, cb, pb, pc, \text{ class } 3 \rangle \quad (7.986)$$

$$\langle a, b, c \mid baa, ca - bab, cb, pb, pc, \text{ class } 3 \rangle \quad (7.987)$$

$$\langle a, b, c \mid baa, ca - p^2a, cb, pb, pc, \text{ class } 3 \rangle \quad (7.988)$$

$$\langle a, b, c \mid baa, ca - bab - p^2a, cb, pb, pc, \text{ class } 3 \rangle \quad (7.989)$$

$$\langle a, b, c \mid baa, ca - \omega bab - p^2a, cb, pb, pc, \text{ class } 3 \rangle \quad (7.990)$$

$$\langle a, b, c \mid baa, ca, cb - p^2a, pb, pc, \text{ class } 3 \rangle \quad (7.991)$$

$$\langle a, b, c \mid baa, ca - bab, cb - p^2a, pb, pc, \text{ class } 3 \rangle \quad (7.992)$$

$$\langle a, b, c \mid baa, ca - p^2a, cb - p^2a, pb, pc, \text{ class } 3 \rangle \quad (7.993)$$

$$\langle a, b, c \mid baa, ca - bab - p^2a, cb - p^2a, pb, pc, \text{ class } 3 \rangle \quad (7.994)$$

$$\langle a, b, c \mid baa, ca - \omega bab - p^2a, cb - p^2a, pb, pc, \text{ class } 3 \rangle \quad (7.995)$$

$$\langle a, b, c \mid baa, ca, cb, pb, pc - bab, \text{ class } 3 \rangle \quad (7.996)$$

$$\langle a, b, c \mid baa, ca - bab, cb, pb, pc - bab, \text{ class } 3 \rangle \quad (7.997)$$

$$\langle a, b, c \mid baa, ca - p^2a, cb, pb, pc - bab, \text{ class } 3 \rangle \quad (7.998)$$

$$\langle a, b, c \mid baa, ca - bab - p^2a, cb, pb, pc - bab, \text{ class } 3 \rangle \quad (7.999)$$

$$\langle a, b, c \mid baa, ca - \omega bab - p^2a, cb, pb, pc - bab, \text{ class } 3 \rangle \quad (7.1000)$$

$$\langle a, b, c \mid baa, ca, cb - p^2a, pb, pc - bab, \text{ class } 3 \rangle \quad (7.1001)$$

$$\langle a, b, c \mid baa, ca - bab, cb - p^2a, pb, pc - bab, \text{ class } 3 \rangle \quad (7.1002)$$

$$\langle a, b, c \mid baa, ca - xbab - p^2a, cb - p^2a, pb, pc - bab, \text{ class } 3 \rangle \ (x \sim ax \text{ if } a^3 = 1) \quad (7.1003)$$

$$\langle a, b, c \mid baa, ca, cb - \omega p^2a, pb, pc - bab, \text{ class } 3 \rangle \ (p = 1 \bmod 3) \quad (7.1004)$$

$$\langle a, b, c \mid baa, ca - bab, cb - \omega p^2a, pb, pc - bab, \text{ class } 3 \rangle \ (p = 1 \bmod 3) \quad (7.1005)$$

$$\langle a, b, c \mid baa, ca - xbab - p^2a, cb - \omega p^2a, pb, pc - bab, \text{ class } 3 \rangle \ (x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.1006)$$

$$\langle a, b, c \mid baa, ca, cb - \omega^2 p^2 a, pb, pc - bab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.1007)$$

$$\langle a, b, c \mid baa, ca - bab, cb - \omega^2 p^2 a, pb, pc - bab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.1008)$$

$$\langle a, b, c \mid baa, ca - xbab - p^2 a, cb - \omega^2 p^2 a, pb, pc - bab, \text{class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.1009)$$

$$\langle a, b, c \mid baa, ca, cb, pb - bab, pc, \text{class } 3 \rangle \quad (7.1010)$$

$$\langle a, b, c \mid baa, ca - bab, cb, pb - bab, pc, \text{class } 3 \rangle \quad (7.1011)$$

$$\langle a, b, c \mid baa, ca - p^2 a, cb, pb - bab, pc, \text{class } 3 \rangle \quad (7.1012)$$

$$\langle a, b, c \mid baa, ca - bab - p^2 a, cb, pb - bab, pc, \text{class } 3 \rangle \quad (7.1013)$$

$$\langle a, b, c \mid baa, ca - \omega bab - p^2 a, cb, pb - bab, pc, \text{class } 3 \rangle \quad (7.1014)$$

$$\langle a, b, c \mid baa, ca, cb - p^2 a, pb - bab, pc, \text{class } 3 \rangle \quad (7.1015)$$

$$\langle a, b, c \mid baa, ca - bab, cb - p^2 a, pb - bab, pc, \text{class } 3 \rangle \quad (7.1016)$$

$$\langle a, b, c \mid baa, ca - \omega bab, cb - p^2 a, pb - bab, pc, \text{class } 3 \rangle \quad (7.1017)$$

$$\langle a, b, c \mid baa, ca - \omega^2 bab, cb - p^2 a, pb - bab, pc, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.1018)$$

$$\langle a, b, c \mid baa, ca - \omega^3 bab, cb - p^2 a, pb - bab, pc, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.1019)$$

$$\langle a, b, c \mid baa, ca - xbab - p^2 a, cb - p^2 a, pb - bab, pc, \text{class } 3 \rangle \quad (7.1020)$$

$$\langle a, b, c \mid baa, ca - xbab - \omega p^2 a, cb - p^2 a, pb - bab, pc, \text{class } 3 \rangle \quad (7.1021)$$

$$\langle a, b, c \mid bab, ca, cb, pb, pc, \text{class } 3 \rangle \quad (7.1022)$$

$$\langle a, b, c \mid bab, ca - p^2 a, cb, pb, pc, \text{class } 3 \rangle \quad (7.1023)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pb, pc, \text{class } 3 \rangle \quad (7.1024)$$

$$\langle a, b, c \mid bab, ca - p^2 a, cb - baa, pb, pc, \text{class } 3 \rangle \quad (7.1025)$$

$$\langle a, b, c \mid bab, ca - \omega p^2 a, cb - baa, pb, pc, \text{class } 3 \rangle \quad (7.1026)$$

$$\langle a, b, c \mid bab, ca, cb - p^2 a, pb, pc, \text{class } 3 \rangle \quad (7.1027)$$

$$\langle a, b, c \mid bab, ca, cb - baa - p^2 a, pb, pc, \text{class } 3 \rangle \quad (7.1028)$$

$$\langle a, b, c \mid bab, ca, cb, pb, pc - baa, \text{class } 3 \rangle \quad (7.1029)$$

$$\langle a, b, c \mid bab, ca - p^2a, cb, pb, pc - baa, \text{class } 3 \rangle \quad (7.1030)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pb, pc - baa, \text{class } 3 \rangle \quad (7.1031)$$

$$\langle a, b, c \mid bab, ca - p^2a, cb - baa, pb, pc - baa, \text{class } 3 \rangle \quad (7.1032)$$

$$\langle a, b, c \mid bab, ca - \omega p^2a, cb - baa, pb, pc - baa, \text{class } 3 \rangle \quad (7.1033)$$

$$\langle a, b, c \mid bab, ca, cb - p^2a, pb, pc - baa, \text{class } 3 \rangle \quad (7.1034)$$

$$\langle a, b, c \mid bab, ca, cb - baa - p^2a, pb, pc - baa, \text{class } 3 \rangle \quad (7.1035)$$

$$\langle a, b, c \mid bab, ca, cb, pb - baa, pc, \text{class } 3 \rangle \quad (7.1036)$$

$$\langle a, b, c \mid bab, ca - p^2a, cb, pb - baa, pc, \text{class } 3 \rangle \quad (7.1037)$$

$$\langle a, b, c \mid bab, ca - xp^2a, cb - baa, pb - baa, pc, \text{class } 3 \rangle \quad (7.1038)$$

$$\langle a, b, c \mid bab, ca, cb - p^2a, pb - baa, pc, \text{class } 3 \rangle \quad (7.1039)$$

$$\langle a, b, c \mid bab, ca, cb - baa - p^2a, pb - baa, pc, \text{class } 3 \rangle \quad (7.1040)$$

$$\langle a, b, c \mid bab, ca, cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1041)$$

$$\langle a, b, c \mid bab, ca - p^2a, cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1042)$$

$$\langle a, b, c \mid bab, ca - xp^2a, cb - baa, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1043)$$

$$\langle a, b, c \mid bab, ca, cb - p^2a, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1044)$$

$$\langle a, b, c \mid bab, ca, cb - baa - p^2a, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1045)$$

$$\langle a, b, c \mid p^2a, ca, cb, pb, pc, \text{class } 3 \rangle \quad (7.1046)$$

$$\langle a, b, c \mid p^2a, ca, cb, pb - baa, pc, \text{class } 3 \rangle \quad (7.1047)$$

$$\langle a, b, c \mid p^2a, ca, cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1048)$$

$$\langle a, b, c \mid p^2a, ca, cb, pb - bab, pc, \text{class } 3 \rangle \quad (7.1049)$$

$$\langle a, b, c \mid p^2a, ca, cb, pb, pc - baa, \text{class } 3 \rangle \quad (7.1050)$$

- $\langle a, b, c \mid p^2a, ca, cb, pb - bab, pc - baa, \text{class } 3 \rangle$ (7.1051)
- $\langle a, b, c \mid p^2a, ca, cb, pb, pc - bab, \text{class } 3 \rangle$ (7.1052)
- $\langle a, b, c \mid p^2a, ca, cb, pb - baa, pc - bab, \text{class } 3 \rangle$ (7.1053)
- $\langle a, b, c \mid p^2a, ca, cb, pb - \omega baa, pc - bab, \text{class } 3 \rangle$ (7.1054)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb, pc, \text{class } 3 \rangle$ (7.1055)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - baa, pc, \text{class } 3 \rangle$ (7.1056)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - \omega baa, pc, \text{class } 3 \rangle$ (7.1057)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - bab, pc, \text{class } 3 \rangle$ (7.1058)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - baa - bab, pc, \text{class } 3 \rangle$ (7.1059)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - \omega baa - bab, pc, \text{class } 3 \rangle$ (7.1060)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb, pc - bab, \text{class } 3 \rangle$ (7.1061)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - baa, pc - bab, \text{class } 3 \rangle$ (7.1062)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - \omega baa, pc - bab, \text{class } 3 \rangle$ (7.1063)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb, pc - baa, \text{class } 3 \rangle$ (7.1064)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - bab, pc - baa, \text{class } 3 \rangle$ (7.1065)
- $\langle a, b, c \mid p^2a, ca, cb - bab, pb - xbab, pc - baa - bab, \text{class } 3 \rangle$ (7.1066)
- $\langle a, b, c \mid p^2a, ca, cb - baa, pb, pc, \text{class } 3 \rangle$ (7.1067)
- $\langle a, b, c \mid p^2a, ca, cb - baa, pb - baa, pc, \text{class } 3 \rangle$ (7.1068)
- $\langle a, b, c \mid p^2a, ca, cb - baa, pb - \omega baa, pc, \text{class } 3 \rangle$ (7.1069)
- $\langle a, b, c \mid p^2a, ca, cb - baa, pb - bab, pc, \text{class } 3 \rangle$ (7.1070)
- $\langle a, b, c \mid p^2a, ca, cb - baa, pb - baa - bab, pc, \text{class } 3 \rangle$ (7.1071)
- $\langle a, b, c \mid p^2a, ca, cb - baa, pb - \omega baa - bab, pc, \text{class } 3 \rangle$ (7.1072)

$$\langle a, b, c \mid p^2a, ca, cb - baa, pb, pc - bab, \text{class } 3 \rangle \quad (7.1073)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa, pb - baa, pc - bab, \text{class } 3 \rangle \quad (7.1074)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa, pb - \omega baa, pc - bab, \text{class } 3 \rangle \quad (7.1075)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa, pb - \omega^2 baa, pc - bab, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.1076)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa, pb - \omega^3 baa, pc - bab, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.1077)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa, pb, pc - baa, \text{class } 3 \rangle \quad (7.1078)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa, pb - bab, pc - baa, \text{class } 3 \rangle \quad (7.1079)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa, pb - xbab, pc - baa - bab, \text{class } 3 \rangle \quad (7.1080)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa - bab, pb, pc, \text{class } 3 \rangle \quad (7.1081)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa - bab, pb - bab, pc, \text{class } 3 \rangle \quad (7.1082)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa - bab, pb - \omega bab, pc, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.1083)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa - bab, pb - baa - xbab, pc, \text{class } 3 \rangle \quad (x \sim 1 - x) \quad (7.1084)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa - bab, pb - \omega baa - xbab, pc, \text{class } 3 \rangle \quad (x \sim \omega - x) \quad (7.1085)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa - bab, pb - xbaa, pc - bab, \text{class } 3 \rangle \quad (\text{all } x) \quad (7.1086)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa - bab, pb - xbab, pc - baa - ybab, \text{class } 3 \rangle \quad (y \neq \frac{1}{2}, (x, y) \sim (-x, 1 - y)) \quad (7.1087)$$

$$\langle a, b, c \mid p^2a, ca, cb - baa - bab, pb - xbab, pc - baa - \frac{1}{2}bab, \text{class } 3 \rangle \quad (x \sim -x) \quad (7.1088)$$

$$\langle a, b, c \mid p^2a, ca - bab, cb, pb, pc, \text{class } 3 \rangle \quad (7.1089)$$

$$\langle a, b, c \mid p^2a, ca - bab, cb, pb - baa, pc, \text{class } 3 \rangle \quad (7.1090)$$

$$\langle a, b, c \mid p^2a, ca - bab, cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1091)$$

$$\langle a, b, c \mid p^2a, ca - bab, cb, pb - bab, pc, \text{class } 3 \rangle \quad (7.1092)$$

$$\langle a, b, c \mid p^2a, ca - bab, cb, pb, pc - baa, \text{class } 3 \rangle \quad (7.1093)$$

$$\langle a, b, c \mid p^2a, ca - bab, cb, pb - bab, pc - baa, \text{class } 3 \rangle \quad (7.1094)$$

- $\langle a, b, c \mid p^2a, ca - bab, cb, pb - \omega bab, pc - baa, \text{class } 3 \rangle (p = 1 \bmod 3)$ (7.1095)
- $\langle a, b, c \mid p^2a, ca - bab, cb, pb - \omega^2 bab, pc - baa, \text{class } 3 \rangle (p = 1 \bmod 3)$ (7.1096)
- $\langle a, b, c \mid p^2a, ca - bab, cb, pb - xbaa, pc - bab, \text{class } 3 \rangle$ (7.1097)
- $\langle a, b, c \mid p^2a, ca - bab, cb - \omega baa, pb, pc, \text{class } 3 \rangle$ (7.1098)
- $\langle a, b, c \mid p^2a, ca - bab, cb - \omega baa, pb - bab, pc, \text{class } 3 \rangle$ (7.1099)
- $\langle a, b, c \mid p^2a, ca - bab, cb - \omega baa, pb - \omega bab, pc, \text{class } 3 \rangle (p = 1 \bmod 4)$ (7.1100)
- $\langle a, b, c \mid p^2a, ca - bab, cb - \omega baa, pb - baa - xbab, pc, \text{class } 3 \rangle (x \sim -x)$ (7.1101)
- $\langle a, b, c \mid p^2a, ca - bab, cb - \omega baa, pb - \omega baa - xbab, pc, \text{class } 3 \rangle (x \sim -x)$ (7.1102)
- $\langle a, b, c \mid p^2a, ca - bab, cb - \omega baa, pb - xbaa, pc - bab, \text{class } 3 \rangle$ (7.1103)
- $\langle a, b, c \mid p^2a, ca - bab, cb - \omega baa, pb - xbab, pc - baa, \text{class } 3 \rangle (x \sim -x)$ (7.1104)
- $\langle a, b, c \mid p^2a, ca - bab, cb - \omega baa, pb - ybab, pc - xbaa - bab, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (-x, -y))$ (7.1105)
- $\langle a, b, c \mid p^2a - baa, ca, cb, pb, pc - bab, \text{class } 3 \rangle$ (7.1106)
- $\langle a, b, c \mid p^2a - baa, ca, cb, pb - xbab, pc, \text{class } 3 \rangle$ (7.1107)
- $\langle a, b, c \mid p^2a - baa, ca, cb - baa, pb, pc - bab, \text{class } 3 \rangle$ (7.1108)
- $\langle a, b, c \mid p^2a - baa, ca, cb - baa, pb, pc - \omega bab, \text{class } 3 \rangle (p = 1 \bmod 3)$ (7.1109)
- $\langle a, b, c \mid p^2a - baa, ca, cb - baa, pb, pc - \omega^2 bab, \text{class } 3 \rangle (p = 1 \bmod 3)$ (7.1110)
- $\langle a, b, c \mid p^2a - baa, ca, cb - baa, pb - xbab, pc, \text{class } 3 \rangle$ (7.1111)
- $\langle a, b, c \mid p^2a - baa, ca, cb - bab, pb, pc - bab, \text{class } 3 \rangle$ (7.1112)
- $\langle a, b, c \mid p^2a - baa, ca, cb - bab, pb - xbab, pc, \text{class } 3 \rangle$ (7.1113)
- $\langle a, b, c \mid p^2a - baa, ca, cb - baa - bab, pb, pc - xbab, \text{class } 3 \rangle (x \sim -x)$ (7.1114)
- $\langle a, b, c \mid p^2a - baa, ca, cb - baa - bab, pb - xbab, pc, \text{class } 3 \rangle$ (7.1115)
- $\langle a, b, c \mid p^2a - baa, ca, cb - \omega baa - bab, pb, pc - xbab, \text{class } 3 \rangle (x \neq 0, x \sim -x)$ (7.1116)

$$\langle a, b, c \mid p^2a - baa, ca, cb - \omega baa - bab, pb - xbab, pc, \text{ class } 3 \rangle \quad (7.1117)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb, pb, pc - bab, \text{ class } 3 \rangle \quad (7.1118)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb, pb - xbab, pc, \text{ class } 3 \rangle \quad (7.1119)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - baa, pb, pc - xbab, \text{ class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.1120)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - \omega baa, pb, pc - xbab, \text{ class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.1121)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - \omega^2 baa, pb, pc - xbab, \text{ class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1122)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - \omega^3 baa, pb, pc - xbab, \text{ class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1123)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - baa, pb - xbab, pc, \text{ class } 3 \rangle \quad (7.1124)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - \omega baa, pb - xbab, pc, \text{ class } 3 \rangle \quad (7.1125)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - \omega^2 baa, pb - xbab, pc, \text{ class } 3 \rangle (p = 1 \pmod{4}) \quad (7.1126)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - \omega^3 baa, pb - xbab, pc, \text{ class } 3 \rangle (p = 1 \pmod{4}) \quad (7.1127)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - xbaa - bab, pb, pc - ybab, \text{ class } 3 \rangle (y \neq 0, (x, y) \sim (x, -y)) \quad (7.1128)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - xbaa - bab, pb - ybab, pc, \text{ class } 3 \rangle \quad (7.1129)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - xbaa - \omega bab, pb, pc - ybab, \text{ class } 3 \rangle (y \neq 0, (x, y) \sim (x, -y)) \quad (7.1130)$$

$$\langle a, b, c \mid p^2a - baa, ca - bab, cb - xbaa - \omega bab, pb - ybab, pc, \text{ class } 3 \rangle \quad (7.1131)$$

$$\langle a, b, c \mid p^2a - bab, ca, cb, pb, pc, \text{ class } 3 \rangle \quad (7.1132)$$

$$\langle a, b, c \mid p^2a - bab, ca, cb, pb - baa, pc, \text{ class } 3 \rangle \quad (7.1133)$$

$$\langle a, b, c \mid p^2a - bab, ca, cb, pb - \omega baa, pc, \text{ class } 3 \rangle \quad (7.1134)$$

$$\langle a, b, c \mid p^2a - bab, ca, cb, pb, pc - baa, \text{ class } 3 \rangle \quad (7.1135)$$

$$\langle a, b, c \mid p^2a - bab, ca - baa, cb, pb, pc, \text{ class } 3 \rangle \quad (7.1136)$$

$$\langle a, b, c \mid p^2a - bab, ca - baa, cb, pb - baa, pc, \text{ class } 3 \rangle \quad (7.1137)$$

$$\langle a, b, c \mid p^2a - bab, ca - baa, cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1138)$$

$$\langle a, b, c \mid p^2a - bab, ca - baa, cb, pb, pc - baa, \text{class } 3 \rangle \quad (7.1139)$$

$$\langle a, b, c \mid p^2a - bab, ca - bab, cb, pb, pc, \text{class } 3 \rangle \quad (7.1140)$$

$$\langle a, b, c \mid p^2a - bab, ca - bab, cb, pb - baa, pc, \text{class } 3 \rangle \quad (7.1141)$$

$$\langle a, b, c \mid p^2a - bab, ca - bab, cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1142)$$

$$\langle a, b, c \mid p^2a - bab, ca - bab, cb, pb, pc - baa, \text{class } 3 \rangle \quad (7.1143)$$

$$\langle a, b, c \mid p^2a - bab, ca - bab, cb, pb, pc - \omega baa, \text{class } 3 \rangle \ (p = 1 \bmod 4) \quad (7.1144)$$

$$\langle a, b, c \mid p^2a - bab, ca, cb - baa, pb, pc, \text{class } 3 \rangle \quad (7.1145)$$

$$\langle a, b, c \mid p^2a - bab, ca, cb - baa, pb - baa, pc, \text{class } 3 \rangle \quad (7.1146)$$

$$\langle a, b, c \mid p^2a - bab, ca, cb - baa, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1147)$$

$$\langle a, b, c \mid p^2a - bab, ca, cb - baa, pb, pc - xbaa, \text{class } 3 \rangle \ (x \neq 0, x \sim -x) \quad (7.1148)$$

$$\langle a, b, c \mid p^2a - bab, ca - bab, cb - baa, pb - xbaa, pc, \text{class } 3 \rangle \ (\text{all } x) \quad (7.1149)$$

$$\langle a, b, c \mid p^2a - bab, ca - bab, cb - baa, pb, pc - xbaa, \text{class } 3 \rangle \ (x \neq 0, x \sim -x) \quad (7.1150)$$

$$\langle a, b, c \mid p^2a - bab, ca - \omega bab, cb - baa, pb - xbaa, pc, \text{class } 3 \rangle \quad (7.1151)$$

$$\langle a, b, c \mid p^2a - bab, ca - \omega bab, cb - baa, pb, pc - xbaa, \text{class } 3 \rangle \ (x \neq 0, x \sim -x) \quad (7.1152)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca, cb, pb, pc, \text{class } 3 \rangle \quad (7.1153)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca, cb, pb - baa, pc, \text{class } 3 \rangle \quad (7.1154)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca, cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1155)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca, cb, pb, pc - baa, \text{class } 3 \rangle \quad (7.1156)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - baa, cb, pb, pc, \text{class } 3 \rangle \quad (7.1157)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - baa, cb, pb - baa, pc, \text{class } 3 \rangle \quad (7.1158)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - baa, cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1159)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - baa, cb, pb, pc - baa, \text{ class } 3 \rangle \quad (7.1160)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - bab, cb, pb, pc, \text{ class } 3 \rangle \quad (7.1161)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - bab, cb, pb - baa, pc, \text{ class } 3 \rangle \quad (7.1162)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - bab, cb, pb - \omega baa, pc, \text{ class } 3 \rangle \quad (7.1163)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - bab, cb, pb, pc - baa, \text{ class } 3 \rangle \quad (7.1164)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - bab, cb, pb, pc - \omega baa, \text{ class } 3 \rangle \ (p = 1 \bmod 4) \quad (7.1165)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca, cb - baa, pb, pc, \text{ class } 3 \rangle \quad (7.1166)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca, cb - baa, pb - baa, pc, \text{ class } 3 \rangle \quad (7.1167)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca, cb - baa, pb - \omega baa, pc, \text{ class } 3 \rangle \quad (7.1168)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca, cb - baa, pb, pc - xbaa, \text{ class } 3 \rangle \ (x \neq 0, x \sim -x) \quad (7.1169)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - bab, cb - baa, pb - xbaa, pc, \text{ class } 3 \rangle \quad (7.1170)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - bab, cb - baa, pb, pc - xbaa, \text{ class } 3 \rangle \ (x \neq 0, x \sim -x) \quad (7.1171)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - \omega bab, cb - baa, pb - xbaa, pc, \text{ class } 3 \rangle \quad (7.1172)$$

$$\langle a, b, c \mid p^2a - \omega bab, ca - \omega bab, cb - baa, pb, pc - xbaa, \text{ class } 3 \rangle \ (x \neq 0, x \sim -x) \quad (7.1173)$$

8.69 Descendants of 6.100

3 algebras

$$\langle a, b, c \mid baa, caa, cb, pb - \omega ca, pc - ba, \text{ class } 3 \rangle \quad (7.1174)$$

$$\langle a, b, c \mid baa, caa, cb - p^2a, pb - \omega ca, pc - ba, \text{ class } 3 \rangle \quad (7.1175)$$

$$\langle a, b, c \mid baa, p^2a, cb, pb - \omega ca, pc - ba, \text{ class } 3 \rangle \quad (7.1176)$$

8.70 Descendants of 6.101

$3(p-1)/2$ algebras

$$\langle a, b, c \mid baa, caa, cb, pb - xca, pc - ba - ca, \text{ class } 3 \rangle \ (1 + 4x \text{ not a square}) \quad (7.1177)$$

$$\langle a, b, c \mid baa, caa, cb - p^2a, pb - xca, pc - ba - ca, \text{ class } 3 \rangle \ (1 + 4x \text{ not a square}) \quad (7.1178)$$

$$\langle a, b, c \mid baa, p^2a, cb, pb - xca, pc - ba - ca, \text{ class } 3 \rangle \ (1 + 4x \text{ not a square}) \quad (7.1179)$$

8.71 Descendants of 6.102

$p + 3$ algebras

$$\langle a, b, c \mid baa, cb - xp^2b, pa - ca, pc - ba, \text{class } 3 \rangle \quad (7.1180)$$

$$\langle a, b, c \mid p^2b, cb, pa - ca, pc - ba, \text{class } 3 \rangle \quad (7.1181)$$

$$\langle a, b, c \mid p^2b - baa, cb, pa - ca, pc - ba, \text{class } 3 \rangle \quad (7.1182)$$

$$\langle a, b, c \mid p^2b - \omega baa, cb, pa - ca, pc - ba, \text{class } 3 \rangle \quad (7.1183)$$

8.72 Descendants of 6.103

$p + 3$ algebras

$$\langle a, b, c \mid bab, cb, pa - ba, pc - ca, \text{class } 3 \rangle \quad (7.1184)$$

$$\langle a, b, c \mid bab, cb - p^2b, pa - ba, pc - ca, \text{class } 3 \rangle \quad (7.1185)$$

$$\langle a, b, c \mid p^2b - xbab, cb, pa - ba, pc - ca, \text{class } 3 \rangle \quad (7.1186)$$

$$\langle a, b, c \mid p^2b + bab, cb, pa - ba, pc - ca - bab, \text{class } 3 \rangle \quad (7.1187)$$

8.73 Descendants of 6.104

$5p + 24 + 3 \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid bab, bac, caa, cab, cac, cbb, cbc, pa - baa, pb, pc, \text{class } 3 \rangle \quad (7.1188)$$

$$\langle a, b, c \mid baa, bab, caa, cab + bac, cac, cbb, cbc, pa - bac, pb, pc, \text{class } 3 \rangle \quad (7.1189)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac + bab, cbb, cbc, pa - bab, pb, pc, \text{class } 3 \rangle \quad (7.1190)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac + \omega bab, cbb, cbc, pa - bab, pb, pc, \text{class } 3 \rangle \quad (7.1191)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac, cbb, cbc, pa - bab, pb, pc, \text{class } 3 \rangle \quad (7.1192)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac, cbb, cbc, pa - \omega bab, pb, pc, \text{class } 3 \rangle \quad (7.1193)$$

$$\langle a, b, c \mid baa, bac, caa - bab, cab, cac, cbb, cbc, pa - bab, pb, pc, \text{class } 3 \rangle \quad (7.1194)$$

$$\langle a, b, c \mid baa, bac, caa - bab, cab, cac, cbb, cbc, pa - \omega bab, pb, pc, \text{class } 3 \rangle \quad (7.1195)$$

$$\langle a, b, c \mid baa, bac, caa, cab - bab, cac - xbab, cbb, cbc, pa - bab, pb, pc, \text{class } 3 \rangle \quad (x \neq -2) \quad (7.1196)$$

$$\langle a, b, c \mid baa, bac, caa, cab - bab, cac - \frac{1}{4}bab, cbb, cbc, pa - \omega bab, pb, pc, \text{class } 3 \rangle \quad (7.1197)$$

$$\langle a, b, c \mid baa, bac, caa, cab - bab, cac + 2bab, cbb, cbc, pa - bab, pb, pc, \text{ class } 3 \rangle \quad (7.1198)$$

$$\langle a, b, c \mid baa, bac, caa - bab, cab - bab, cac + 2bab, cbb, cbc, pa - bab, pb, pc, \text{ class } 3 \rangle \quad (7.1199)$$

$$\langle a, b, c \mid baa, bab, bac, caa, cab, cac, cbc, pa - cbb, pb, pc, \text{ class } 3 \rangle \quad (7.1200)$$

$$\langle a, b, c \mid bab, bac, caa, cab, cac, cbb - baa, cbc, pa - baa, pb, pc, \text{ class } 3 \rangle \quad (7.1201)$$

$$\langle a, b, c \mid baa, bab, bac, cab, cac, cbb - caa, cbc, pa - caa, pb, pc, \text{ class } 3 \rangle \quad (7.1202)$$

$$\langle a, b, c \mid baa, bab, bac, cab, cac, cbb - \omega caa, cbc, pa - caa, pb, pc, \text{ class } 3 \rangle \quad (7.1203)$$

$$\langle a, b, c \mid baa, bab, caa - xbac, cab, cac, cbb - bac, cbc, pa - bac, pb, pc, \text{ class } 3 \rangle \quad (7.1204)$$

$$\langle a, b, c \mid bab, bac - baa, caa + 2baa, cab, cac, cbb - baa, cbc, pa - baa, pb, pc, \text{ class } 3 \rangle \quad (7.1205)$$

$$\langle a, b, c \mid bab, bac - baa, caa + 2baa, cab, cac, cbb - baa, cbc, pa - \omega baa, pb, pc, \text{ class } 3 \rangle \quad (7.1206)$$

$$\langle a, b, c \mid baa, bab, bac, caa, cab, cbb - cac, cbc, pa - cac, pb, pc, \text{ class } 3 \rangle \quad (7.1207)$$

$$\langle a, b, c \mid baa, bab, bac, caa, cab, cbb - cac, cbc, pa - \omega cac, pb, pc, \text{ class } 3 \rangle \quad (7.1208)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac + bab, cbb + bab, cbc, pa - bab, pb, pc, \text{ class } 3 \rangle \quad (7.1209)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac + bab, cbb + bab, cbc, pa - \omega bab, pb, pc, \text{ class } 3 \rangle \quad (7.1210)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac + \omega bab, cbb + \omega bab, cbc, pa - bab, pb, pc, \text{ class } 3 \rangle \quad (7.1211)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac + \omega bab, cbb + \omega bab, cbc, pa - \omega bab, pb, pc, \text{ class } 3 \rangle \quad (7.1212)$$

$$\langle a, b, c \mid bab - xbaa, bac, caa, cab, cac - baa, cbb - baa, cbc, pa - baa, pb, pc, \text{ class } 3 \rangle (x \sim ax \text{ if } a^3 = 1) \quad (7.1213)$$

$$\langle a, b, c \mid bab - xbaa, bac, caa, cab, cac - \omega baa, cbb - \omega baa, cbc, pa - baa, pb, pc, \text{ class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.1214)$$

$$\langle a, b, c \mid bab - xbaa, bac, caa, cab, cac - \omega^2 baa, cbb - \omega^2 baa, cbc, pa - baa, pb, pc, \text{ class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.1215)$$

$$\langle a, b, c \mid bab - xbaa, bac, caa, cab, cac - baa, cbb - baa, cbc, pa - \omega baa, pb, pc, \text{ class } 3 \rangle (x \sim ax \text{ if } a^3 = 1) \quad (7.1216)$$

$$\langle a, b, c \mid bab - xbaa, bac, caa, cab, cac - \omega baa, cbb - \omega baa, cbc, pa - \omega baa, pb, pc, \text{class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.1217)$$

$$\langle a, b, c \mid bab - xbaa, bac, caa, cab, cac - \omega^2 baa, cbb - \omega^2 baa, cbc, pa - \omega baa, pb, pc, \text{class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.1218)$$

$$\langle a, b, c \mid bab, bac, caa, cab, cac, cbb, cbc, pa, pb, pc, \text{class } 3 \rangle \quad (7.1219)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac - bab, cbb, cbc, pa, pb, pc, \text{class } 3 \rangle \quad (7.1220)$$

$$\langle a, b, c \mid baa, bac, caa, cab, cac - \omega bab, cbb, cbc, pa, pb, pc, \text{class } 3 \rangle \quad (7.1221)$$

$$\langle a, b, c \mid baa, bac, caa - bab, cab, cac, cbb, cbc, pa, pb, pc, \text{class } 3 \rangle \quad (7.1222)$$

$$\langle a, b, c \mid bab, bac, caa, cab, cac - baa, cbb - baa, cbc - xbaa, pa, pb, pc, \text{class } 3 \rangle (x \sim ax \text{ if } a^3 = 1) \quad (7.1223)$$

$$\langle a, b, c \mid bab, bac, caa, cab, cac - baa, cbb - \omega baa, cbc - xbaa, pa, pb, pc, \text{class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.1224)$$

$$\langle a, b, c \mid bab, bac, caa, cab, cac - baa, cbb - \omega^2 baa, cbc - xbaa, pa, pb, pc, \text{class } 3 \rangle (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.1225)$$

$$\langle a, b, c \mid bab - baa, bac, caa - baa, cab, cac + baa, cbb + baa, cbc - baa, pa, pb, pc, \text{class } 3 \rangle \quad (7.1226)$$

8.74 Descendants of 6.105

$11 + 5 \gcd(p - 1, 3) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid baa, bab, bac, cac, pa - cb, pb, pc, \text{class } 3 \rangle \quad (7.1227)$$

$$\langle a, b, c \mid baa, bab, bac, cac, pa - cb - caa, pb, pc, \text{class } 3 \rangle \quad (7.1228)$$

$$\langle a, b, c \mid baa, bab, bac, cac, pa - cb, pb, pc - caa, \text{class } 3 \rangle \quad (7.1229)$$

$$\langle a, b, c \mid baa, bab, bac, cac, pa - cb, pb, pc - \omega caa, \text{class } 3 \rangle \quad (7.1230)$$

$$\langle a, b, c \mid baa, bab, bac, cac, pa - cb, pb - caa, pc, \text{class } 3 \rangle \quad (7.1231)$$

$$\langle a, b, c \mid baa, bab, bac, caa, pa - cb, pb, pc, \text{class } 3 \rangle \quad (7.1232)$$

$$\langle a, b, c \mid baa, bab, bac, caa, pa - cb, pb, pc - cac, \text{class } 3 \rangle \quad (7.1233)$$

$$\langle a, b, c \mid baa, bab, bac, caa, pa - cb, pb - cac, pc, \text{class } 3 \rangle \quad (7.1234)$$

$$\langle a, b, c \mid baa, bab, bac, caa, pa - cb, pb - \omega cac, pc, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1235)$$

$$\langle a, b, c \mid baa, bab, bac, caa, pa - cb, pb - \omega^2 cac, pc, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1236)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, pa - cb, pb, pc, \text{class } 3 \rangle \quad (7.1237)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, pa - cb, pb, pc - baa, \text{class } 3 \rangle \quad (7.1238)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, pa - cb, pb, pc - xbaa, \text{class } 3 \rangle \quad (x = \omega, \omega^2, \omega^3, \omega^4, p = 1 \bmod 5) \quad (7.1239)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, pa - cb, pb - baa, pc, \text{class } 3 \rangle \quad (7.1240)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, pa - cb, pb - \omega baa, pc, \text{class } 3 \rangle \quad (7.1241)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, pa - cb, pb - xbaa, pc, \text{class } 3 \rangle \quad (x = \omega^2, \omega^3, \omega^4, \omega^5, p = 1 \bmod 3) \quad (7.1242)$$

$$\langle a, b, c \mid baa, bac, caa, cac + bab, pa - cb, pb, pc, \text{class } 3 \rangle \quad (7.1243)$$

$$\langle a, b, c \mid baa, bac, caa, cac + bab, pa - cb, pb, pc - bab, \text{class } 3 \rangle \quad (7.1244)$$

$$\langle a, b, c \mid baa, bac, caa, cac + bab, pa - cb, pb, pc - \omega bab, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1245)$$

$$\langle a, b, c \mid baa, bac, caa, cac + bab, pa - cb, pb, pc - \omega^2 bab, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1246)$$

$$\langle a, b, c \mid baa, bac, caa, cac + bab, pa - cb, pb - bab, pc - bab, \text{class } 3 \rangle \quad (7.1247)$$

$$\langle a, b, c \mid baa, bac, caa, cac + \omega bab, pa - cb, pb, pc, \text{class } 3 \rangle \quad (7.1248)$$

$$\langle a, b, c \mid baa, bac, caa, cac + \omega bab, pa - cb, pb, pc - bab, \text{class } 3 \rangle \quad (7.1249)$$

$$\langle a, b, c \mid baa, bac, caa, cac + \omega bab, pa - cb, pb, pc - \omega bab, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1250)$$

$$\langle a, b, c \mid baa, bac, caa, cac + \omega bab, pa - cb, pb, pc - \omega^2 bab, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1251)$$

8.75 Descendants of 6.106

$p^2 + 10p + 32 + \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

- $$\langle a, b, c \mid caa, cab, cac, cbc, pa - ba, pb, pc - xcbb, \text{ class } 3 \rangle \quad (7.1252)$$
- $$\langle a, b, c \mid caa, cab, cac, cbc, pa - ba, pb - cbb, pc - xcbb, \text{ class } 3 \rangle \quad (7.1253)$$
- $$\langle a, b, c \mid caa, cab, cbb - cac, cbc, pa - ba, pb, pc - xcac, \text{ class } 3 \rangle \quad (7.1254)$$
- $$\langle a, b, c \mid caa, cab, cbb - cac, cbc, pa - ba, pb - cac, pc - xcac, \text{ class } 3 \rangle \quad (7.1255)$$
- $$\langle a, b, c \mid cab, cac, cbb - caa, cbc, pa - ba, pb, pc - xcaa, \text{ class } 3 \rangle \quad (7.1256)$$
- $$\langle a, b, c \mid cab, cac, cbb - caa, cbc, pa - ba, pb - caa, pc - xcaa, \text{ class } 3 \rangle \quad (7.1257)$$
- $$\langle a, b, c \mid cab, cac, cbb - \omega caa, cbc, pa - ba, pb, pc - xcaa, \text{ class } 3 \rangle \quad (7.1258)$$
- $$\langle a, b, c \mid cab, cac, cbb - \omega caa, cbc, pa - ba, pb - caa, pc - xcaa, \text{ class } 3 \rangle \quad (7.1259)$$
- $$\langle a, b, c \mid cab, cac - caa, cbb - caa, cbc, pa - ba, pb - ycaa, pc - xcaa, \text{ class } 3 \rangle ((x, y) \sim (x, -y)) \quad (7.1260)$$
- $$\langle a, b, c \mid cab, cac - caa, cbb - \omega caa, cbc, pa - ba, pb - ycaa, pc - xcaa, \text{ class } 3 \rangle ((x, y) \sim (x, -y)) \quad (7.1261)$$
- $$\langle a, b, c \mid caa, cac, cbb, cbc, pa - ba, pb, pc, \text{ class } 3 \rangle \quad (7.1262)$$
- $$\langle a, b, c \mid caa, cac, cbb, cbc, pa - ba, pb, pc - cab, \text{ class } 3 \rangle \quad (7.1263)$$
- $$\langle a, b, c \mid caa, cac, cbb, cbc, pa - ba, pb - cab, pc, \text{ class } 3 \rangle \quad (7.1264)$$
- $$\langle a, b, c \mid caa, cac, cbb, cbc, pa - ba, pb - cab, pc - cab, \text{ class } 3 \rangle \quad (7.1265)$$
- $$\langle a, b, c \mid caa, cac, cbb, cbc - cab, pa - ba, pb, pc, \text{ class } 3 \rangle \quad (7.1266)$$
- $$\langle a, b, c \mid caa, cac, cbb, cbc - cab, pa - ba, pb - cab, pc, \text{ class } 3 \rangle \quad (7.1267)$$
- $$\langle a, b, c \mid caa, cac, cbb, cbc - cab, pa - ba, pb - \omega cab, pc, \text{ class } 3 \rangle \quad (7.1268)$$
- $$\langle a, b, c \mid caa, cac, cbb, cbc - cab, pa - ba, pb - xcb, pc - cab, \text{ class } 3 \rangle \quad (7.1269)$$
- $$\langle a, b, c \mid caa, cab, cac, cbb, pa - ba, pb, pc, \text{ class } 3 \rangle \quad (7.1270)$$
- $$\langle a, b, c \mid caa, cab, cac, cbb, pa - ba, pb, pc - cbc, \text{ class } 3 \rangle \quad (7.1271)$$
- $$\langle a, b, c \mid caa, cab, cac, cbb, pa - ba, pb - cbc, pc, \text{ class } 3 \rangle \quad (7.1272)$$

- $\langle a, b, c \mid caa, cab, cac, cbb, pa - ba, pb - \omega cbc, pc, \text{class } 3 \rangle$ (7.1273)
- $\langle a, b, c \mid caa, cab, cac, cbb, pa - ba - cbc, pb, pc, \text{class } 3 \rangle$ (7.1274)
- $\langle a, b, c \mid caa, cab, cac, cbb, pa - ba - cbc, pb, pc - cbc, \text{class } 3 \rangle$ (7.1275)
- $\langle a, b, c \mid caa, cab, cac, cbb, pa - ba - cbc, pb - cbc, pc, \text{class } 3 \rangle$ (7.1276)
- $\langle a, b, c \mid caa, cab, cac, cbb, pa - ba - cbc, pb - \omega cbc, pc, \text{class } 3 \rangle$ (7.1277)
- $\langle a, b, c \mid cab, cac, cbb, cbc - caa, pa - ba, pb, pc, \text{class } 3 \rangle$ (7.1278)
- $\langle a, b, c \mid cab, cac, cbb, cbc - caa, pa - ba, pb, pc - caa, \text{class } 3 \rangle$ (7.1279)
- $\langle a, b, c \mid cab, cac, cbb, cbc - caa, pa - ba, pb, pc - \omega caa, \text{class } 3 \rangle$ (7.1280)
- $\langle a, b, c \mid cab, cac, cbb, cbc - caa, pa - ba, pb - caa, pc, \text{class } 3 \rangle$ (7.1281)
- $\langle a, b, c \mid cab, cac, cbb, cbc - caa, pa - ba, pb - \omega caa, pc, \text{class } 3 \rangle$ (7.1282)
- $\langle a, b, c \mid cab, cac, cbb, cbc - caa, pa - ba, pb - \omega^2 caa, pc, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.1283)
- $\langle a, b, c \mid cab, cac, cbb, cbc - caa, pa - ba, pb - \omega^3 caa, pc, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.1284)
- $\langle a, b, c \mid caa, cab, cbb, cbc, pa - ba, pb, pc, \text{class } 3 \rangle$ (7.1285)
- $\langle a, b, c \mid caa, cab, cbb, cbc, pa - ba - cac, pb, pc, \text{class } 3 \rangle$ (7.1286)
- $\langle a, b, c \mid caa, cab, cbb, cbc, pa - ba - \omega cac, pb, pc, \text{class } 3 \rangle$ (7.1287)
- $\langle a, b, c \mid caa, cab, cbb, cbc, pa - ba, pb, pc - cac, \text{class } 3 \rangle$ (7.1288)
- $\langle a, b, c \mid caa, cab, cbb, cbc, pa - ba - cac, pb, pc - cac, \text{class } 3 \rangle$ (7.1289)
- $\langle a, b, c \mid caa, cab, cbb, cbc, pa - ba - \omega cac, pb, pc - cac, \text{class } 3 \rangle$ (7.1290)
- $\langle a, b, c \mid caa, cab, cbb, cbc, pa - ba, pb - cac, pc, \text{class } 3 \rangle$ (7.1291)
- $\langle a, b, c \mid cab, cac, cbb, cbc, pa - ba, pb, pc, \text{class } 3 \rangle$ (7.1292)
- $\langle a, b, c \mid cab, cac, cbb, cbc, pa - ba, pb, pc - caa, \text{class } 3 \rangle$ (7.1293)
- $\langle a, b, c \mid cab, cac, cbb, cbc, pa - ba, pb, pc - \omega caa, \text{class } 3 \rangle$ (7.1294)

$$\langle a, b, c \mid cab, cac, cbb, cbc, pa - ba, pb - caa, pc, \text{ class } 3 \rangle \quad (7.1295)$$

$$\langle a, b, c \mid cab, cac - caa, cbb, cbc, pa - ba, pb, pc, \text{ class } 3 \rangle \quad (7.1296)$$

$$\langle a, b, c \mid cab, cac - caa, cbb, cbc, pa - ba, pb, pc - caa, \text{ class } 3 \rangle \quad (7.1297)$$

$$\langle a, b, c \mid cab, cac - caa, cbb, cbc, pa - ba, pb, pc - \omega caa, \text{ class } 3 \rangle \quad (7.1298)$$

$$\langle a, b, c \mid cab, cac - caa, cbb, cbc, pa - ba, pb - caa, pc, \text{ class } 3 \rangle \quad (7.1299)$$

$$\langle a, b, c \mid cab, cac - caa, cbb, cbc, pa - ba, pb - \omega caa, pc, \text{ class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.1300)$$

$$\langle a, b, c \mid cab, cac - caa, cbb, cbc, pa - ba, pb - \omega^2 caa, pc, \text{ class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.1301)$$

8.76 Descendants of 6.108

$(p-1)(12 + \gcd(p-1, 3))/2 + \gcd(p-1, 4)/2$ algebras

$$\langle a, b, c \mid baa, bab, pa - ca, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1302)$$

$$\langle a, b, c \mid baa, bac, pa - ca, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1303)$$

$$\langle a, b, c \mid baa, bac, pa - ca - bab, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1304)$$

$$\langle a, b, c \mid baa, bac, pa - ca - \omega bab, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1305)$$

$$\langle a, b, c \mid baa, bac, pa - ca, pb - cb, pc - bab, \text{ class } 3 \rangle \quad (7.1306)$$

$$\langle a, b, c \mid baa, bab, pa - ca, pb - xcb, pc, \text{ class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1307)$$

$$\langle a, b, c \mid bab, bac - baa, pa - ca, pb + 2cb, pc, \text{ class } 3 \rangle \quad (7.1308)$$

$$\langle a, b, c \mid baa, bac, pa - ca, pb - xcb, pc, \text{ class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1309)$$

$$\langle a, b, c \mid baa, bac, pa - ca - bab, pb - xcb, pc, \text{ class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1310)$$

$$\langle a, b, c \mid baa, bac, pa - ca - \omega bab, pb - xcb, pc, \text{ class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1311)$$

$$\langle a, b, c \mid baa, bac, pa - ca, pb - xcb, pc - bab, \text{ class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1312)$$

$$\langle a, b, c \mid bab, bac, pa - ca, pb - xcb, pc, \text{ class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1313)$$

$$\langle a, b, c \mid bab, bac, pa - ca, pb - xcb - baa, pc, \text{ class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1314)$$

$$\langle a, b, c \mid bab, bac, pa - ca, pb - xcb - \omega baa, pc, \text{class } 3 \rangle (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1315)$$

$$\langle a, b, c \mid bab, bac, pa - ca, pb - xcb, pc - baa, \text{class } 3 \rangle (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1316)$$

$$\langle a, b, c \mid bab - baa, bac, pa - ca, pb - xcb, pc, \text{class } 3 \rangle (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1317)$$

$$\langle a, b, c \mid bab - baa, bac, pa - ca, pb - xcb - baa, pc, \text{class } 3 \rangle (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1318)$$

$$\langle a, b, c \mid bab - baa, bac, pa - ca, pb - xcb - \omega baa, pc, \text{class } 3 \rangle (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1319)$$

$$\langle a, b, c \mid bab - baa, bac, pa - ca, pb - xcb, pc - baa, \text{class } 3 \rangle (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.1320)$$

$$\langle a, b, c \mid bab - baa, bac, pa - ca, pb - xcb, pc - \omega baa, \text{class } 3 \rangle (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}, p = 1 \pmod{3}) \quad (7.1321)$$

$$\langle a, b, c \mid bab - baa, bac, pa - ca, pb - xcb, pc - \omega^2 baa, \text{class } 3 \rangle (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}, p = 1 \pmod{3}) \quad (7.1322)$$

$$\langle a, b, c \mid baa, bab, pa - ca, pb + cb, pc, \text{class } 3 \rangle \quad (7.1323)$$

$$\langle a, b, c \mid baa, cab, pa - ca, pb + cb, pc, \text{class } 3 \rangle \quad (7.1324)$$

$$\langle a, b, c \mid baa, cab, pa - ca - bab, pb + cb, pc, \text{class } 3 \rangle \quad (7.1325)$$

$$\langle a, b, c \mid baa, cab, pa - ca - \omega bab, pb + cb, pc, \text{class } 3 \rangle \quad (7.1326)$$

$$\langle a, b, c \mid baa, cab, pa - ca, pb + cb, pc - bab, \text{class } 3 \rangle \quad (7.1327)$$

$$\langle a, b, c \mid bab - baa, cab, pa - ca, pb + cb, pc, \text{class } 3 \rangle \quad (7.1328)$$

$$\langle a, b, c \mid bab - baa, cab, pa - ca, pb + cb - baa, pc, \text{class } 3 \rangle \quad (7.1329)$$

$$\langle a, b, c \mid bab - baa, cab, pa - ca, pb + cb - \omega baa, pc, \text{class } 3 \rangle (p = 1 \pmod{4}) \quad (7.1330)$$

$$\langle a, b, c \mid bab - baa, cab, pa - ca, pb + cb, pc - baa, \text{class } 3 \rangle \quad (7.1331)$$

$$\langle a, b, c \mid bab - baa, cab, pa - ca, pb + cb, pc - \omega baa, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.1332)$$

$$\langle a, b, c \mid bab - baa, cab, pa - ca, pb + cb, pc - \omega^2 baa, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.1333)$$

8.77 Descendants of 6.109

$7 + 2 \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid baa, bab, pa - ca - cb, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1334)$$

$$\langle a, b, c \mid baa, caa, pa - ca - cb, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1335)$$

$$\langle a, b, c \mid baa, caa, pa - ca - cb, pb - cb - bab, pc, \text{ class } 3 \rangle \quad (7.1336)$$

$$\langle a, b, c \mid baa, caa, pa - ca - cb, pb - cb - \omega bab, pc, \text{ class } 3 \rangle \quad (7.1337)$$

$$\langle a, b, c \mid baa, caa, pa - ca - cb, pb - cb, pc - bab, \text{ class } 3 \rangle \quad (7.1338)$$

$$\langle a, b, c \mid baa, caa, pa - ca - cb, pb - cb, pc - \omega bab, \text{ class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1339)$$

$$\langle a, b, c \mid baa, caa, pa - ca - cb, pb - cb, pc - \omega^2 bab, \text{ class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1340)$$

$$\langle a, b, c \mid bab, caa, pa - ca - cb, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1341)$$

$$\langle a, b, c \mid bab, caa, pa - ca - cb, pb - cb - baa, pc, \text{ class } 3 \rangle \quad (7.1342)$$

$$\langle a, b, c \mid bab, caa, pa - ca - cb, pb - cb - \omega baa, pc, \text{ class } 3 \rangle \quad (7.1343)$$

$$\langle a, b, c \mid bab, caa, pa - ca - cb, pb - cb, pc - baa, \text{ class } 3 \rangle \quad (7.1344)$$

$$\langle a, b, c \mid bab, caa, pa - ca - cb, pb - cb, pc - \omega baa, \text{ class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1345)$$

$$\langle a, b, c \mid bab, caa, pa - ca - cb, pb - cb, pc - \omega^2 baa, \text{ class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1346)$$

8.78 Descendants of 6.110

$2 + \gcd(p - 1, 3) + \gcd(p - 1, 4)/2$ algebras

$$\langle a, b, c \mid baa, bab, pa - \omega cb, pb - ca, pc, \text{ class } 3 \rangle \quad (7.1347)$$

$$\langle a, b, c \mid baa, cbb, pa - \omega cb, pb - ca, pc, \text{ class } 3 \rangle \quad (7.1348)$$

$$\langle a, b, c \mid baa, cbb, pa - \omega cb, pb - ca, pc - bab, \text{ class } 3 \rangle \quad (7.1349)$$

$$\langle a, b, c \mid baa, cbb, pa - \omega cb, pb - ca, pc - \omega bab, \text{ class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1350)$$

$$\langle a, b, c \mid baa, cbb, pa - \omega cb, pb - ca, pc - \omega^2 bab, \text{ class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.1351)$$

$$\langle a, b, c \mid baa, cbb, pa - \omega cb, pb - ca - bab, pc, \text{ class } 3 \rangle \quad (7.1352)$$

$$\langle a, b, c \mid baa, cbb, pa - \omega cb, pb - ca - \omega bab, pc, \text{ class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.1353)$$

8.79 Descendants of 6.111

$(p-1)(4 + \gcd(p-1, 3))/2$ algebras

$$\langle a, b, c \mid baa, bab, pa - xcb, pb - ca - cb, pc, \text{class } 3 \rangle (1 + 4x \text{ not a square}) \quad (7.1354)$$

$$\langle a, b, c \mid baa, bac, pa - xcb, pb - ca - cb, pc, \text{class } 3 \rangle (1 + 4x \text{ not a square}) \quad (7.1355)$$

$$\langle a, b, c \mid baa, bac, pa - xcb, pb - ca - cb - bab, pc, \text{class } 3 \rangle (1 + 4x \text{ not a square}) \quad (7.1356)$$

$$\langle a, b, c \mid baa, bac, pa - xcb, pb - ca - cb - \omega bab, pc, \text{class } 3 \rangle (1 + 4x \text{ not a square}) \quad (7.1357)$$

$$\langle a, b, c \mid baa, bac, pa - xcb, pb - ca - cb, pc - bab, \text{class } 3 \rangle (1 + 4x \text{ not a square}) \quad (7.1358)$$

$$\langle a, b, c \mid baa, bac, pa - xcb, pb - ca - cb, pc - \omega bab, \text{class } 3 \rangle (1 + 4x \text{ not a square, } p = 1 \pmod{3}) \quad (7.1359)$$

$$\langle a, b, c \mid baa, bac, pa - xcb, pb - ca - cb, pc - \omega^2 bab, \text{class } 3 \rangle (1 + 4x \text{ not a square, } p = 1 \pmod{3}) \quad (7.1360)$$

8.80 Descendants of 6.112

$p^2 + 4p + 3 + 2 \gcd(p-1, 3)$ algebras

$$\langle a, b, c \mid bab, cbb, pa - ba, pb - ca, pc, \text{class } 3 \rangle \quad (7.1361)$$

$$\langle a, b, c \mid bab, cbb, pa - ba - cbc, pb - ca, pc, \text{class } 3 \rangle \quad (7.1362)$$

$$\langle a, b, c \mid bab, cbb, pa - ba - \omega cbc, pb - ca, pc, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.1363)$$

$$\langle a, b, c \mid bab, cbb, pa - ba - \omega^2 cbc, pb - ca, pc, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.1364)$$

$$\langle a, b, c \mid bab, cbb, pa - ba - xcbc, pb - ca, pc - cbc, \text{class } 3 \rangle \quad (7.1365)$$

$$\langle a, b, c \mid bab, cbc, pa - ba, pb - ca, pc - xcbb, \text{class } 3 \rangle \quad (7.1366)$$

$$\langle a, b, c \mid bab, cbc, pa - ba, pb - ca - cbb, pc - xcbb, \text{class } 3 \rangle \quad (7.1367)$$

$$\langle a, b, c \mid bab, cbc - cbb, pa - ba, pb - ca - xcbb, pc - ycbb, \text{class } 3 \rangle \quad (7.1368)$$

$$\langle a, b, c \mid cbb, cbc, pa - ba, pb - ca, pc, \text{class } 3 \rangle \quad (7.1369)$$

$$\langle a, b, c \mid cbb, cbc - bab, pa - ba, pb - ca, pc, \text{class } 3 \rangle \quad (7.1370)$$

$$\langle a, b, c \mid cbb, cbc - \omega bab, pa - ba, pb - ca, pc, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.1371)$$

$$\langle a, b, c \mid cbb, cbc - \omega^2 bab, pa - ba, pb - ca, pc, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.1372)$$

$$\langle a, b, c \mid cbb - bab, cbc - xbab, pa - ba, pb - ca, pc, \text{class } 3 \rangle (x \sim -x) \quad (7.1373)$$

$$\langle a, b, c \mid cbb - \omega bab, cbc - xbab, pa - ba, pb - ca, pc, \text{class } 3 \rangle (x \sim -x) \quad (7.1374)$$

8.81 Descendants of 6.113

$5p + 4$ algebras

$$\langle a, b, c \mid bab, cac, pa - ba, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1375)$$

$$\langle a, b, c \mid bab, cac, pa - ba, pb - cb, pc - caa, \text{ class } 3 \rangle \quad (7.1376)$$

$$\langle a, b, c \mid bab, cac, pa - ba, pb - cb, pc - \omega caa, \text{ class } 3 \rangle \quad (7.1377)$$

$$\langle a, b, c \mid bab, cac, pa - ba, pb - cb - caa, pc - xcaa, \text{ class } 3 \rangle \quad (7.1378)$$

$$\langle a, b, c \mid bab, cac, pa - ba, pb - cb - \omega caa, pc - xcaa, \text{ class } 3 \rangle \quad (7.1379)$$

$$\langle a, b, c \mid bab, caa, pa - ba - xcac, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1380)$$

$$\langle a, b, c \mid bab, caa, pa - ba - xcac, pb - cb, pc - cac, \text{ class } 3 \rangle \quad (7.1381)$$

$$\langle a, b, c \mid caa, cac, pa - ba, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1382)$$

$$\langle a, b, c \mid caa, cac - xbab, pa - ba, pb - cb, pc, \text{ class } 3 \rangle \quad (x \neq 0) \quad (7.1383)$$

$$\langle a, b, c \mid caa - bab, cac + 2bab, pa - ba, pb - cb, pc, \text{ class } 3 \rangle \quad (7.1384)$$

8.82 Descendants of 6.114

$4p - 4$ algebras

$$\langle a, b, c \mid bac, pa - ba, pb - cb, pc + ba - ca, \text{ class } 3 \rangle \quad (7.1385)$$

$$\langle a, b, c \mid bac - bab, pa - ba, pb - cb, pc + ba - ca - xbab, \text{ class } 3 \rangle \quad (7.1386)$$

$$\langle a, b, c \mid bac, pa - ba, pb - cb, pc - 3ba - ca, \text{ class } 3 \rangle \quad (7.1387)$$

$$\langle a, b, c \mid bac + bab, pa - ba, pb - cb, pc - 3ba - ca, \text{ class } 3 \rangle \quad (7.1388)$$

$$\langle a, b, c \mid bac - ybab, pa - ba, pb - cb, pc - xba - ca, \text{ class } 3 \rangle \quad (x \neq -1, 3) \quad (7.1389)$$

In algebra 7.1389, (for a given $x \neq -1, 3$) y and y' define isomorphic algebras if the ratios $1 : y$ and $1 : y'$ are in the same orbit of ratios $\alpha : \beta$ under the action

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow A \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

where A equals

$$\begin{pmatrix} x-1 & 1 \\ -1 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} x^2-2x & x-1 \\ 1-x & -1 \end{pmatrix} \text{ or } \begin{pmatrix} (1+\gamma x)(\gamma x-2\gamma+1) & \gamma(\gamma x+2-\gamma) \\ -\gamma(\gamma x+2-\gamma) & -(-1+\gamma)(\gamma+1) \end{pmatrix}$$

with $\gamma \neq -1$ and γ not a root of $\gamma^2 + (x-1)\gamma + 1 = 0$. (Note that the ratio $1 : 0$ is in the same orbit as the ratio $0 : 1$.)

8.83 Descendants of 6.115

3 algebras

$$\langle a, b, c \mid bac, cac, pa - ba, pb - ca, pc - cb, \text{class } 3 \rangle \quad (7.1390)$$

$$\langle a, b, c \mid bac, cac - bab, pa - ba, pb - ca, pc - cb, \text{class } 3 \rangle \quad (7.1391)$$

$$\langle a, b, c \mid bac, cac - \omega bab, pa - ba, pb - ca, pc - cb, \text{class } 3 \rangle \quad (7.1392)$$

8.84 Descendants of 6.116

3 algebras

$$\langle a, b, c \mid bab, pa - ba, pb - ca, pc + cb, \text{class } 3 \rangle \quad (7.1393)$$

$$\langle a, b, c \mid bab, pa - ba - cac, pb - ca, pc + cb, \text{class } 3 \rangle \quad (7.1394)$$

$$\langle a, b, c \mid bab, pa - ba - \omega cac, pb - ca, pc + cb, \text{class } 3 \rangle \quad (7.1395)$$

8.85 Descendants of 6.117

$p + 1$ algebras

$$\langle a, b, c \mid bab, pa - ba - xcac, pb - ca, pc - \omega ba + cb, \text{class } 3 \rangle \quad (7.1396)$$

$$\langle a, b, c \mid cac, pa - ba, pb - ca, pc - \omega ba + cb, \text{class } 3 \rangle \quad (7.1397)$$

8.86 Descendants of 6.118

$11 + 4 \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid ca, cb, baaa, baab, pa, pb, pc, \text{class } 4 \rangle \quad (7.1398)$$

$$\langle a, b, c \mid ca, cb, baaa, baab, pa, pb - babb, pc, \text{class } 4 \rangle \quad (7.1399)$$

$$\langle a, b, c \mid ca, cb, baaa, baab, pa - babb, pb, pc, \text{class } 4 \rangle \quad (7.1400)$$

$$\langle a, b, c \mid ca, cb, baaa, baab, pa - \omega babb, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1401)$$

$$\langle a, b, c \mid ca, cb, baaa, baab, pa - \omega^2 babb, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1402)$$

$$\langle a, b, c \mid ca, cb, baaa, baab, pa, pb, pc - babb, \text{class } 4 \rangle \quad (7.1403)$$

$$\langle a, b, c \mid ca - babb, cb, baaa, baab, pa, pb, pc, \text{class } 4 \rangle \quad (7.1404)$$

$$\langle a, b, c \mid ca - babb, cb, baaa, baab, pa, pb - babb, pc, \text{class } 4 \rangle \quad (7.1405)$$

$$\langle a, b, c \mid ca - babb, cb, baaa, baab, pa - babb, pb, pc, \text{class } 4 \rangle \quad (7.1406)$$

$$\langle a, b, c \mid ca - babb, cb, baaa, baab, pa - \omega babb, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1407)$$

$$\langle a, b, c \mid ca - babb, cb, baaa, baab, pa - \omega^2 babb, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1408)$$

$$\langle a, b, c \mid ca - babb, cb, baaa, baab, pa, pb, pc - babb, \text{class } 4 \rangle \quad (7.1409)$$

$$\langle a, b, c \mid ca, cb, baab, babb + baaa, pa, pb, pc, \text{class } 4 \rangle \quad (7.1410)$$

$$\langle a, b, c \mid ca, cb, baab, babb + baaa, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (7.1411)$$

$$\langle a, b, c \mid ca, cb, baab, babb + baaa, pa, pb - \omega baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1412)$$

$$\langle a, b, c \mid ca, cb, baab, babb + baaa, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1413)$$

$$\langle a, b, c \mid ca, cb, baab, babb + baaa, pa - baaa, pb - baaa, pc, \text{class } 4 \rangle \quad (7.1414)$$

$$\langle a, b, c \mid ca, cb, baab, babb + baaa, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (7.1415)$$

$$\langle a, b, c \mid ca, cb, baab, babb + \omega baaa, pa, pb, pc, \text{class } 4 \rangle \quad (7.1416)$$

$$\langle a, b, c \mid ca, cb, baab, babb + \omega baaa, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (7.1417)$$

$$\langle a, b, c \mid ca, cb, baab, babb + \omega baaa, pa, pb - \omega baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1418)$$

$$\langle a, b, c \mid ca, cb, baab, babb + \omega baaa, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1419)$$

$$\langle a, b, c \mid ca, cb, baab, babb + \omega baaa, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (7.1420)$$

8.87 Descendants of 6.119

$(p - 1)/2 + 3 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid ca, cb, pa - bab, pb, pc, \text{class } 4 \rangle \quad (7.1421)$$

$$\langle a, b, c \mid ca, cb, pa - bab - baaa, pb, pc, \text{class } 4 \rangle \quad (7.1422)$$

$$\langle a, b, c \mid ca, cb, pa - bab - \omega baaa, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1423)$$

$$\langle a, b, c \mid ca, cb, pa - bab, pb - baaa, pc, \text{class } 4 \rangle \quad (7.1424)$$

$$\langle a, b, c \mid ca, cb, pa - bab, pb - \omega baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1425)$$

$$\langle a, b, c \mid ca, cb, pa - bab, pb - \omega^2 baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1426)$$

$$\langle a, b, c \mid ca, cb, pa - bab, pb, pc - baaa, \text{class } 4 \rangle \quad (7.1427)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - bab, pb, pc - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1428)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - bab - baaa, pb, pc, \text{class } 4 \rangle \quad (7.1429)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - bab - \omega baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.1430)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - bab, pb - baaa, pc, \text{class } 4 \rangle \quad (7.1431)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - bab, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1432)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - bab, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1433)$$

8.88 Descendants of 6.120

$(p-1)/2 + 3 + 2 \gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b, c \mid ca, cb, pa - \omega bab, pb, pc, \text{class } 4 \rangle \quad (7.1434)$$

$$\langle a, b, c \mid ca, cb, pa - \omega bab - baaa, pb, pc, \text{class } 4 \rangle \quad (7.1435)$$

$$\langle a, b, c \mid ca, cb, pa - \omega bab - \omega baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.1436)$$

$$\langle a, b, c \mid ca, cb, pa - \omega bab, pb - baaa, pc, \text{class } 4 \rangle \quad (7.1437)$$

$$\langle a, b, c \mid ca, cb, pa - \omega bab, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1438)$$

$$\langle a, b, c \mid ca, cb, pa - \omega bab, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1439)$$

$$\langle a, b, c \mid ca, cb, pa - \omega bab, pb, pc - baaa, \text{class } 4 \rangle \quad (7.1440)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - \omega bab, pb, pc - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1441)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - \omega bab - baaa, pb, pc, \text{class } 4 \rangle \quad (7.1442)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - \omega bab - \omega baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.1443)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - \omega bab, pb - baaa, pc, \text{class } 4 \rangle \quad (7.1444)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - \omega bab, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1445)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - \omega bab, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1446)$$

8.89 Descendants of 6.121

$2p + 4 + 2 \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid ca, cb, pa - baa, pb, pc, \text{ class } 4 \rangle \quad (7.1447)$$

$$\langle a, b, c \mid ca, cb, pa - baa - babb, pb, pc, \text{ class } 4 \rangle \quad (7.1448)$$

$$\langle a, b, c \mid ca, cb, pa - baa - \omega babb, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1449)$$

$$\langle a, b, c \mid ca, cb, pa - baa - \omega^2 babb, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1450)$$

$$\langle a, b, c \mid ca, cb, pa - baa - xbabb, pb - babb, pc, \text{ class } 4 \rangle \quad (7.1451)$$

$$\langle a, b, c \mid ca, cb, pa - baa, pb, pc - babb, \text{ class } 4 \rangle \quad (7.1452)$$

$$\langle a, b, c \mid ca - babb, cb, pa - baa, pb, pc, \text{ class } 4 \rangle \quad (7.1453)$$

$$\langle a, b, c \mid ca - babb, cb, pa - baa - babb, pb, pc, \text{ class } 4 \rangle \quad (7.1454)$$

$$\langle a, b, c \mid ca - babb, cb, pa - baa - \omega babb, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1455)$$

$$\langle a, b, c \mid ca - babb, cb, pa - baa - \omega^2 babb, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1456)$$

$$\langle a, b, c \mid ca - babb, cb, pa - baa - xbabb, pb - babb, pc, \text{ class } 4 \rangle \quad (7.1457)$$

$$\langle a, b, c \mid ca - babb, cb, pa - baa, pb, pc - babb, \text{ class } 4 \rangle \quad (7.1458)$$

8.90 Descendants of 6.122

$p + 2$ algebras

$$\langle a, b, c \mid ca, cb, pa - baa, pb + bab, pc, \text{ class } 4 \rangle \quad (7.1459)$$

$$\langle a, b, c \mid ca, cb, pa - baa, pb + bab, pc - baab, \text{ class } 4 \rangle \quad (7.1460)$$

$$\langle a, b, c \mid ca, cb, pa - baa, pb + bab - baab, pc, \text{ class } 4 \rangle \quad (7.1461)$$

$$\langle a, b, c \mid ca, cb, pa - baa - baab, pb + bab - xbaab, pc, \text{ class } 4 \rangle \quad (x \neq 0) \quad (7.1462)$$

8.91 Descendants of 6.125

$p + 1$ algebras

$$\langle a, b, c \mid ca, cb, pa + bab, pb + \omega baa, pc - baaa, \text{ class } 4 \rangle \quad (7.1463)$$

$$\langle a, b, c \mid ca, cb, pa + bab - xbaaa, pb + \omega baa - ybaaa, pc, \text{ class } 4 \rangle \quad ((x, y) \sim (x', y') \text{ if } y^2 - \omega x^2 = y'^2 - \omega x'^2) \quad (7.1464)$$

8.92 Descendants of 6.127

$3p + 4 + 6 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

- $$\langle a, b, c \mid ca, cb, pa, pb, pc - bab, \text{ class } 4 \rangle \quad (7.1465)$$
- $$\langle a, b, c \mid ca, cb, pa - baaa, pb, pc - bab, \text{ class } 4 \rangle \quad (7.1466)$$
- $$\langle a, b, c \mid ca, cb, pa, pb - baaa, pc - bab, \text{ class } 4 \rangle \quad (7.1467)$$
- $$\langle a, b, c \mid ca, cb, pa, pb - \omega baaa, pc - bab, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1468)$$
- $$\langle a, b, c \mid ca, cb, pa, pb - \omega^2 baaa, pc - bab, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1469)$$
- $$\langle a, b, c \mid ca, cb, pa, pb, pc - bab - baaa, \text{ class } 4 \rangle \quad (7.1470)$$
- $$\langle a, b, c \mid ca, cb, pa - baaa, pb, pc - bab - baaa, \text{ class } 4 \rangle \quad (7.1471)$$
- $$\langle a, b, c \mid ca, cb, pa - \omega baaa, pb, pc - bab - baaa, \text{ class } 4 \rangle \quad (7.1472)$$
- $$\langle a, b, c \mid ca, cb, pa - \omega^2 baaa, pb, pc - bab - baaa, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.1473)$$
- $$\langle a, b, c \mid ca, cb, pa - \omega^3 baaa, pb, pc - bab - baaa, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.1474)$$
- $$\langle a, b, c \mid ca, cb, pa, pb - baaa, pc - bab - baaa, \text{ class } 4 \rangle \quad (7.1475)$$
- $$\langle a, b, c \mid ca, cb, pa, pb - \omega baaa, pc - bab - baaa, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1476)$$
- $$\langle a, b, c \mid ca, cb, pa, pb - \omega^2 baaa, pc - bab - baaa, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1477)$$
- $$\langle a, b, c \mid ca, cb - baaa, pa, pb, pc - bab, \text{ class } 4 \rangle \quad (7.1478)$$
- $$\langle a, b, c \mid ca, cb - baaa, pa - baaa, pb, pc - bab, \text{ class } 4 \rangle \quad (7.1479)$$
- $$\langle a, b, c \mid ca, cb - baaa, pa - \omega baaa, pb, pc - bab, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1480)$$
- $$\langle a, b, c \mid ca, cb - baaa, pa - \omega^2 baaa, pb, pc - bab, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1481)$$
- $$\langle a, b, c \mid ca, cb - baaa, pa, pb - baaa, pc - bab, \text{ class } 4 \rangle \quad (7.1482)$$
- $$\langle a, b, c \mid ca, cb - baaa, pa, pb - \omega baaa, pc - bab, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1483)$$
- $$\langle a, b, c \mid ca, cb - baaa, pa, pb - \omega^2 baaa, pc - bab, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1484)$$
- $$\langle a, b, c \mid ca, cb - baaa, pa, pb - xbaaa, pc - bab - baaa, \text{ class } 4 \rangle \quad (x \sim -x) \quad (7.1485)$$

$$\langle a, b, c \mid ca, cb - baaa, pa - xbaaa, pb, pc - bab - baaa, \text{class } 4 \rangle (x \neq 0) \quad (7.1486)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa, pb, pc - bab, \text{class } 4 \rangle \quad (7.1487)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa - baaa, pb, pc - bab, \text{class } 4 \rangle \quad (7.1488)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa - \omega baaa, pb, pc - bab, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1489)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa - \omega^2 baaa, pb, pc - bab, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1490)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa, pb - baaa, pc - bab, \text{class } 4 \rangle \quad (7.1491)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa, pb - \omega baaa, pc - bab, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1492)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa, pb - \omega^2 baaa, pc - bab, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1493)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa, pb - xbaaa, pc - bab - baaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1494)$$

$$\langle a, b, c \mid ca, cb - \omega baaa, pa - xbaaa, pb, pc - bab - baaa, \text{class } 4 \rangle (x \neq 0) \quad (7.1495)$$

8.93 Descendants of 6.131

$15 + (p + 10) \gcd(p - 1, 3) + \gcd(p - 1, 4) + \gcd(p - 1, 7)$ algebras

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa, pb, pc, \text{class } 4 \rangle \quad (7.1496)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa, pb, pc - babb, \text{class } 4 \rangle \quad (7.1497)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa, pb - babb, pc, \text{class } 4 \rangle \quad (7.1498)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa, pb - babb, pc - babb, \text{class } 4 \rangle \quad (7.1499)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa - babb, pb, pc, \text{class } 4 \rangle \quad (7.1500)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa - \omega babb, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1501)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa - \omega^2 babb, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1502)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa - babb, pb - babb, pc, \text{class } 4 \rangle \quad (7.1503)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa - \omega babb, pb - babb, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1504)$$

$$\langle a, b, c \mid baaa, baab, ca - bab, cb, pa - \omega^2 babb, pb - babb, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1505)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb, pa, pb, pc, \text{ class } 4 \rangle \quad (7.1506)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb, pa - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1507)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1508)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1509)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb, pa, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1510)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1511)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa, pb, pc, \text{ class } 4 \rangle \quad (7.1512)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1513)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa - xbaaa, pb, pc, \text{ class } 4 \rangle \quad (x = \omega^i, i = 1, 2, 3, 4, 5, 6, p = 1 \bmod 7) \quad (7.1514)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1515)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (7.1516)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x = \omega^i, i = 2, 3, 4, 5, p = 1 \bmod 3) \quad (7.1517)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1518)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa, pb, pc - \omega baaa, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1519)$$

$$\langle a, b, c \mid baab, babb, ca - bab, cb - baaa, pa, pb, pc - \omega^2 baaa, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1520)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa, pb, pc, \text{ class } 4 \rangle \quad (7.1521)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1522)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1523)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1524)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa - baaa, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \sim -x) \quad (7.1525)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa - \omega baaa, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \sim -x, p = 1 \bmod 3) \quad (7.1526)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa - \omega^2baaa, pb - xbaaa, pc, \text{ class } 4 \rangle (x \sim -x, p = 1 \bmod 3) \quad (7.1527)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1528)$$

$$\langle a, b, c \mid baab, babb - baaa, ca - bab, cb, pa, pb, pc - \omega baaa, \text{ class } 4 \rangle (p = 1 \bmod 4) \quad (7.1529)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa, pb, pc, \text{ class } 4 \rangle \quad (7.1530)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1531)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1532)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa, pb - \omega^2baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1533)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa - baaa, pb - xbaaa, pc, \text{ class } 4 \rangle (x \sim -x) \quad (7.1534)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa - \omega baaa, pb - xbaaa, pc, \text{ class } 4 \rangle (x \sim -x, p = 1 \bmod 3) \quad (7.1535)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa - \omega^2baaa, pb - xbaaa, pc, \text{ class } 4 \rangle (x \sim -x, p = 1 \bmod 3) \quad (7.1536)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1537)$$

$$\langle a, b, c \mid baab, babb - \omega baaa, ca - bab, cb, pa, pb, pc - \omega baaa, \text{ class } 4 \rangle (p = 1 \bmod 4) \quad (7.1538)$$

$$\langle a, b, c \mid baaa, babb, ca - bab, cb, pa, pb, pc, \text{ class } 4 \rangle \quad (7.1539)$$

$$\langle a, b, c \mid baaa, babb, ca - bab, cb, pa, pb - baab, pc, \text{ class } 4 \rangle \quad (7.1540)$$

$$\langle a, b, c \mid baaa, babb, ca - bab, cb, pa - baab, pb, pc, \text{ class } 4 \rangle \quad (7.1541)$$

$$\langle a, b, c \mid baaa, babb, ca - bab, cb, pa - baab, pb - baab, pc, \text{ class } 4 \rangle \quad (7.1542)$$

$$\langle a, b, c \mid baaa, babb, ca - bab, cb, pa - baab, pb - \omega baab, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1543)$$

$$\langle a, b, c \mid baaa, babb, ca - bab, cb, pa - baab, pb - \omega^2baab, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1544)$$

$$\langle a, b, c \mid baaa, babb, ca - bab, cb, pa, pb, pc - baab, \text{ class } 4 \rangle \quad (7.1545)$$

$$\langle a, b, c \mid baaa, babb, ca - bab, cb, pa, pb, pc - \omega baab, \text{ class } 4 \rangle \quad (7.1546)$$

8.94 Descendants of 6.132

$(p + 1 + 3(p + 1) \gcd(p - 1, 3) + \gcd(p - 1, 4))/2$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb, pc, \text{ class } 4 \rangle \quad (7.1547)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1548)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab - \omega baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1549)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1550)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1551)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1552)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1553)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb, pc - \omega baaa, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1554)$$

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb, pc - \omega^2 baaa, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1555)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - bab, pb, pc - xbaaa, \text{ class } 4 \rangle \quad (x \sim -x) \quad (7.1556)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa - bab, pb, pc - xbaaa, \text{ class } 4 \rangle \quad (x \sim -x, p = 1 \pmod{3}) \quad (7.1557)$$

$$\langle a, b, c \mid ca - bab, cb - \omega^2 baaa, pa - bab, pb, pc - xbaaa, \text{ class } 4 \rangle \quad (x \sim -x, p = 1 \pmod{3}) \quad (7.1558)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - bab, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0) \quad (7.1559)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa - bab, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0, p = 1 \pmod{3}) \quad (7.1560)$$

$$\langle a, b, c \mid ca - bab, cb - \omega^2 baaa, pa - bab, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0, p = 1 \pmod{3}) \quad (7.1561)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - bab - xbaaa, pb, pc, \text{ class } 4 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^6 = 1) \quad (7.1562)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa - bab - xbaaa, pb, pc, \text{ class } 4 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.1563)$$

$$\langle a, b, c \mid ca - bab, cb - \omega^2 baaa, pa - bab - xbaaa, pb, pc, \text{ class } 4 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.1564)$$

8.95 Descendants of 6.133

$(p + 1 + 3(p + 1) \gcd(p - 1, 3) + \gcd(p - 1, 4))/2$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb, pc, \text{ class } 4 \rangle \quad (7.1565)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1566)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab - \omega baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1567)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1568)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1569)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1570)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1571)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb, pc - \omega baaa, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1572)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb, pc - \omega^2 baaa, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1573)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - \omega bab, pb, pc - xbaaa, \text{ class } 4 \rangle \quad (x \sim -x) \quad (7.1574)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa - \omega bab, pb, pc - xbaaa, \text{ class } 4 \rangle \quad (x \sim -x, p = 1 \pmod{3}) \quad (7.1575)$$

$$\langle a, b, c \mid ca - bab, cb - \omega^2 baaa, pa - \omega bab, pb, pc - xbaaa, \text{ class } 4 \rangle \quad (x \sim -x, p = 1 \pmod{3}) \quad (7.1576)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - \omega bab, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0) \quad (7.1577)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa - \omega bab, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0, p = 1 \pmod{3}) \quad (7.1578)$$

$$\langle a, b, c \mid ca - bab, cb - \omega^2 baaa, pa - \omega bab, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0, p = 1 \pmod{3}) \quad (7.1579)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - \omega bab - xbaaa, pb, pc, \text{ class } 4 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^6 = 1) \quad (7.1580)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa - \omega bab - xbaaa, pb, pc, \text{ class } 4 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.1581)$$

$$\langle a, b, c \mid ca - bab, cb - \omega^2 baaa, pa - \omega bab - xbaaa, pb, pc, \text{ class } 4 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.1582)$$

8.96 Descendants of 6.134

$3p - 1 + \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa - baa, pb, pc - xbabb, \text{ class } 4 \rangle \quad (7.1583)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa, pb - babb, pc - xbabb, \text{ class } 4 \rangle \quad (7.1584)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa - babb, pb, pc - babb, \text{ class } 4 \rangle \quad (7.1585)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa - \omega babb, pb, pc - babb, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1586)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa - \omega^2 babb, pb, pc - babb, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1587)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baa - xbabb, pb - babb, pc - babb, \text{ class } 4 \rangle \quad (x \neq 0) \quad (7.1588)$$

8.97 Descendants of 6.135

$p^2 + 2p + 3 + \gcd(p - 1, 3) + \gcd(p - 1, 4) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa, pb - bab, pc, \text{ class } 4 \rangle \quad (7.1589)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - bab - baaa, pc, \text{ class } 4 \rangle \quad (7.1590)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - bab - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1591)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - bab - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1592)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baaa, pb - bab - xbaaa, pc, \text{ class } 4 \rangle \quad (7.1593)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa, pb - bab, pc, \text{ class } 4 \rangle \quad (7.1594)$$

$$\langle a, b, c \mid ca - bab, cb - xbaaa, pa, pb - bab, pc, \text{ class } 4 \rangle \quad (x = \omega^i, i = 1, 2, 3, 4, p = 1 \pmod{5}) \quad (7.1595)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa, pb - bab - xbaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.1596)$$

$$\langle a, b, c \mid ca - bab, cb - ybaaa, pa, pb - bab - xbaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0, y = \omega^i, i = 1, 2, 3, 4, (x, y) \sim (ax, y) \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.1597)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - xbaaa, pb - bab - ybaaa, pc, \text{ class } 4 \rangle \quad (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^5 = 1) \quad (7.1598)$$

In the presentation below we have $x \neq 0, z = \omega, \omega^2, \omega^3, \omega^4, (x, y, z) \sim (ax, a^3y, z)$ if $a^5 = 1, p = 1 \pmod{5}$:

$$\langle a, b, c \mid ca - bab, cb - zbaaa, pa - xbaaa, pb - bab - ybaaa, pc, \text{ class } 4 \rangle \quad (7.1599)$$

$$\langle a, b, c \mid ca - bab, cb - xbaaa, pa, pb - bab, pc - baaa, \text{ class } 4 \rangle \quad (x \sim ax \text{ if } a^4 = 1) \quad (7.1600)$$

$$\langle a, b, c \mid ca - bab, cb - xbaaa, pa, pb - bab, pc - \omega baaa, \text{class } 4 \rangle (x \sim ax \text{ if } a^4 = 1) \quad (7.1601)$$

$$\langle a, b, c \mid ca - bab, cb - xbaaa, pa, pb - bab, pc - \omega^2 baaa, \text{class } 4 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1602)$$

$$\langle a, b, c \mid ca - bab, cb - xbaaa, pa, pb - bab, pc - \omega^3 baaa, \text{class } 4 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1603)$$

$$\langle a, b, c \mid ca - bab, cb, pa + baa, pb - bab, pc, \text{class } 4 \rangle \quad (7.1604)$$

$$\langle a, b, c \mid ca - bab, cb, pa + baa, pb - bab - baab, pc, \text{class } 4 \rangle \quad (7.1605)$$

$$\langle a, b, c \mid ca - bab, cb, pa + baa, pb - bab, pc - baab, \text{class } 4 \rangle \quad (7.1606)$$

$$\langle a, b, c \mid ca - bab, cb, pa + baa, pb - bab, pc - \omega baab, \text{class } 4 \rangle \quad (7.1607)$$

8.98 Descendants of 6.138

$p(p+1)/2$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa - baa - ybabb, pb - baa, pc - xbabb, \text{class } 4 \rangle ((x, y) \sim (x, -y)) \quad (7.1608)$$

8.99 Descendants of 6.139

$(p+3)/2 + \gcd(p-1, 3)$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa, pb - baa, pc, \text{class } 4 \rangle \quad (7.1609)$$

$$\langle a, b, c \mid ca - bab, cb, pa - babb, pb - baa, pc, \text{class } 4 \rangle \quad (7.1610)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega babb, pb - baa, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1611)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^2 babb, pb - baa, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1612)$$

$$\langle a, b, c \mid ca - bab, cb, pa - xbabb, pb - baa, pc - babb, \text{class } 4 \rangle (x \sim -x) \quad (7.1613)$$

8.100 Descendants of 6.140

$(3p+1)/2$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - baa, pc - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1614)$$

In the following presentation we pick a unique y such that $y^2 = 2$.

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - baa - xbaaa, pc - ybaaa, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (-x, y), p = \pm 1 \pmod{8}) \quad (7.1615)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - baa, pc - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1616)$$

In the following presentation we pick a unique y such that $\omega y^2 = 2$.

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - baa - xbaaa, pc - ybaaa, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (-x, y), p = \pm 3 \pmod{8}) \quad (7.1617)$$

8.101 Descendants of 6.142

$p(p + 1)/2$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa - baa - ybabb, pb - \omega baa, pc - xbabb, \text{class } 4 \rangle ((x, y) \sim (x, -y)) \quad (7.1618)$$

8.102 Descendants of 6.143

$(p + 3)/2 + \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega baa, pc, \text{class } 4 \rangle \quad (7.1619)$$

$$\langle a, b, c \mid ca - bab, cb, pa - babb, pb - \omega baa, pc, \text{class } 4 \rangle \quad (7.1620)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega babb, pb - \omega baa, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1621)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^2 babb, pb - \omega baa, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1622)$$

$$\langle a, b, c \mid ca - bab, cb, pa - xbabb, pb - \omega baa, pc - babb, \text{class } 4 \rangle (x \sim -x) \quad (7.1623)$$

8.103 Descendants of 6.144

$(3p + 1)/2$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - \omega baa, pc - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1624)$$

In the following presentation we pick a unique y such that $y^2 = 2\omega^3$:

$$\langle a, b, c \mid ca - bab, cb, pa - bab, pb - \omega baa - xbaaa, pc - ybaaa, \text{class } 4 \rangle (x, y) \sim (-x, y), p = \pm 3 \pmod{8} \quad (7.1625)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - \omega baa, pc - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1626)$$

In the following presentation we pick a unique y such that $y^2 = 2\omega^2$:

$$\langle a, b, c \mid ca - bab, cb, pa - \omega bab, pb - \omega baa - xbaaa, pc - ybaaa, \text{class } 4 \rangle (x, y) \sim (-x, y), p = \pm 1 \pmod{8} \quad (7.1627)$$

8.104 Descendants of 6.146

$2p^2 + 3p$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa, pb - xbaaa, pc - bab, \text{class } 4 \rangle \quad (7.1628)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - xbaaa, pc - bab - baaa, \text{class } 4 \rangle \quad (7.1629)$$

$$\langle a, b, c \mid ca - bab, cb, pa - baaa, pb, pc - bab, \text{class } 4 \rangle \quad (7.1630)$$

$$\langle a, b, c \mid ca - bab, cb, pa - xbaaa, pb, pc - bab - baaa, \text{class } 4 \rangle (x \neq 0) \quad (7.1631)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa, pb - xbaaa, pc - bab - ybaaa, \text{class } 4 \rangle ((x, y) \sim (x, -y)) \quad (7.1632)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - xbaaa, pb, pc - bab, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.1633)$$

$$\langle a, b, c \mid ca - bab, cb - baaa, pa - xbaaa, pb, pc - bab - ybaaa, \text{class } 4 \rangle (x, y \neq 0, (x, y) \sim (-x, -y)) \quad (7.1634)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa, pb - xbaaa, pc - bab - ybaaa, \text{class } 4 \rangle ((x, y) \sim (x, -y)) \quad (7.1635)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa - xbaaa, pb, pc - bab, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.1636)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baaa, pa - xbaaa, pb, pc - bab - ybaaa, \text{class } 4 \rangle (x, y \neq 0, (x, y) \sim (-x, -y)) \quad (7.1637)$$

8.105 Descendants of 6.148

$p^3 + p^2 + p + (p + 2) \gcd(p - 1, 3) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid ca - bab, cb, pa, pb, pc - baa, \text{class } 4 \rangle \quad (7.1638)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb, pc - baa - babb, \text{class } 4 \rangle \quad (7.1639)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb, pc - baa - \omega babb, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1640)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb, pc - baa - \omega^2 babb, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1641)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - babb, pc - baa, \text{class } 4 \rangle \quad (7.1642)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega babb, pc - baa, \text{class } 4 \rangle (p = 1 \pmod{5}) \quad (7.1643)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega^2 babb, pc - baa, \text{class } 4 \rangle (p = 1 \pmod{5}) \quad (7.1644)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega^3 babb, pc - baa, \text{class } 4 \rangle (p = 1 \pmod{5}) \quad (7.1645)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega^4 babb, pc - baa, \text{class } 4 \rangle (p = 1 \pmod{5}) \quad (7.1646)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - babb, pc - baa - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.1647)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega babb, pc - baa - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.1648)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega^2 babb, pc - baa - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.1649)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega^3 babb, pc - baa - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.1650)$$

$$\langle a, b, c \mid ca - bab, cb, pa, pb - \omega^4 babb, pc - baa - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1, p = 1 \pmod{5}) \quad (7.1651)$$

$$\langle a, b, c \mid ca - bab, cb, pa - babb, pb, pc - baa - xbabb, \text{class } 4 \rangle (x \sim -x) \quad (7.1652)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega babb, pb, pc - baa - xbabb, \text{class } 4 \rangle (x \sim -x) \quad (7.1653)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^2 babb, pb, pc - baa - xbabb, \text{class } 4 \rangle (x \sim -x, p = 1 \pmod{3}) \quad (7.1654)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^3 babb, pb, pc - baa - xbabb, \text{class } 4 \rangle (x \sim -x, p = 1 \pmod{3}) \quad (7.1655)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^4 babb, pb, pc - baa - xbabb, \text{class } 4 \rangle (x \sim -x, p = 1 \pmod{3}) \quad (7.1656)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^5 babb, pb, pc - baa - xbabb, \text{class } 4 \rangle (x \sim -x, p = 1 \pmod{3}) \quad (7.1657)$$

$$\langle a, b, c \mid ca - bab, cb, pa - babb, pb - xbabb, pc - baa - ybabb, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^6 = 1) \quad (7.1658)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega babb, pb - xbabb, pc - baa - ybabb, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^6 = 1) \quad (7.1659)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^2 babb, pb - xbabb, pc - baa - ybabb, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.1660)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^3 babb, pb - xbabb, pc - baa - ybabb, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.1661)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^4 babb, pb - xbabb, pc - baa - ybabb, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.1662)$$

$$\langle a, b, c \mid ca - bab, cb, pa - \omega^5 babb, pb - xbabb, pc - baa - ybabb, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.1663)$$

$$\langle a, b, c \mid ca - bab - babb, cb, pa - xbabb, pb, pc - baa - ybabb, \text{class } 4 \rangle ((x, y) \sim (x, -y)) \quad (7.1664)$$

$$\langle a, b, c \mid ca - bab - babb, cb, pa - xbabb, pb - ybabb, pc - baa - zbabb, \text{class } 4 \rangle (y \neq 0, (x, y, z) \sim (x, -y, -z)) \quad (7.1665)$$

$$\langle a, b, c \mid ca - bab - \omega babb, cb, pa - xbabb, pb, pc - baa - ybabb, \text{class } 4 \rangle ((x, y) \sim (x, -y)) \quad (7.1666)$$

$$\langle a, b, c \mid ca - bab - \omega babb, cb, pa - xbabb, pb - ybabb, pc - baa - zbabb, \text{class } 4 \rangle (y \neq 0, (x, y, z) \sim (x, -y, -z)) \quad (7.1667)$$

8.106 Descendants of 6.150

Algebra 6.150 has $4p + 14 + (p^2/2 + 2p + 13/2) \gcd(p-1, 3) + \gcd(p-1, 4)$ descendants of order p^7 . It has presentation

$$\langle a, b, c \mid ca - baa, cb, pa, pb, pc, \text{class } 3 \rangle$$

and so if L is a descendant of 6.150 of order p^7 then the commutator structure of L is determined by the classification of nilpotent Lie algebras over \mathbb{Z}_p . So we can assume that one of the following sets of commutator relations holds.

$$\begin{aligned} baaa &= baab = ca - baa = cb = 0, \\ baaa &= baab = ca - baa - babb = cb = 0, \\ baaa &= baab = babb, ca - baa = cb = 0, \\ babb &= -baaa, baab = ca - baa = cb = 0, \\ babb &= -\omega baaa, baab = ca - baa = cb = 0, \\ baaa &= babb = ca - baa = cb = 0, \\ babb &= baab, baaa = ca - baa = cb = 0, \\ baaa &= baab, babb = xbaaa, ca - baa = cb = 0 \ (x \neq 0, 1). \end{aligned}$$

For each of these cases we obtain a generator d for L_4 (d equals one of $baaa$, $baab$, $babb$) and we write

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = Ad$$

where A is a 3×1 matrix over \mathbb{Z}_p . In each case the isomorphism classes of algebras are given by the orbits of the matrices A under a given action by a group of automorphisms. I was able to “solve” the problem in every case, providing presentations with fewer parameters, and explicit relatively simple equivalence relations on the parameter sets. However in four of the cases the equivalence classes are slightly more complex than usual. For example in one case the equivalence classes for a parameter y were $\{\pm y, \pm \frac{\omega}{y}\}$. These four cases are 3,4,5,8, and these are described below.

8.106.1 Case 3

If $baaa = baab = babb$, $ca - baa = cb = 0$ then L_4 is generated by $baaa$ and the action on A is

$$A \rightarrow \alpha^{-4} \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \alpha & -\gamma \\ 0 & 0 & \alpha^2 \end{pmatrix} A$$

and

$$A \rightarrow -\alpha^{-4} \begin{pmatrix} 0 & \alpha & \gamma \\ \alpha & 0 & -\gamma \\ 0 & 0 & \alpha^2 \end{pmatrix} A.$$

8.106.2 Case 4

If $babb + baaa = baab = ca - baa = cb = 0$ then L_4 is generated by $baaa$ and the action on A is

$$A \rightarrow \alpha^{-4} \begin{pmatrix} \pm\alpha & 0 & \gamma \\ 0 & \alpha & \varepsilon \\ 0 & 0 & \alpha^2 \end{pmatrix} A$$

and

$$A \rightarrow \alpha^{-4} \begin{pmatrix} 0 & \pm\alpha & \gamma \\ \alpha & 0 & \varepsilon \\ 0 & 0 & \alpha^2 \end{pmatrix} A.$$

8.106.3 Case 5

If $babb + \omega baaa = baab = ca - baa = cb = 0$ then L_4 is generated by $baaa$ and the action on A is

$$A \rightarrow \alpha^{-4} \begin{pmatrix} \pm\alpha & 0 & \gamma \\ 0 & \alpha & \varepsilon \\ 0 & 0 & \alpha^2 \end{pmatrix} A$$

and

$$A \rightarrow \omega^{-2}\alpha^{-4} \begin{pmatrix} 0 & \pm\alpha & \gamma \\ \omega\alpha & 0 & \varepsilon \\ 0 & 0 & \omega\alpha^2 \end{pmatrix} A.$$

8.106.4 Case 8

If $baaa = baab, babb = xbaaa, ca - baa = cb = 0$ where $x \neq 0, 1$ then L_4 is generated by $baaa$. (If we set $x = 0$ we have Case 7, and if we set $x = 1$ we have Case 3.) The action on A is

$$A \rightarrow \alpha^{-4} \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \alpha & \varepsilon \\ 0 & 0 & \alpha^2 \end{pmatrix} A$$

and

$$A \rightarrow x^{-2}\alpha^{-4} \begin{pmatrix} 0 & \alpha & \gamma \\ x\alpha & 0 & \varepsilon \\ 0 & 0 & -x\alpha^2 \end{pmatrix} A.$$

8.106.5 The presentations

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa, pb, pc, \text{class } 4 \rangle \quad (7.1668)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa - babb, pb, pc, \text{class } 4 \rangle \quad (7.1669)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa - \omega babb, pb, pc, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.1670)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa - \omega^2 babb, pb, pc, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.1671)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa, pb - babb, pc, \text{class } 4 \rangle \quad (7.1672)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa - babb, pb - babb, pc, \text{class } 4 \rangle \quad (7.1673)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa - \omega babb, pb - babb, pc, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.1674)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa - \omega^2 babb, pb - babb, pc, \text{class } 4 \rangle \ (p = 1 \pmod{3}) \quad (7.1675)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa, pb, pc - babb, \text{class } 4 \rangle \quad (7.1676)$$

$$\langle a, b, c \mid baaa, baab, ca - baa, cb, pa, pb, pc - \omega babb, \text{class } 4 \rangle \quad (7.1677)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa, pb, pc, \text{class } 4 \rangle \quad (7.1678)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa, pb - babb, pc, \text{class } 4 \rangle \quad (7.1679)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa, pb - \omega babb, pc, \text{class } 4 \rangle \quad (7.1680)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa, pb - \omega^2 babb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.1681)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa, pb - \omega^3 babb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.1682)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa - babb, pb, pc, \text{class } 4 \rangle \quad (7.1683)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa - \omega babb, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1684)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa - \omega^2 babb, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1685)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa - babb, pb - xbabb, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.1686)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa - \omega babb, pb - xbabb, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.1687)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa - \omega^2 babb, pb - xbabb, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.1688)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa, pb, pc - babb, \text{class } 4 \rangle \quad (7.1689)$$

$$\langle a, b, c \mid baaa, baab, ca - baa - babb, cb, pa, pb, pc - \omega babb, \text{class } 4 \rangle \quad (7.1690)$$

$$\langle a, b, c \mid baab - baaa, babb - baaa, ca - baa, cb, pa - xbaaa, pb - ybaaa, pc - zbaaa, \text{class } 4 \rangle (\text{Case 3 above}) \quad (7.1691)$$

$$\langle a, b, c \mid babb + baaa, baab, ca - baa, cb, pa - xbaaa, pb - ybaaa, pc - zbaaa, \text{class } 4 \rangle (\text{Case 4 above}) \quad (7.1692)$$

$$\langle a, b, c \mid babb + \omega baaa, baab, ca - baa, cb, pa - xbaaa, pb - ybaaa, pc - zbaaa, \text{class } 4 \rangle (\text{Case 5 above}) \quad (7.1693)$$

$$\langle a, b, c \mid baaa, babb, ca - baa, cb, pa, pb, pc, \text{class } 4 \rangle \quad (7.1694)$$

$$\langle a, b, c \mid baaa, babb, ca - baa, cb, pa, pb - baab, pc, \text{class } 4 \rangle \quad (7.1695)$$

$$\langle a, b, c \mid baaa, babb, ca - baa, cb, pa - baab, pb - baab, pc, \text{class } 4 \rangle \quad (7.1696)$$

$$\langle a, b, c \mid baaa, babb, ca - baa, cb, pa - baab, pb - \omega baab, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1697)$$

$$\langle a, b, c \mid baaa, babb, ca - baa, cb, pa - baab, pb - \omega^2 baab, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1698)$$

$$\langle a, b, c \mid baaa, babb, ca - baa, cb, pa, pb, pc - baab, \text{ class } 4 \rangle \quad (7.1699)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa, pb, pc, \text{ class } 4 \rangle \quad (7.1700)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa, pb - baab, pc, \text{ class } 4 \rangle \quad (7.1701)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa, pb - \omega baab, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1702)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa, pb - \omega^2 baab, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1703)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa - baab, pb - xbaab, pc, \text{ class } 4 \rangle \quad (7.1704)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa - \omega baab, pb - xbaab, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1705)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa - \omega^2 baab, pb - xbaab, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1706)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa, pb, pc - baab, \text{ class } 4 \rangle \quad (7.1707)$$

$$\langle a, b, c \mid babb - baab, baaa, ca - baa, cb, pa, pb, pc - \omega baab, \text{ class } 4 \rangle \quad (7.1708)$$

$$\langle a, b, c \mid baab - baaa, babb - xbaaa, ca - baa, cb, pa - ybaaa, pb - zbaaa, pc - tbaaa, \text{ class } 4 \rangle (\text{Case 8 above}) \quad (7.1709)$$

8.107 Descendants of 6.151

$p^2 + p + 2 + (p + 1) \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - baa, pb, pc, \text{ class } 4 \rangle \quad (7.1710)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - babb, pb, pc, \text{ class } 4 \rangle \quad (7.1711)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega babb, pb, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1712)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega^2 babb, pb, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1713)$$

$$\langle a, b, c \mid ca - baa - babb, cb, pa - baa - xbabb, pb, pc, \text{ class } 4 \rangle \quad (7.1714)$$

$$\langle a, b, c \mid ca - baa - \omega babb, cb, pa - baa - xbabb, pb, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1715)$$

$$\langle a, b, c \mid ca - baa - \omega^2 babb, cb, pa - baa - xbabb, pb, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1716)$$

$$\langle a, b, c \mid ca - baa - xbabb, cb, pa - baa - ybabb, pb - babb, pc, \text{ class } 4 \rangle \quad (7.1717)$$

$$\langle a, b, c \mid ca - baa - xbabb, cb, pa - baa, pb, pc - babb, \text{ class } 4 \rangle (x \sim -x) \quad (7.1718)$$

$$\langle a, b, c \mid ca - baa - xbabb, cb, pa - baa, pb, pc - \omega babb, \text{ class } 4 \rangle (x \sim -x) \quad (7.1719)$$

8.108 Descendants of 6.152

$(p^2 + 2p + 3 + 2 \gcd(p - 1, 3) + (p + 1) \gcd(p - 1, 4))/2$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb, pc, \text{ class } 4 \rangle \quad (7.1720)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1721)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1722)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1723)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab - baaa, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \sim ax \text{ if } a^4 = 1) \quad (7.1724)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab - \omega baaa, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1725)$$

$$\langle a, b, c \mid ca - baa, cb - baaa, pa - bab - xbaaa, pb - ybaaa, pc, \text{ class } 4 \rangle \quad ((x, y) \sim (x, ay) \text{ if } a^4 = 1) \quad (7.1726)$$

$$\langle a, b, c \mid ca - baa, cb - \omega baaa, pa - bab - xbaaa, pb - ybaaa, pc, \text{ class } 4 \rangle \quad ((x, y) \sim (x, ay) \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1727)$$

$$\langle a, b, c \mid ca - baa, cb - xbaaa, pa - bab, pb, pc - baaa, \text{ class } 4 \rangle \quad (x \sim -x) \quad (7.1728)$$

$$\langle a, b, c \mid ca - baa, cb - xbaaa, pa - bab, pb, pc - \omega baaa, \text{ class } 4 \rangle \quad (x \sim -x) \quad (7.1729)$$

8.109 Descendants of 6.153

$p(p + 1)/2$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab - ybaaa, pb, pc - xbaaa, \text{ class } 4 \rangle \quad (x \neq 0, (x, y) \sim (x, -y)) \quad (7.1730)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \sim -x) \quad (7.1731)$$

8.110 Descendants of 6.154

$(p^2 + 2p + 3 + 2 \gcd(p - 1, 3) + (p + 1) \gcd(p - 1, 4))/2$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb, pc, \text{ class } 4 \rangle \quad (7.1732)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1733)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1734)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1735)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab - baaa, pb - xbaaa, pc, \text{ class } 4 \rangle \quad (x \sim ax \text{ if } a^4 = 1) \quad (7.1736)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab - \omega baaa, pb - xbaaa, pc, \text{class } 4 \rangle (x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1737)$$

$$\langle a, b, c \mid ca - baa, cb - baaa, pa - \omega bab - xbaaa, pb - ybaaa, pc, \text{class } 4 \rangle ((x, y) \sim (x, ay) \text{ if } a^4 = 1) \quad (7.1738)$$

$$\langle a, b, c \mid ca - baa, cb - \omega baaa, pa - \omega bab - xbaaa, pb - ybaaa, pc, \text{class } 4 \rangle ((x, y) \sim (x, ay) \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1739)$$

$$\langle a, b, c \mid ca - baa, cb - xbaaa, pa - \omega bab, pb, pc - baaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1740)$$

$$\langle a, b, c \mid ca - baa, cb - xbaaa, pa - \omega bab, pb, pc - \omega baaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1741)$$

8.111 Descendants of 6.155

$p(p+1)/2$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab - ybaaa, pb, pc - xbaaa, \text{class } 4 \rangle (x \neq 0, (x, y) \sim (x, -y)) \quad (7.1742)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb - xbaaa, pc, \text{class } 4 \rangle (x \sim -x) \quad (7.1743)$$

8.112 Descendants of 6.156

p algebras

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb + baa, pc - xbaaa, \text{class } 4 \rangle (x \neq 0, -1, x \sim -1 - x) \quad (7.1744)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab - xbaaa, pb + baa, pc, \text{class } 4 \rangle (x \sim -x) \quad (7.1745)$$

8.113 Descendants of 6.157

$2p-1$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb + \omega baa, pc - xbaaa, \text{class } 4 \rangle \quad (7.1746)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab - xbaaa, pb + \omega baa, pc, \text{class } 4 \rangle (x \sim -x) \quad (7.1747)$$

$$\langle a, b, c \mid ca - baa, cb, pa - bab, pb + \omega baa - xbaaa, pc + \omega baaa, \text{class } 4 \rangle (x \sim -x) \quad (7.1748)$$

8.114 Descendants of 6.158

p algebras

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab, pb + \omega baa, pc - xbaaa, \text{class } 4 \rangle (x \neq 0, -\omega, x \sim -\omega - x) \quad (7.1749)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega bab - xbaaa, pb + \omega baa, pc, \text{class } 4 \rangle (x \sim -x) \quad (7.1750)$$

8.115 Descendants of 6.159

$(p^3 + p^2)/2$ algebras

$$\langle a, b, c \mid ca - baa - xbabb, cb, pa - baa, pb - baa, pc - ybabb, \text{class } 4 \rangle (y \neq 0, (x, y) \sim (-x, y)) \quad (7.1751)$$

$$\langle a, b, c \mid ca - baa - xbabb, cb, pa - baa - ybabb, pb - baa - zbabb, pc, \text{class } 4 \rangle ((x, y, z) \sim (-x, -y, -z)) \quad (7.1752)$$

8.116 Descendants of 6.160

$(p^3 + p^2)/2$ algebras

$$\langle a, b, c \mid ca - baa - xbabb, cb, pa - baa, pb - \omega baa, pc - ybabb, \text{class } 4 \rangle (y \neq 0, (x, y) \sim (-x, y)) \quad (7.1753)$$

$$\langle a, b, c \mid ca - baa - xbabb, cb, pa - baa - ybabb, pb - \omega baa - zbabb, pc, \text{class } 4 \rangle ((x, y, z) \sim (-x, -y, -z)) \quad (7.1754)$$

8.117 Descendants of 6.160a

$(p + 3)/2$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - baa, pb + bab, pc - xbaab, \text{class } 4 \rangle (x \sim -1 - x) \quad (7.1755)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - baab, pb + bab, pc - baab, \text{class } 4 \rangle \quad (7.1756)$$

8.118 Descendants of 6.161

$2p - 1$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb + bab, pc - xbaaa, \text{class } 4 \rangle \quad (7.1757)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb + bab - xbaaa, pc + baaa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.1758)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - bab, pb + bab - xbaaa, pc - 2baaa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.1759)$$

8.119 Descendants of 6.162

$2p - 1$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb + bab, pc - xbaaa, \text{class } 4 \rangle \quad (7.1760)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb + bab - xbaaa, pc + \omega^{-1}baaa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.1761)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \omega bab, pb + bab - xbaaa, pc - 2\omega^{-1}baaa, \text{class } 4 \rangle (x \neq 0, x \sim -x) \quad (7.1762)$$

8.120 Descendants of 6.163

Algebras 6.163 – 6.167 give a classification of algebras of order p^6 with presentations

$$\langle a, b, c \mid ca - baa, cb, pa - \lambda baa - \mu bab, pb + \nu baa + \xi bab, pc, \text{class } 3 \rangle$$

with $\lambda, \mu, \nu, \xi \neq 0$. Most of these algebras are terminal, and we need a slightly different classification of these algebras from that given in the classification of nilpotent Lie rings of order p^6 , so as to classify the capable ones. It turns out that $\frac{5}{2}p - \frac{9}{2} + \frac{1}{2} \gcd(p-1, 4)$ of these algebras are capable, and that they have a total of $\frac{1}{2}p^3 + 2p^2 - 5p + \frac{1}{2} + \frac{p}{2} \gcd(p-1, 4)$ descendants of order p^7 and p -class 4.

If we write the parameters λ, μ, ν, ξ above in a matrix $A = \begin{pmatrix} \lambda & \mu \\ \nu & \xi \end{pmatrix}$ then A and B give isomorphic algebras if

$$B = \begin{pmatrix} \frac{\lambda}{\alpha\beta} & \frac{\mu}{\beta^2} \\ \frac{\nu}{\alpha^2} & \frac{\xi}{\alpha\beta} \end{pmatrix} \text{ or } \begin{pmatrix} \frac{\xi}{\alpha\beta} & \frac{\nu}{\beta^2} \\ \frac{\mu}{\alpha^2} & \frac{\lambda}{\alpha\beta} \end{pmatrix}.$$

So we can take $\lambda = 1$ and $\mu = 1$ or ω (or any other fixed integer which is not a square mod p). Given these values of λ, μ it turns out that the algebra is terminal unless $\xi = 1$ or $\xi = \mu\nu$. We choose a set of representatives for the equivalence classes (as defined above) containing matrices of the form

$$\begin{pmatrix} 1 & 1 \\ \nu & 1 \end{pmatrix}, \begin{pmatrix} 1 & \omega \\ \nu & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ \nu & \nu \end{pmatrix}, \begin{pmatrix} 1 & \omega \\ \nu & \omega\nu \end{pmatrix}.$$

Corresponding to these choices we obtain two families of descendants of order p^7 with the following presentations involving parameters y, z, t :

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \mu bab - ybaaa, pb + \nu baa + bab - zbaaa, pc - tbaaa, \text{class } 4 \rangle (\mu = 1, \omega), \quad (7.1763)$$

$$\langle a, b, c \mid ca - baa, cb, pa - baa - \mu bab - ybaaa, pb + \nu baa + \mu\nu bab - zbaaa, pc - tbaaa, \text{class } 4 \rangle (\mu = 1, \omega). \quad (7.1764)$$

For the family 7.1763, (y, z, t) and (y', z', t') gives isomorphic algebras if

$$(y', z', t') = (\pm y + \gamma t + \gamma\mu^{-1} + \varepsilon, \pm z + \varepsilon t - \nu\gamma\mu^{-1} + \nu\varepsilon - 2\varepsilon\mu^{-1}, t)$$

for some γ, ε , and where if $\mu\nu = 1$ we need $\gamma = \mu\varepsilon$, or if

$$(y', z', t') = (-\alpha z - \gamma t + \varepsilon - \nu\gamma + 2\gamma\mu^{-1}, -\alpha^{-1}y - \varepsilon t - \gamma\mu^{-2} - \varepsilon\mu^{-1}, -t - \nu + \mu^{-1})$$

for some γ, ε , where $\alpha^2\nu = \mu$, and where if $\mu\nu = 1$ we need $\gamma = \mu\varepsilon$.

For the family 7.1764 we can assume that $\mu\nu \neq 1$. In this case (y, z, t) and (y', z', t') gives isomorphic algebras if

$$(y', z', t') = (\pm y + \mu\varepsilon t + 2\mu, \pm z + \varepsilon t - 2\mu\varepsilon, t),$$

for some ε , or (when $\mu\nu = -1$ and $p = 1 \pmod{4}$, and $\alpha^2 = -\mu^2$),

$$(y', z', t') = (-\alpha z - \mu\varepsilon t - 2\varepsilon, \alpha^{-1}y - \varepsilon t + 2\nu\varepsilon, t)$$

for some ε .

8.121 Descendants of 6.168

$p^3 + 2p^2 + 2p + 2 + \gcd(p-1, 3)$ algebras

$$\langle a, b, c \mid ca - baa, cb, pa, pb, pc - baa, \text{class } 4 \rangle \quad (7.1765)$$

$$\langle a, b, c \mid ca - baa, cb, pa - babb, pb, pc - baa, \text{class } 4 \rangle \quad (7.1766)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega babb, pb, pc - baa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1767)$$

$$\langle a, b, c \mid ca - baa, cb, pa - \omega^2 babb, pb, pc - baa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.1768)$$

$$\langle a, b, c \mid ca - baa, cb, pa - xbabb, pb, pc - baa - babb, \text{class } 4 \rangle (x \sim -x) \quad (7.1769)$$

$$\langle a, b, c \mid ca - baa, cb, pa - xbabb, pb, pc - baa - \omega babb, \text{class } 4 \rangle (x \sim -x) \quad (7.1770)$$

$$\langle a, b, c \mid ca - baa, cb, pa - ybabb, pb - babb, pc - baa - xbabb, \text{class } 4 \rangle ((x, y) \sim (x, -y)) \quad (7.1771)$$

$$\langle a, b, c \mid ca - baa, cb, pa - ybabb, pb - \omega babb, pc - baa - xbabb, \text{class } 4 \rangle ((x, y) \sim (x, -y)) \quad (7.1772)$$

$$\langle a, b, c \mid ca - baa - babb, cb, pa - zbabb, pb - xbabb, pc - baa - ybabb, \text{class } 4 \rangle (((x, y, z) \sim (x, y, -z))) \quad (7.1773)$$

$$\langle a, b, c \mid ca - baa - \omega babb, cb, pa - zbabb, pb - xbabb, pc - baa - ybabb, \text{class } 4 \rangle (((x, y, z) \sim (x, y, -z))) \quad (7.1774)$$

8.122 Descendants of 6.172

$(p^4 + p^2)/2$ algebras

$$\langle a, b, c \mid ca - baa, cb - ybaaa, pa - zbaaa, pb - tbaaa, pc - baa - bab - xbaaa, \text{class } 4 \rangle (x \neq 0, (x, y, z, t) \sim (-x, y, t, z)) \quad (7.1775)$$

$$\langle a, b, c \mid ca - baa, cb - ybaaa, pa - zbaaa, pb - xbaaa, pc - baa - bab, \text{class } 4 \rangle ((x, y, z) \sim (z, y, x)) \quad (7.1776)$$

8.123 Descendants of 6.173

Algebra 6.173 has $3p + 3 + (p^2 + 2p + 3) \gcd(p - 1, 3)/2$ descendants of order p^7 . Algebra 6.173 has presentation

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa, pb, pc, \text{class } 3 \rangle.$$

If L is a descendant of 6.173 of order p^7 then the commutator structure of L is given by the classification of nilpotent Lie algebras over \mathbb{Z}_p of order p^7 . The commutator relations take the form

$$ca = bab, cb = \omega baa, baab = \lambda baaa, babb = \mu baaa$$

for some parameters λ, μ . Provided $\alpha^2 + 2\alpha\beta\lambda + \beta^2\mu \neq 0$, the parameters λ, μ give the same algebra (up to isomorphism) as the pair

$$\frac{\pm(\omega\alpha\beta + \alpha^2\lambda + \omega\beta^2\lambda + \alpha\beta\mu)}{\alpha^2 + 2\alpha\beta\lambda + \beta^2\mu}, \frac{\omega^2\beta^2 + 2\omega\alpha\beta\lambda + \alpha^2\mu}{\alpha^2 + 2\alpha\beta\lambda + \beta^2\mu}.$$

There are $p + 2$ orbits of pairs λ, μ under this action.

We pick a set representative pairs λ, μ for these orbits, and get the following presentations for the descendants of 6.173 of order p^7 :

$$\langle a, b, c \mid ca - bab, cb - \omega baa, baab - \lambda baaa, babb - \mu baaa, pa - ybaaa, pb - zbaaa, pc - tbaaa, \text{class } 4 \rangle. \quad (7.1777)$$

For each pair λ, μ we compute the set of pairs (α, β) which fixes λ, μ under one of the \pm actions given above. It turns out that we need to treat the pair $\lambda = \mu = 0$ separately from the other pairs.

If $\lambda = \mu = 0$, then (y, z, t) and (y', z', t') give isomorphic algebras if

$$\begin{pmatrix} y' \\ z' \\ t' \end{pmatrix} = \frac{1}{\alpha^4} \begin{pmatrix} \pm\alpha & 0 & 0 \\ 0 & \alpha & \gamma \\ 0 & 0 & \pm\alpha^2 \end{pmatrix} \begin{pmatrix} y \\ z \\ t \end{pmatrix}$$

for some α, γ with $\alpha \neq 0$.

In all other cases we can assume that if $t \neq 0$ then $y = z = 0$. For non-zero t we have

$$t \sim \frac{\pm t}{(\alpha^2 + 2\alpha\beta\lambda + \beta^2\mu)}$$

for some (α, β) which fixes λ, μ under one of the \pm actions given above. If $t = 0$ then (y, z) and (y', z') give isomorphic algebras if

$$\begin{pmatrix} y' \\ z' \end{pmatrix} = \frac{1}{(\alpha^2 - \omega\beta^2)(\alpha^2 + 2\alpha\beta\lambda + \beta^2\mu)} \begin{pmatrix} \pm\alpha & \pm\beta \\ \omega\beta & \alpha \end{pmatrix} \begin{pmatrix} y \\ z \end{pmatrix}$$

for some (α, β) which fixes λ, μ under one of the \pm actions given above.

8.124 Descendants of 6.174

$(p^3 + p^2 + p + 1)/4$ algebras

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - bab - ybaaa, pb, pc - xbaaa, \text{ class 4} \rangle (x \neq 0, (x, y) \sim (-x, y) \sim (x, -y)) \quad (7.1778)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - bab, pb - xbaaa, pc, \text{ class 4} \rangle (x \sim -x) \quad (7.1779)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - zbaa - bab - ybaaa, pb, pc - xbaaa, \text{ class 4} \rangle (x, z \neq 0, ((x, y, z) \sim (-x, y, -z) \sim (x, -y, z))) \quad (7.1780)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - xbaa - bab, pb - ybaaa, pc, \text{ class 4} \rangle (x \neq 0, (x, y) \sim (-x, y) \sim (x, -y)) \quad (7.1781)$$

8.125 Descendants of 6.175

$(p^3 + p^2 + p + 1)/4$ algebras

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \omega bab - ybaaa, pb, pc - xbaaa, \text{ class 4} \rangle (x \neq 0, (x, y) \sim (-x, y) \sim (x, -y)) \quad (7.1782)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \omega bab, pb - xbaaa, pc, \text{ class 4} \rangle (x \sim -x) \quad (7.1783)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - zbaa - \omega bab - ybaaa, pb, pc - xbaaa, \text{ class 4} \rangle (x, z \neq 0, ((x, y, z) \sim (-x, y, -z) \sim (x, -y, z))) \quad (7.1784)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - xbaa - \omega bab, pb - ybaaa, pc, \text{ class 4} \rangle (x \neq 0, (x, y) \sim (-x, y) \sim (x, -y)) \quad (7.1785)$$

8.126 Descendants of 6.176

$p(p-1) + p \gcd(p-1, 4)/2$ algebras

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa, pb, pc - xbabb, \text{class } 4 \rangle \quad (7.1786)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa, pb - ybabb, pc - xbabb, \text{class } 4 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^4 = 1) \quad (7.1787)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - xbabb, pb, pc - babb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.1788)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - ybabb, pb - xbabb, pc - babb, \text{class } 4 \rangle (x, y \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^4 = 1) \quad (7.1789)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \omega baa, pb, pc - xbabb, \text{class } 4 \rangle (p = 1 \pmod{4}) \quad (7.1790)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \omega baa, pb - ybabb, pc - xbabb, \text{class } 4 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1791)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \omega baa - xbabb, pb, pc - \omega babb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1792)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \omega baa - ybabb, pb - xbabb, pc - \omega babb, \text{class } 4 \rangle (x, y \neq 0, (x, y) \sim (ax, a^3y) \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.1793)$$

8.127 Descendants of 6.178

Algebra 6.178 has $(3p^2 - 1)/2$ descendants of order p^7 . Algebra 6.178 has presentation

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \lambda baa - \mu bab, pb - \nu baa - \xi bab, pc, \text{class } 3 \rangle,$$

where we write $A = \begin{pmatrix} \lambda & \mu \\ \nu & \xi \end{pmatrix}$, and A ranges over a set of representatives for the orbits of non-singular 2×2 matrices under the action

$$A \rightarrow \frac{1}{\det P} P A P^{-1}$$

as P ranges over non-singular matrices

$$P = \begin{pmatrix} \alpha & \beta \\ \pm\omega\beta & \pm\alpha \end{pmatrix}.$$

These algebras are terminal unless $\xi = -\lambda$. The number of orbits of non-singular matrices with $\xi = -\lambda$ is $(3p-1)/2$. The matrices split up into one orbit of size $p-1$ (matrices $\begin{pmatrix} 0 & y \\ \omega y & 0 \end{pmatrix}$), $p-1$ orbits of size $(p^2-1)/2$ (including two orbits of elements $\begin{pmatrix} x & y \\ -\omega y & -x \end{pmatrix}$), and $(p-1)/2$ orbits of size p^2-1 . All orbits contain matrices where $\lambda = 0$ or $\lambda = 1$.

It is possible to choose orbit representatives of the following 6 types:

1. $\begin{pmatrix} 0 & 1 \\ \omega & 0 \end{pmatrix},$

2. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ when $p = 1 \pmod{4}$,
3. $\begin{pmatrix} 0 & 1 \\ -\omega & 0 \end{pmatrix}$,
4. one representative $\begin{pmatrix} 1 & \mu \\ -\omega\mu & -1 \end{pmatrix}$ ($\mu \neq 0$) which is not in the same orbit as $\begin{pmatrix} 0 & 1 \\ -\omega & 0 \end{pmatrix}$ when $p = 3 \pmod{4}$,
5. $p - 3$ representatives $\begin{pmatrix} 0 & \mu \\ \nu & 0 \end{pmatrix}$ ($\nu \neq \pm\omega\mu$), and
6. $(p - 1)/2$ representatives of the form $\begin{pmatrix} 1 & \mu \\ \nu & -1 \end{pmatrix}$ ($\nu \neq -\omega\mu$).

We then obtain the following presentations for the descendants of 6.178. The first four cases of the matrix A are straightforward.

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - bab, pb - \omega baa, pc - xbaaa, \text{ class } 4 \rangle (x \sim -x) \quad (7.1794)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa, pb + bab, pc - xbaab, \text{ class } 4 \rangle (x \sim -x, p = 1 \pmod{4}) \quad (7.1795)$$

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - bab, pb + \omega baa, pc - xbaaa, \text{ class } 4 \rangle (x \sim -x) \quad (7.1796)$$

For the one matrix $\begin{pmatrix} 1 & \mu \\ -\omega\mu & -1 \end{pmatrix}$ ($\mu \neq 0$) (when $p = 3 \pmod{4}$) we have:

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - \mu bab, pb + \omega\mu baa + bab, pc - xbaaa, \text{ class } 4 \rangle (x \sim -x, p = 3 \pmod{4}) \quad (7.1797)$$

For the $p - 3$ matrices $\begin{pmatrix} 0 & \mu \\ \nu & 0 \end{pmatrix}$ ($\nu \neq \pm\omega\mu$) we have:

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \mu bab, pb - \nu baa, pc - xbaaa, \text{ class } 4 \rangle (x \sim -x).$$

But we have extra descendants if $(\omega\mu + 2\nu)(2\omega\nu + \mu^{-1}\nu^2)$ is a square. If $\omega\mu + 2\nu = 0$ then we have,

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \mu bab - xbaaa, pb - \nu baa, pc, \text{ class } 4 \rangle (x \sim -x),$$

If $2\omega\nu + \mu^{-1}\nu^2 = 0$ we have

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \mu bab, pb - \nu baa - xbaaa, pc, \text{ class } 4 \rangle (x \neq 0, x \sim -x),$$

and if $(\omega\mu + 2\nu)(2\omega\nu + \mu^{-1}\nu^2) = y^2 \neq 0$ then for one such value y we have

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \mu bab, pb - \nu baa - xbaaa, pc - ybaaa, \text{ class } 4 \rangle (x \neq 0, x \sim -x).$$

We combine these four possibilities into a single presentation

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - \mu bab - xbaaa, pb - \nu baa - ybaaa, pc - zbaaa, \text{ class } 4 \rangle \quad (7.1798)$$

For the $(p - 1)/2$ matrices $\begin{pmatrix} 1 & \mu \\ \nu & -1 \end{pmatrix}$ ($\nu \neq -\omega\mu$) the situation is even more complicated. First we have

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - \mu bab, pb - \nu baa + bab, pc - xbaab, \text{ class } 4 \rangle$$

But if $(1 + yz) \left(2(\omega y + z)^2 + \omega(1 + yz) \right)$ is a square we have an additional $p - 1$ descendants. It is not that easy to prove, but $(1 + yz) \left(2(\omega y + z)^2 + \omega(1 + yz) \right)$ cannot equal zero, under the assumption that A is not in the same orbit as a matrix

with $(1, 1)$ entry equal to zero. If $(1 + yz) \left(2(\omega y + z)^2 + \omega(1 + yz) \right) = x^2 \neq 0$, then if $x - \omega\mu - \nu = \omega\mu^2 + 2\mu\nu + 1 = 0$ we have

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - \mu bab - ybaab, pb - \nu baa + bab, pc - xbaab, \text{class } 4 \rangle (y \neq 0, y \sim -y),$$

but if one of $x - \omega\mu - \nu, \omega\mu^2 + 2\mu\nu + 1$ is non-zero we have

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - \mu bab, pb - \nu baa + bab - ybaab, pc - xbaab, \text{class } 4 \rangle (y \neq 0, y \sim -y).$$

And similarly for $-x, x + \omega\mu + \nu = \omega\mu^2 + 2\mu\nu + 1 = 0$ we have

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - \mu bab - ybaab, pb - \nu baa + bab, pc + xbaab, \text{class } 4 \rangle (y \neq 0, y \sim -y),$$

but if one of $x + \omega\mu + \nu, \omega\mu^2 + 2\mu\nu + 1$ is non-zero we have

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - \mu bab, pb - \nu baa + bab - ybaab, pc + xbaab, \text{class } 4 \rangle (y \neq 0, y \sim -y).$$

We combine these five possibilities into a single presentation

$$\langle a, b, c \mid ca - bab, cb - \omega baa, pa - baa - \mu bab - xbaab, pb - \nu baa + bab - ybaab, pc - zbaab, \text{class } 4 \rangle \quad (7.1799)$$

8.128 Descendants of 6.179

$(p^4 + p^2)/2$ algebras

$$\langle a, b, c \mid ca - bab, cb - \omega baa - xbaaa, pa - ybaaa, pb - zbaaa, pc - bab - tbaaa, \text{class } 4 \rangle ((x, y, z, t) \sim (x, -y, z, -t)) \quad (7.1800)$$

8.129 Descendants of 6.182

4 algebras

$$\langle a, b, c \mid ba, ca, cb, p^2b, pc, \text{class } 4 \rangle \quad (7.1801)$$

$$\langle a, b, c \mid ba, ca, cb - p^3a, p^2b, pc, \text{class } 4 \rangle \quad (7.1802)$$

$$\langle a, b, c \mid ba, ca - p^3a, cb, p^2b, pc, \text{class } 4 \rangle \quad (7.1803)$$

$$\langle a, b, c \mid ba - p^3a, ca, cb, p^2b, pc, \text{class } 4 \rangle \quad (7.1804)$$

8.130 Descendants of 6.183

2 algebras

$$\langle a, b, c \mid ba - p^2a, ca, cb, p^2b, pc, \text{class } 4 \rangle \quad (7.1805)$$

$$\langle a, b, c \mid ba - p^2a, ca, cb - p^3a, p^2b, pc, \text{class } 4 \rangle \quad (7.1806)$$

8.131 Descendants of 6.184

9 algebras

- $$\langle a, b, c \mid baa, bab, ca, cb, pb, pc, \text{ class } 4 \rangle \quad (7.1807)$$
- $$\langle a, b, c \mid baa, bab, ca - p^3a, cb, pb, pc, \text{ class } 4 \rangle \quad (7.1808)$$
- $$\langle a, b, c \mid baa, bab, ca, cb - p^3a, pb, pc, \text{ class } 4 \rangle \quad (7.1809)$$
- $$\langle a, b, c \mid baa - p^3a, bab, ca, cb, pb, pc, \text{ class } 4 \rangle \quad (7.1810)$$
- $$\langle a, b, c \mid baa - p^3a, bab, ca, cb - p^3a, pb, pc, \text{ class } 4 \rangle \quad (7.1811)$$
- $$\langle a, b, c \mid baa, bab - p^3a, ca, cb, pb, pc, \text{ class } 4 \rangle \quad (7.1812)$$
- $$\langle a, b, c \mid baa, bab - p^3a, ca - p^3a, cb, pb, pc, \text{ class } 4 \rangle \quad (7.1813)$$
- $$\langle a, b, c \mid baa, bab - \omega p^3a, ca, cb, pb, pc, \text{ class } 4 \rangle \quad (7.1814)$$
- $$\langle a, b, c \mid baa, bab - \omega p^3a, ca - p^3a, cb, pb, pc, \text{ class } 4 \rangle \quad (7.1815)$$

8.132 Descendants of 6.187

$11 + 4 \gcd(p - 1, 3) + 2 \gcd(p - 1, 4)$ algebras

- $$\langle a, b, c \mid bab, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1816)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1817)$$
- $$\langle a, b, c \mid bab - \omega baaa, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1818)$$
- $$\langle a, b, c \mid bab - \omega^2 baaa, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1819)$$
- $$\langle a, b, c \mid bab - \omega^3 baaa, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1820)$$
- $$\langle a, b, c \mid bab, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1821)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1822)$$
- $$\langle a, b, c \mid bab - \omega baaa, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.1823)$$
- $$\langle a, b, c \mid bab - \omega^2 baaa, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1824)$$
- $$\langle a, b, c \mid bab - \omega^3 baaa, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1825)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb, pc, \text{ class } 4 \rangle \quad (7.1826)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1827)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - \omega baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1828)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1829)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1830)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, p^2a, pb, pc, \text{ class } 4 \rangle \quad (7.1831)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, p^2a, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1832)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, p^2a, pb - \omega baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1833)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1834)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, p^2a, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1835)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a, pb, pc, \text{ class } 4 \rangle \quad (7.1836)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1837)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a, pb - \omega baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1838)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1839)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1840)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, p^2a, pb, pc, \text{ class } 4 \rangle \quad (7.1841)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, p^2a, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.1842)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, p^2a, pb - \omega baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1843)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.1844)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, p^2a, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.1845)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, p^2a, pb, pc - \omega baaa, \text{ class } 4 \rangle \quad (7.1846)$$

8.133 Descendants of 6.188

$3 + 2 \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a - baaa, pb, pc, \text{class } 4 \rangle \quad (7.1847)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb, pc, \text{class } 4 \rangle \quad (7.1848)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - baaa, pc, \text{class } 4 \rangle \quad (7.1849)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - \omega baaa, pc, \text{class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1850)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - \omega^2 baaa, pc, \text{class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1851)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb, pc - baaa, \text{class } 4 \rangle \quad (7.1852)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - baaa, pc - baaa, \text{class } 4 \rangle \quad (7.1853)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - \omega baaa, pc - baaa, \text{class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1854)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - \omega^2 baaa, pc - baaa, \text{class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1855)$$

8.134 Descendants of 6.189

$(5p + 7)/2$ algebras

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - baa - xbaaa, pc, \text{class } 4 \rangle \quad (x \sim -x) \quad (7.1856)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a - baaa, pb - baa, pc, \text{class } 4 \rangle \quad (7.1857)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a - xbaaa, pb - baa, pc, \text{class } 4 \rangle \quad (7.1858)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - baa, pc - baaa, \text{class } 4 \rangle \quad (7.1859)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a - baaa, pb - baa, pc - baaa, \text{class } 4 \rangle \quad (7.1860)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a - xbaaa, pb - baa, pc - baaa, \text{class } 4 \rangle \quad (7.1861)$$

8.135 Descendants of 6.190

2 algebras

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - baa, pc, \text{class } 4 \rangle \quad (7.1862)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a - baaa, pb - baa, pc, \text{class } 4 \rangle \quad (7.1863)$$

8.136 Descendants of 6.191

$(5p + 7)/2$ algebras

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - \omega baa - xbaaa, pc, \text{class } 4 \rangle (x \sim -x) \quad (7.1864)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a - baaa, pb - \omega baa, pc, \text{class } 4 \rangle \quad (7.1865)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a - xbaaa, pb - \omega baa, pc, \text{class } 4 \rangle \quad (7.1866)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a, pb - \omega baa, pc - baaa, \text{class } 4 \rangle \quad (7.1867)$$

$$\langle a, b, c \mid bab, ca, cb, p^2a - baaa, pb - \omega baa, pc - baaa, \text{class } 4 \rangle \quad (7.1868)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, p^2a - xbaaa, pb - \omega baa, pc - baaa, \text{class } 4 \rangle \quad (7.1869)$$

8.137 Descendants of 6.192

2 algebras

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a, pb - \omega baa, pc, \text{class } 4 \rangle \quad (7.1870)$$

$$\langle a, b, c \mid bab, ca, cb - baa, p^2a - baaa, pb - \omega baa, pc, \text{class } 4 \rangle \quad (7.1871)$$

8.138 Descendants of 6.197

$11 + 4 \gcd(p - 1, 3) + 2 \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid baa, ca, cb, p^2a - babb, pb, pc, \text{class } 4 \rangle \quad (7.1872)$$

$$\langle a, b, c \mid baa, ca, cb, p^2a - \omega babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1873)$$

$$\langle a, b, c \mid baa, ca, cb, p^2a - \omega^2 babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1874)$$

$$\langle a, b, c \mid baa, ca - babb, cb, p^2a - babb, pb, pc, \text{class } 4 \rangle \quad (7.1875)$$

$$\langle a, b, c \mid baa, ca - babb, cb, p^2a - \omega babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1876)$$

$$\langle a, b, c \mid baa, ca - babb, cb, p^2a - \omega^2 babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1877)$$

$$\langle a, b, c \mid baa - babb, ca, cb, p^2a - babb, pb, pc, \text{class } 4 \rangle \quad (7.1878)$$

$$\langle a, b, c \mid baa - babb, ca, cb, p^2a - \omega babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1879)$$

$$\langle a, b, c \mid baa - babb, ca, cb, p^2a - \omega^2 babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1880)$$

$$\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a - babb, pb, pc, \text{class } 4 \rangle \quad (7.1881)$$

- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a - \omega babb, pb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.1882)
- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a - \omega^2 babb, pb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.1883)
- $\langle a, b, c \mid baa, ca, cb, p^2a, pb, pc, \text{class } 4 \rangle$ (7.1884)
- $\langle a, b, c \mid baa, ca, cb, p^2a, pb, pc - babb, \text{class } 4 \rangle$ (7.1885)
- $\langle a, b, c \mid baa, ca, cb, p^2a, pb - babb, pc, \text{class } 4 \rangle$ (7.1886)
- $\langle a, b, c \mid baa, ca - babb, cb, p^2a, pb, pc, \text{class } 4 \rangle$ (7.1887)
- $\langle a, b, c \mid baa, ca - babb, cb, p^2a, pb, pc - babb, \text{class } 4 \rangle$ (7.1888)
- $\langle a, b, c \mid baa, ca - babb, cb, p^2a, pb - babb, pc, \text{class } 4 \rangle$ (7.1889)
- $\langle a, b, c \mid baa - babb, ca, cb, p^2a, pb, pc, \text{class } 4 \rangle$ (7.1890)
- $\langle a, b, c \mid baa - babb, ca, cb, p^2a, pb, pc - babb, \text{class } 4 \rangle$ (7.1891)
- $\langle a, b, c \mid baa - babb, ca, cb, p^2a, pb - babb, pc, \text{class } 4 \rangle$ (7.1892)
- $\langle a, b, c \mid baa - babb, ca, cb, p^2a, pb - \omega babb, pc, \text{class } 4 \rangle$ (7.1893)
- $\langle a, b, c \mid baa - babb, ca, cb, p^2a, pb - \omega^2 babb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{4}$) (7.1894)
- $\langle a, b, c \mid baa - babb, ca, cb, p^2a, pb - \omega^3 babb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{4}$) (7.1895)
- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a, pb, pc, \text{class } 4 \rangle$ (7.1896)
- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a, pb, pc - babb, \text{class } 4 \rangle$ (7.1897)
- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a, pb, pc - \omega babb, \text{class } 4 \rangle$ (7.1898)
- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a, pb - babb, pc, \text{class } 4 \rangle$ (7.1899)
- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a, pb - \omega babb, pc, \text{class } 4 \rangle$ (7.1900)
- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a, pb - \omega^2 babb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{4}$) (7.1901)
- $\langle a, b, c \mid baa - babb, ca - babb, cb, p^2a, pb - \omega^3 babb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{4}$) (7.1902)

8.139 Descendants of 6.198

4 + gcd($p - 1, 3$) algebras

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a - babb, pb, pc, \text{ class } 4 \rangle \quad (7.1903)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a - \omega babb, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1904)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a - \omega^2 babb, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.1905)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a, pb, pc, \text{ class } 4 \rangle \quad (7.1906)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a, pb, pc - babb, \text{ class } 4 \rangle \quad (7.1907)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a, pb - babb, pc, \text{ class } 4 \rangle \quad (7.1908)$$

$$\langle a, b, c \mid baa, ca - bab, cb, p^2a, pb - babb, pc - babb, \text{ class } 4 \rangle \quad (7.1909)$$

8.140 Descendants of 6.207

5 algebras

$$\langle a, b, c \mid baa, ca, cb, pb - ba, pc, \text{ class } 4 \rangle \quad (7.1910)$$

$$\langle a, b, c \mid baa, ca, cb - p^3a, pb - ba, pc, \text{ class } 4 \rangle \quad (7.1911)$$

$$\langle a, b, c \mid baa - p^3a, ca, cb, pb - ba, pc, \text{ class } 4 \rangle \quad (7.1912)$$

$$\langle a, b, c \mid baa - p^3a, ca, cb - p^3a, pb - ba, pc, \text{ class } 4 \rangle \quad (7.1913)$$

$$\langle a, b, c \mid baa, ca - p^3a, cb, pb - ba, pc, \text{ class } 4 \rangle \quad (7.1914)$$

8.141 Descendants of 6.212

4 algebras

$$\langle a, b, c \mid ca, cb, pb, pc - ba, \text{ class } 4 \rangle \quad (7.1915)$$

$$\langle a, b, c \mid ca - p^3a, cb, pb, pc - ba, \text{ class } 4 \rangle \quad (7.1916)$$

$$\langle a, b, c \mid ca, cb - p^3a, pb, pc - ba, \text{ class } 4 \rangle \quad (7.1917)$$

$$\langle a, b, c \mid ca, cb - \omega p^3a, pb, pc - ba, \text{ class } 4 \rangle \quad (7.1918)$$

8.142 Descendants of 6.215

3 algebras

$$\langle a, b, c \mid ca - p^2a, cb, pb, pc - ba, \text{class } 4 \rangle \quad (7.1919)$$

$$\langle a, b, c \mid ca - p^2a, cb - p^3a, pb, pc - ba, \text{class } 4 \rangle \quad (7.1920)$$

$$\langle a, b, c \mid ca - p^2a, cb - \omega p^3a, pb, pc - ba, \text{class } 4 \rangle \quad (7.1921)$$

8.143 Descendants of 6.216

4 algebras

$$\langle a, b, c \mid baa, bab, ca, cb, pa, pb, \text{class } 4 \rangle \quad (7.1922)$$

$$\langle a, b, c \mid baa, bab, ca, cb - p^3c, pa, pb, \text{class } 4 \rangle \quad (7.1923)$$

$$\langle a, b, c \mid baa, bab - p^3c, ca, cb, pa, pb, \text{class } 4 \rangle \quad (7.1924)$$

$$\langle a, b, c \mid baa, bab - p^3c, ca - p^3c, cb, pa, pb, \text{class } 4 \rangle \quad (7.1925)$$

8.144 Descendants of 6.218

$11 + 4 \gcd(p - 1, 3) + 2 \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid bab, ca, cb, pa, pb, p^2c, \text{class } 4 \rangle \quad (7.1926)$$

$$\langle a, b, c \mid bab, ca, cb, pa, pb, p^2c - baaa, \text{class } 4 \rangle \quad (7.1927)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, pa, pb, p^2c, \text{class } 4 \rangle \quad (7.1928)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, pa, pb, p^2c - baaa, \text{class } 4 \rangle \quad (7.1929)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, pa, pb, p^2c, \text{class } 4 \rangle \quad (7.1930)$$

$$\langle a, b, c \mid bab, ca, cb - baaa, pa, pb, p^2c - baaa, \text{class } 4 \rangle \quad (7.1931)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, pa, pb, p^2c, \text{class } 4 \rangle \quad (7.1932)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baaa, pa, pb, p^2c - baaa, \text{class } 4 \rangle \quad (7.1933)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - \omega baaa, pa, pb, p^2c - baaa, \text{class } 4 \rangle \quad (7.1934)$$

$$\langle a, b, c \mid bab, ca, cb, pa, pb - baaa, p^2c, \text{class } 4 \rangle \quad (7.1935)$$

$$\langle a, b, c \mid bab - baaa, ca, cb, pa, pb - baaa, p^2c, \text{class } 4 \rangle \quad (7.1936)$$

- (7.1937)
- (7.1938)
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- (7.1942)
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- (7.1947)
- (7.1948)
- (7.1949)
- (7.1950)
- (7.1951)
- (7.1952)
- (7.1953)
- (7.1954)
- (7.1955)
- (7.1956)

8.145 Descendants of 6.222

$2p + 5 + 3 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

- $$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, p^2c, \text{ class } 4 \rangle \quad (7.1957)$$
- $$\langle a, b, c \mid bab, ca, cb - baa - baaa, pa, pb, p^2c, \text{ class } 4 \rangle \quad (7.1958)$$
- $$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, p^2c - baaa, \text{ class } 4 \rangle \quad (7.1959)$$
- $$\langle a, b, c \mid bab, ca, cb - baa - baaa, pa, pb, p^2c - baaa, \text{ class } 4 \rangle \quad (7.1960)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa, pb, p^2c, \text{ class } 4 \rangle \quad (7.1961)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa, pb, p^2c - baaa, \text{ class } 4 \rangle \quad (7.1962)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa, pb, p^2c - \omega baaa, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1963)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa, pb, p^2c - \omega^2 baaa, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1964)$$
- $$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - baaa, p^2c, \text{ class } 4 \rangle \quad (7.1965)$$
- $$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega baaa, p^2c, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1966)$$
- $$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega^2 baaa, p^2c, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1967)$$
- $$\langle a, b, c \mid bab, ca, cb - baa, pa - baaa, pb, p^2c, \text{ class } 4 \rangle \quad (7.1968)$$
- $$\langle a, b, c \mid bab, ca, cb - baa - baaa, pa, pb - xbaaa, p^2c, \text{ class } 4 \rangle \quad (x \neq 0) \quad (7.1969)$$
- $$\langle a, b, c \mid bab, ca, cb - baa - baaa, pa - baaa, pb, p^2c, \text{ class } 4 \rangle \quad (7.1970)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa - baaa, pa - baaa, pb, p^2c, \text{ class } 4 \rangle \quad (7.1971)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa - \omega baaa, pb, p^2c, \text{ class } 4 \rangle \quad (7.1972)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa - \omega^2 baaa, pb, p^2c, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1973)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa - \omega^3 baaa, pb, p^2c, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.1974)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa, pb - baaa, p^2c, \text{ class } 4 \rangle \quad (7.1975)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa, pb - \omega baaa, p^2c, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1976)$$
- $$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa, pb - \omega^2 baaa, p^2c, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.1977)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa - xbaaa, pb - baaa, p^2c, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.1978)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa - xbaaa, pb - \omega baaa, p^2c, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.1979)$$

$$\langle a, b, c \mid bab - baaa, ca, cb - baa, pa - xbaaa, pb - \omega^2 baaa, p^2c, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.1980)$$

8.146 Descendants of 6.228

$p + 1$ algebras

$$\langle a, b, c \mid ca, cb - xp^3c, pa - ba, pb, \text{class } 4 \rangle \quad (7.1981)$$

$$\langle a, b, c \mid ca - p^3c, cb, pa - ba, pb, \text{class } 4 \rangle \quad (7.1982)$$

8.147 Descendants of 6.231

$p^2 + 5p + 14 + (p + 17) \gcd(p - 1, 3) + 2 \gcd(p - 1, 4) + \gcd(p - 1, 7) + \gcd(p - 1, 8)$ algebras

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa, pb, pc, \text{class } 4 \rangle \quad (7.1983)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - babb, pb, pc, \text{class } 4 \rangle \quad (7.1984)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - \omega babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1985)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - \omega^2 babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1986)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa, pb - babb, pc, \text{class } 4 \rangle \quad (7.1987)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - babb, pb - babb, pc, \text{class } 4 \rangle \quad (7.1988)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - \omega babb, pb - babb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1989)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa - \omega^2 babb, pb - babb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1990)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac, pa, pb, pc - babb, \text{class } 4 \rangle \quad (7.1991)$$

$$\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa, pb, pc, \text{class } 4 \rangle \quad (7.1992)$$

$$\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa - babb, pb, pc, \text{class } 4 \rangle \quad (7.1993)$$

$$\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa - \omega babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1994)$$

$$\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa - \omega^2 babb, pb, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.1995)$$

- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa, pb - babb, pc, \text{class } 4 \rangle$ (7.1996)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa - babb, pb - babb, pc, \text{class } 4 \rangle$ (7.1997)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa - \omega babb, pb - babb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.1998)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa - \omega^2 babb, pb - babb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.1999)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa, pb, pc - babb, \text{class } 4 \rangle$ (7.2000)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa, pb - babb, pc - babb, \text{class } 4 \rangle$ (7.2001)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa, pb - \omega babb, pc - babb, \text{class } 4 \rangle$ ($p = 1 \pmod{4}$) (7.2002)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa, pb, pc - \omega babb, \text{class } 4 \rangle$ (7.2003)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa, pb - babb, pc - \omega babb, \text{class } 4 \rangle$ (7.2004)
- $\langle a, b, c \mid cb, baa, bac, caa - babb, cac, pa, pb - \omega babb, pc - \omega babb, \text{class } 4 \rangle$ ($p = 1 \pmod{4}$) (7.2005)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa, pb, pc, \text{class } 4 \rangle$ (7.2006)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - babb, pb, pc, \text{class } 4 \rangle$ (7.2007)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - \omega babb, pb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2008)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - \omega^2 babb, pb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2009)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa, pb - babb, pc, \text{class } 4 \rangle$ (7.2010)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - babb, pb - babb, pc, \text{class } 4 \rangle$ (7.2011)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - \omega babb, pb - babb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2012)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - \omega^2 babb, pb - babb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2013)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa, pb, pc - babb, \text{class } 4 \rangle$ (7.2014)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - babb, pb, pc - babb, \text{class } 4 \rangle$ (7.2015)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - \omega babb, pb, pc - babb, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2016)
- $\langle a, b, c \mid cb, baa, bac - babb, caa, cac, pa - \omega^2 babb, pb, pc - babb, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2017)

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- (7.2038)

- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa, pb, pc, \text{ class } 4 \rangle$ (7.2039)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa, pb, pc - babb, \text{ class } 4 \rangle$ (7.2040)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa, pb - babb, pc, \text{ class } 4 \rangle$ (7.2041)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa, pb - babb, pc - babb, \text{ class } 4 \rangle$ (7.2042)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - babb, pb, pc, \text{ class } 4 \rangle$ (7.2043)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega babb, pb, pc, \text{ class } 4 \rangle$ (7.2044)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^2 babb, pb, pc, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2045)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^3 babb, pb, pc, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2046)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^4 babb, pb, pc, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2047)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^5 babb, pb, pc, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2048)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - babb, pb, pc - babb, \text{ class } 4 \rangle$ (7.2049)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega babb, pb, pc - babb, \text{ class } 4 \rangle$ (7.2050)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^2 babb, pb, pc - babb, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2051)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^3 babb, pb, pc - babb, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2052)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^4 babb, pb, pc - babb, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2053)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^5 babb, pb, pc - babb, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2054)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - babb, pb - babb, pc, \text{ class } 4 \rangle$ (7.2055)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega babb, pb - babb, pc, \text{ class } 4 \rangle$ (7.2056)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^2 babb, pb - babb, pc, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2057)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^3 babb, pb - babb, pc, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2058)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^4 babb, pb - babb, pc, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2059)
- $\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^5 babb, pb - babb, pc, \text{ class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2060)

$$\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - babb, pb - babb, pc - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^6 = 1) \quad (7.2061)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega babb, pb - babb, pc - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^6 = 1) \quad (7.2062)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^2 babb, pb - babb, pc - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.2063)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^3 babb, pb - babb, pc - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.2064)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^4 babb, pb - babb, pc - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.2065)$$

$$\langle a, b, c \mid cb, baa, bac, caa, cac - babb, pa - \omega^5 babb, pb - babb, pc - xbabb, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^6 = 1, p = 1 \pmod{3}) \quad (7.2066)$$

$$\langle a, b, c \mid cb, baa - babb, bac, caa, cac - babb, pa, pb, pc, \text{class } 4 \rangle \quad (7.2067)$$

$$\langle a, b, c \mid cb, baa - babb, bac, caa, cac - babb, pa, pb, pc - babb, \text{class } 4 \rangle \quad (7.2068)$$

$$\langle a, b, c \mid cb, baa - babb, bac, caa, cac - babb, pa, pb, pc - xbabb, \text{class } 4 \rangle (x = \omega^i, i = 1, 2, 3, 4, 5, 6, p = 1 \pmod{7}) \quad (7.2069)$$

$$\langle a, b, c \mid cb, baa - babb, bac, caa, cac - babb, pa, pb - xbabb, pc, \text{class } 4 \rangle (x = \omega^i, 0 \leq i < \gcd(p-1, 8)) \quad (7.2070)$$

In the following presentation we have $x = \omega^i, 0 \leq i < \gcd(p-1, 8), y \neq 0, (x, y) \sim (x, ay)$ if $a^8 = 1$:

$$\langle a, b, c \mid cb, baa - babb, bac, caa, cac - babb, pa, pb - xbabb, pc - ybabb, \text{class } 4 \rangle \quad (7.2071)$$

In the following presentation we have $x = \omega^i, 0 \leq i < 2 \gcd(p-1, 3)$:

$$\langle a, b, c \mid cb, baa - babb, bac, caa, cac - babb, pa - xbabb, pb, pc, \text{class } 4 \rangle \quad (7.2072)$$

In the following presentation we have $x = \omega^i, 0 \leq i < 2 \gcd(p-1, 3), y \neq 0, (x, y) \sim (x, ay)$ if $a^6 = 1$:

$$\langle a, b, c \mid cb, baa - babb, bac, caa, cac - babb, pa - xbabb, pb, pc - ybabb, \text{class } 4 \rangle \quad (7.2073)$$

In the following presentation we have $x = \omega^i, 0 \leq i < 2 \gcd(p-1, 3), y \neq 0, (x, y, z) \sim (x, ay, \pm a^2 z)$ if $a^3 = 1$:

$$\langle a, b, c \mid cb, baa - babb, bac, caa, cac - babb, pa - xbabb, pb - ybabb, pc - zbabb, \text{class } 4 \rangle \quad (7.2074)$$

8.148 Descendants of 6.256

$20 + (p + 11) \gcd(p - 1, 3) + 4 \gcd(p - 1, 4)$ algebras

- $$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb, pc, \text{ class } 4 \rangle \quad (7.2075)$$
- $$\langle a, b, c \mid cb, bab, bac, caa, cac, pa - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.2076)$$
- $$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.2077)$$
- $$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2078)$$
- $$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2079)$$
- $$\langle a, b, c \mid cb, bab, bac, caa, cac, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.2080)$$
- $$\langle a, b, c \mid cb - baaa, bab, bac, caa, cac, pa, pb, pc, \text{ class } 4 \rangle \quad (7.2081)$$
- $$\langle a, b, c \mid cb - baaa, bab, bac, caa, cac, pa - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.2082)$$
- $$\langle a, b, c \mid cb - baaa, bab, bac, caa, cac, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.2083)$$
- $$\langle a, b, c \mid cb - baaa, bab, bac, caa, cac, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2084)$$
- $$\langle a, b, c \mid cb - baaa, bab, bac, caa, cac, pa, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2085)$$
- $$\langle a, b, c \mid cb - baaa, bab, bac, caa, cac, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.2086)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa, pb, pc, \text{ class } 4 \rangle \quad (7.2087)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.2088)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa - \omega baaa, pb, pc, \text{ class } 4 \rangle \quad (7.2089)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa - \omega^2 baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.2090)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa - \omega^3 baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.2091)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.2092)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2093)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2094)$$
- $$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.2095)$$

- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa, pb, pc, \text{class } 4 \rangle$ (7.2096)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa - baaa, pb, pc, \text{class } 4 \rangle$ (7.2097)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa - \omega baaa, pb, pc, \text{class } 4 \rangle$ (7.2098)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa - \omega^2 baaa, pb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{4}$) (7.2099)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa - \omega^3 baaa, pb, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{4}$) (7.2100)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa, pb - baaa, pc, \text{class } 4 \rangle$ (7.2101)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa, pb - \omega baaa, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2102)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2103)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa, pb, pc - baaa, \text{class } 4 \rangle$ (7.2104)
- $\langle a, b, c \mid cb - baaa, bab - baaa, bac, caa, cac, pa, pb, pc - \omega baaa, \text{class } 4 \rangle$ (7.2105)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa, pb, pc, \text{class } 4 \rangle$ (7.2106)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa - baaa, pb, pc, \text{class } 4 \rangle$ (7.2107)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa, pb - baaa, pc, \text{class } 4 \rangle$ (7.2108)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa, pb - \omega baaa, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2109)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2110)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa, pb - baaa, pc - baaa, \text{class } 4 \rangle$ (7.2111)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa, pb - \omega baaa, pc - baaa, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2112)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa, pb - \omega^2 baaa, pc - baaa, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.2113)
- $\langle a, b, c \mid cb, bab, bac - baaa, caa, cac, pa, pb, pc - baaa, \text{class } 4 \rangle$ (7.2114)
- $\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa - baaa, pb, pc, \text{class } 4 \rangle$ (7.2115)
- $\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa - \omega baaa, pb, pc, \text{class } 4 \rangle$ (7.2116)
- $\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa, pb, pc, \text{class } 4 \rangle$ (7.2117)

$$\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (7.2118)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa, pb - \omega baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2119)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2120)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (7.2121)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa, pb - baaa, pc - baaa, \text{class } 4 \rangle \quad (7.2122)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa, pb - \omega baaa, pc - baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2123)$$

$$\langle a, b, c \mid cb, bab, bac, caa, cac - baaa, pa, pb - \omega^2 baaa, pc - baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2124)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa, pb, pc, \text{class } 4 \rangle \quad (7.2125)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (7.2126)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa - \omega baaa, pb, pc, \text{class } 4 \rangle \quad (7.2127)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa - \omega^2 baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2128)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa - \omega^3 baaa, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2129)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (7.2130)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa, pb, pc - \omega baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2131)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa, pb, pc - \omega^2 baaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2132)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa, pb - baaa, pc - xbaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.2133)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa, pb - \omega baaa, pc - xbaaa, \text{class } 4 \rangle (x \sim -x, p = 1 \bmod 3) \quad (7.2134)$$

$$\langle a, b, c \mid cb, bab - baaa, bac, caa, cac - baaa, pa, pb - \omega^2 baaa, pc - xbaaa, \text{class } 4 \rangle (x \sim -x, p = 1 \bmod 3) \quad (7.2135)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa, pb, pc, \text{class } 4 \rangle \quad (7.2136)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (7.2137)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa - \omega baaa, pb, pc, \text{class } 4 \rangle \quad (7.2138)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa - \omega^2 baaa, pb, pc, \text{ class } 4 \rangle (p = 1 \bmod 4) \quad (7.2139)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa - \omega^3 baaa, pb, pc, \text{ class } 4 \rangle (p = 1 \bmod 4) \quad (7.2140)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.2141)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa, pb, pc - \omega baaa, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.2142)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa, pb, pc - \omega^2 baaa, \text{ class } 4 \rangle (p = 1 \bmod 3) \quad (7.2143)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa, pb - baaa, pc - xbaaa, \text{ class } 4 \rangle (x \sim -x) \quad (7.2144)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa, pb - \omega baaa, pc - xbaaa, \text{ class } 4 \rangle (x \sim -x, p = 1 \bmod 3) \quad (7.2145)$$

$$\langle a, b, c \mid cb, bab - \omega baaa, bac, caa, cac - baaa, pa, pb - \omega^2 baaa, pc - xbaaa, \text{ class } 4 \rangle (x \sim -x, p = 1 \bmod 3) \quad (7.2146)$$

8.149 Descendants of 6.261

$4p + 2 + (p^2 + 3p + 1) \gcd(p - 1, 3) + (p + 1) \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa, pb, pc, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}) \quad (7.2147)$$

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa - baaa, pb, pc, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}) \quad (7.2148)$$

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa, pb, pc - baaa, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}) \quad (7.2149)$$

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - baaa, pc, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}) \quad (7.2150)$$

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}, p = 1 \bmod 3) \quad (7.2151)$$

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}, p = 1 \bmod 3) \quad (7.2152)$$

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - baaa, pc - baaa, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}) \quad (7.2153)$$

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - \omega baaa, pc - baaa, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}, p = 1 \bmod 3) \quad (7.2154)$$

$$\langle a, b, c \mid cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - \omega^2 baaa, pc - baaa, \text{ class } 4 \rangle (x \neq -1, -\frac{1}{2}, p = 1 \bmod 3) \quad (7.2155)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa, pb, pc, \text{ class } 4 \rangle \quad (7.2156)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.2157)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.2158)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa - baaa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.2159)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.2160)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2161)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2162)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa, pb - baaa, pc - baaa, \text{ class } 4 \rangle \quad (7.2163)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa, pb - \omega baaa, pc - baaa, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2164)$$

$$\langle a, b, c \mid cb - baa, bab, bac + baaa, caa, cac, pa, pb - \omega^2 baaa, pc - baaa, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2165)$$

$$\langle a, b, c \mid cb - baa, bab, bac + \frac{1}{2} baaa, caa, cac, pa, pb, pc, \text{ class } 4 \rangle \quad (7.2166)$$

$$\langle a, b, c \mid cb - baa, bab, bac + \frac{1}{2} baaa, caa, cac, pa - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.2167)$$

$$\langle a, b, c \mid cb - baa, bab, bac + \frac{1}{2} baaa, caa, cac, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.2168)$$

$$\langle a, b, c \mid cb - baa, bab, bac + \frac{1}{2} baaa, caa, cac, pa, pb - \omega baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2169)$$

$$\langle a, b, c \mid cb - baa, bab, bac + \frac{1}{2} baaa, caa, cac, pa, pb - \omega^2 baaa, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.2170)$$

$$\langle a, b, c \mid cb - baa, bab, bac + \frac{1}{2} baaa, caa, cac, pa, pb, pc - baaa, \text{ class } 4 \rangle \quad (7.2171)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2} baaa, caa, cac, pa, pb, pc, \text{ class } 4 \rangle \quad (7.2172)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2} baaa, caa, cac, pa - baaa, pb, pc, \text{ class } 4 \rangle \quad (7.2173)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2} baaa, caa, cac, pa - \omega baaa, pb, pc, \text{ class } 4 \rangle \quad (7.2174)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2} baaa, caa, cac, pa - \omega^2 baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.2175)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2} baaa, caa, cac, pa - \omega^3 baaa, pb, pc, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.2176)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2} baaa, caa, cac, pa, pb - baaa, pc, \text{ class } 4 \rangle \quad (7.2177)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2}baaa, caa, cac, pa, pb - \omega baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2178)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2}baaa, caa, cac, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2179)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2}baaa, caa, cac, pa, pb, pc - baaa, \text{class } 4 \rangle \quad (7.2180)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2}baaa, caa, cac, pa, pb, pc - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2181)$$

$$\langle a, b, c \mid cb - baa, bab - baaa, bac + \frac{1}{2}baaa, caa, cac, pa, pb, pc - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2182)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa, pb, pc, \text{class } 4 \rangle \quad (7.2183)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa - baaa, pb, pc, \text{class } 4 \rangle \quad (7.2184)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa - \omega baaa, pb, pc, \text{class } 4 \rangle \quad (7.2185)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa - \omega^2 baaa, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.2186)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa - \omega^3 baaa, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.2187)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa, pb - baaa, pc, \text{class } 4 \rangle \quad (7.2188)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa, pb - \omega baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2189)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa, pb - \omega^2 baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2190)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa, pb - ybaaa, pc - baaa, \text{class } 4 \rangle \quad (7.2191)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa, pb - ybaaa, pc - \omega baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2192)$$

$$\langle a, b, c \mid cb - baa, bab - xbaaa, bac, caa, cac, pa, pb - ybaaa, pc - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2193)$$

8.150 Descendants of 6.267

$p + 3 + 3 \gcd(p - 1, 3) + \gcd(p - 1, 4) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid cb, caa, pa - ba, pb, pc, \text{class } 4 \rangle \quad (7.2194)$$

$$\langle a, b, c \mid cb, caa, pa - ba - cacc, pb, pc, \text{class } 4 \rangle \quad (7.2195)$$

$$\langle a, b, c \mid cb, caa, pa - ba - \omega cacc, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2196)$$

$$\langle a, b, c \mid cb, caa, pa - ba - \omega^2 cacc, pb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2197)$$

$$\langle a, b, c \mid cb, caa, pa - ba, pb, pc - cacc, \text{class } 4 \rangle \quad (7.2198)$$

$$\langle a, b, c \mid cb, caa, pa - ba - cacc, pb, pc - cacc, \text{class } 4 \rangle \quad (7.2199)$$

$$\langle a, b, c \mid cb, caa, pa - ba - \omega cacc, pb, pc - cacc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2200)$$

$$\langle a, b, c \mid cb, caa, pa - ba - \omega^2 cacc, pb, pc - cacc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2201)$$

$$\langle a, b, c \mid cb, caa, pa - ba, pb - cacc, pc, \text{class } 4 \rangle \quad (7.2202)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba, pb - cacc, pc, \text{class } 4 \rangle \quad (7.2203)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba, pb - xcacc, pc, \text{class } 4 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \bmod 5) \quad (7.2204)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba, pb, pc, \text{class } 4 \rangle \quad (7.2205)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba, pb, pc - cacc, \text{class } 4 \rangle \quad (7.2206)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba, pb, pc - \omega cacc, \text{class } 4 \rangle \quad (7.2207)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba, pb, pc - \omega^2 cacc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2208)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba, pb, pc - \omega^3 cacc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2209)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba - cacc, pb, pc, \text{class } 4 \rangle \quad (7.2210)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba - \omega cacc, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2211)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba - \omega cacc, pb, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2212)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba - cacc, pb, pc - xcacc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.2213)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba - \omega cacc, pb, pc - xcacc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2214)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - ba - \omega^2 cacc, pb, pc - xcacc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2215)$$

8.151 Descendants of 6.269

$(p + 5)/2$ algebras

$$\langle a, b, c \mid cb, caa, pa - ba - cac, pb, pc, \text{class } 4 \rangle \quad (7.2216)$$

$$\langle a, b, c \mid cb, caa, pa - ba - cac, pb - cacc, pc, \text{class } 4 \rangle \quad (7.2217)$$

$$\langle a, b, c \mid cb, caa, pa - ba - cac, pb - xcacc, pc - cacc, \text{class } 4 \rangle (x \sim -x) \quad (7.2218)$$

8.152 Descendants of 6.271

$(p + 5)/2$ algebras

$$\langle a, b, c \mid cb, caa, pa - ba - \omega cac, pb, pc, \text{class } 4 \rangle \quad (7.2219)$$

$$\langle a, b, c \mid cb, caa, pa - ba - \omega cac, pb - cacc, pc, \text{class } 4 \rangle \quad (7.2220)$$

$$\langle a, b, c \mid cb, caa, pa - ba - \omega cac, pb - xcacc, pc - cacc, \text{class } 4 \rangle (x \sim -x) \quad (7.2221)$$

8.153 Descendants of 6.273

$1 + \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid cb, cac, pa - ba, pb - caa, pc, \text{class } 4 \rangle \quad (7.2222)$$

$$\langle a, b, c \mid cb, cac - caaa, pa - ba, pb - caa, pc, \text{class } 4 \rangle \quad (7.2223)$$

$$\langle a, b, c \mid cb, cac - \omega caaa, pa - ba, pb - caa, pc, \text{class } 4 \rangle \quad (7.2224)$$

$$\langle a, b, c \mid cb, cac - \omega^2 caaa, pa - ba, pb - caa, pc, \text{class } 4 \rangle (p = 1 \pmod{4}) \quad (7.2225)$$

$$\langle a, b, c \mid cb, cac - \omega^3 caaa, pa - ba, pb - caa, pc, \text{class } 4 \rangle (p = 1 \pmod{4}) \quad (7.2226)$$

$$\langle a, b, c \mid cb - caaa, cac, pa - ba, pb - caa, pc, \text{class } 4 \rangle \quad (7.2227)$$

$$\langle a, b, c \mid cb - \omega caaa, cac, pa - ba, pb - caa, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.2228)$$

$$\langle a, b, c \mid cb - \omega^2 caaa, cac, pa - ba, pb - caa, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.2229)$$

8.154 Descendants of 6.274

$p + 2 + (2p + 3) \gcd(p - 1, 3) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc, \text{class } 4 \rangle \quad (7.2230)$$

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc - caaa, \text{class } 4 \rangle \quad (7.2231)$$

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc - \omega caaa, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.2232)$$

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc - \omega^2 caaa, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.2233)$$

$$\langle a, b, c \mid cb, cac - caaa, pa - ba, pb, pc, \text{class } 4 \rangle \quad (7.2234)$$

$$\langle a, b, c \mid cb, cac - caaa, pa - ba, pb, pc - caaa, \text{class } 4 \rangle \quad (7.2235)$$

$$\langle a, b, c \mid cb, cac - caaa, pa - ba, pb, pc - \omega caaa, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.2236)$$

$$\langle a, b, c \mid cb, cac - caaa, pa - ba, pb, pc - \omega^2 caaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2237)$$

$$\langle a, b, c \mid cb - caaa, cac, pa - ba, pb, pc - xcaaa, \text{class } 4 \rangle \quad (7.2238)$$

$$\langle a, b, c \mid cb - \omega caaa, cac, pa - ba, pb, pc - xcaaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2239)$$

$$\langle a, b, c \mid cb - \omega^2 caaa, cac, pa - ba, pb, pc - xcaaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2240)$$

$$\langle a, b, c \mid cb - caaa, cac - caaa, pa - ba, pb, pc - xcaaa, \text{class } 4 \rangle \quad (7.2241)$$

$$\langle a, b, c \mid cb - \omega caaa, cac - caaa, pa - ba, pb, pc - xcaaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2242)$$

$$\langle a, b, c \mid cb - \omega^2 caaa, cac - caaa, pa - ba, pb, pc - xcaaa, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2243)$$

$$\langle a, b, c \mid cb, cac, pa - ba, pb - caaa, pc, \text{class } 4 \rangle \quad (7.2244)$$

$$\langle a, b, c \mid cb - caaa, cac, pa - ba, pb - caaa, pc, \text{class } 4 \rangle \quad (7.2245)$$

$$\langle a, b, c \mid cb - \omega caaa, cac, pa - ba, pb - caaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2246)$$

$$\langle a, b, c \mid cb - \omega^2 caaa, cac, pa - ba, pb - caaa, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2247)$$

$$\langle a, b, c \mid cb, cac - caaa, pa - ba, pb - caaa, pc, \text{class } 4 \rangle \quad (7.2248)$$

$$\langle a, b, c \mid cb, cac - caaa, pa - ba, pb - xcaaa, pc, \text{class } 4 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \bmod 5) \quad (7.2249)$$

$$\langle a, b, c \mid cb - caaa, cac - caaa, pa - ba, pb - xcaaa, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.2250)$$

$$\langle a, b, c \mid cb - \omega caaa, cac - caaa, pa - ba, pb - xcaaa, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2251)$$

$$\langle a, b, c \mid cb - \omega^2 caaa, cac - caaa, pa - ba, pb - xcaaa, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2252)$$

8.155 Descendants of 6.275

1 algebra

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc - caa, \text{class } 4 \rangle \quad (7.2253)$$

8.156 Descendants of 6.276

1 algebra

$$\langle a, b, c \mid cb, cac, pa - ba, pb, pc - \omega caa, \text{class } 4 \rangle \quad (7.2254)$$

8.157 Descendants of 6.277

p algebras

$$\langle a, b, c \mid cb - caa, cac - xcaaa, pa - ba, pb - caa, pc, \text{class } 4 \rangle \quad (7.2255)$$

8.158 Descendants of 6.278

$(5p + 3)/2$ algebras

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb, pc - xcaa, \text{class } 4 \rangle (x \neq 0, -1, 2, \frac{1}{2}) \quad (7.2256)$$

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb, pc - xcaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.2257)$$

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb - caaa, pc - xcaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.2258)$$

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb, pc + caa, \text{class } 4 \rangle \quad (7.2259)$$

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb - caaa, pc + caa, \text{class } 4 \rangle \quad (7.2260)$$

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb, pc - 2caa - xcaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.2261)$$

$$\langle a, b, c \mid cb - caa, cac, pa - ba, pb, pc - \frac{1}{2}caa, \text{class } 4 \rangle \quad (7.2262)$$

$$\langle a, b, c \mid cb - caa, cac - caaa, pa - ba, pb, pc - \frac{1}{2}caa, \text{class } 4 \rangle \quad (7.2263)$$

8.159 Descendants of 6.279

p algebras

$$\langle a, b, c \mid cb - \omega caa, cac - xcaaa, pa - ba, pb - caa, pc, \text{class } 4 \rangle \quad (7.2264)$$

8.160 Descendants of 6.280

$(5p + 3)/2$ algebras

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb, pc - xcaa, \text{class } 4 \rangle (x \neq 0, -\omega, 2\omega, \frac{\omega}{2}) \quad (7.2265)$$

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb, pc - xcaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.2266)$$

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb - caaa, pc - xcaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.2267)$$

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb, pc + \omega caa, \text{class } 4 \rangle \quad (7.2268)$$

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb - caaa, pc + \omega caa, \text{class } 4 \rangle \quad (7.2269)$$

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb, pc - 2\omega caa - xcaaa, \text{class } 4 \rangle (x \sim -x) \quad (7.2270)$$

$$\langle a, b, c \mid cb - \omega caa, cac, pa - ba, pb, pc - \frac{\omega}{2} caa, \text{class } 4 \rangle \quad (7.2271)$$

$$\langle a, b, c \mid cb - \omega caa, cac - caaa, pa - ba, pb, pc - \frac{\omega}{2} caa, \text{class } 4 \rangle \quad (7.2272)$$

8.161 Descendants of 6.281

$2p^2 + 4p + 4 + 2 \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid cb, caa, pa, pb - ba, pc, \text{class } 4 \rangle \quad (7.2273)$$

$$\langle a, b, c \mid cb, caa, pa - cacc, pb - ba, pc, \text{class } 4 \rangle \quad (7.2274)$$

$$\langle a, b, c \mid cb, caa, pa - \omega cacc, pb - ba, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2275)$$

$$\langle a, b, c \mid cb, caa, pa - \omega^2 cacc, pb - ba, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2276)$$

$$\langle a, b, c \mid cb, caa, pa - xcacc, pb - ba, pc - cacc, \text{class } 4 \rangle (x \sim -x) \quad (7.2277)$$

$$\langle a, b, c \mid cb, caa, pa - xcacc, pb - ba, pc - \omega cacc, \text{class } 4 \rangle (x \sim -x) \quad (7.2278)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - xcacc, pb - ba, pc - ycacc, \text{class } 4 \rangle ((x, y) \sim (-x, y)) \quad (7.2279)$$

$$\langle a, b, c \mid cb, caa - \omega cacc, pa - xcacc, pb - ba, pc - ycacc, \text{class } 4 \rangle ((x, y) \sim (-x, y)) \quad (7.2280)$$

$$\langle a, b, c \mid cb, caa, pa, pb - ba - cacc, pc, \text{class } 4 \rangle \quad (7.2281)$$

$$\langle a, b, c \mid cb, caa, pa - cacc, pb - ba - cacc, pc, \text{class } 4 \rangle \quad (7.2282)$$

$$\langle a, b, c \mid cb, caa, pa - \omega cacc, pb - ba - cacc, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2283)$$

$$\langle a, b, c \mid cb, caa, pa - \omega^2 cacc, pb - ba - cacc, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2284)$$

$$\langle a, b, c \mid cb, caa, pa - xcacc, pb - ba - cacc, pc - cacc, \text{class } 4 \rangle (x \sim -x) \quad (7.2285)$$

$$\langle a, b, c \mid cb, caa, pa - xcacc, pb - ba - cacc, pc - \omega cacc, \text{class } 4 \rangle (x \sim -x) \quad (7.2286)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - xcacc, pb - ba - cacc, pc - ycacc, \text{class } 4 \rangle ((x, y) \sim (-x, y)) \quad (7.2287)$$

$$\langle a, b, c \mid cb, caa - \omega cacc, pa - xcacc, pb - ba - cacc, pc - ycacc, \text{class } 4 \rangle ((x, y) \sim (-x, y)) \quad (7.2288)$$

8.162 Descendants of 6.282

$p^3 + p^2 + 2p + 2 + \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid cb, caa, pa, pb - ba - cac, pc, \text{class } 4 \rangle \quad (7.2289)$$

$$\langle a, b, c \mid cb, caa, pa - cacc, pb - ba - cac, pc, \text{class } 4 \rangle \quad (7.2290)$$

$$\langle a, b, c \mid cb, caa, pa - \omega cacc, pb - ba - cac, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.2291)$$

$$\langle a, b, c \mid cb, caa, pa - \omega^2 cacc, pb - ba - cac, pc, \text{class } 4 \rangle (p = 1 \pmod{3}) \quad (7.2292)$$

$$\langle a, b, c \mid cb, caa, pa - xcacc, pb - ba - cac, pc - cacc, \text{class } 4 \rangle (x \sim -x) \quad (7.2293)$$

$$\langle a, b, c \mid cb, caa, pa - xcacc, pb - ba - cac, pc - \omega cacc, \text{class } 4 \rangle (x \sim -x) \quad (7.2294)$$

$$\langle a, b, c \mid cb, caa - cacc, pa - xcacc, pb - ba - cac, pc - ycacc, \text{class } 4 \rangle ((x, y) \sim (-x, y)) \quad (7.2295)$$

$$\langle a, b, c \mid cb, caa - \omega cacc, pa - xcacc, pb - ba - cac, pc - ycacc, \text{class } 4 \rangle ((x, y) \sim (-x, y)) \quad (7.2296)$$

$$\langle a, b, c \mid cb, caa - xcacc, pa - ycacc, pb - ba - cac - cacc, pc - zcacc, \text{class } 4 \rangle \quad (7.2297)$$

8.163 Descendants of 6.289

$6p$ algebras

$$\langle a, b, c \mid cb, cac, pa, pb - ba, pc - xcaaa, \text{class } 4 \rangle \quad (7.2298)$$

$$\langle a, b, c \mid cb, cac - caaa, pa, pb - ba, pc - xcaaa, \text{class } 4 \rangle \quad (7.2299)$$

$$\langle a, b, c \mid cb - caaa, cac, pa, pb - ba, pc - xcaaa, \text{class } 4 \rangle \quad (7.2300)$$

$$\langle a, b, c \mid cb - caaa, cac - caaa, pa, pb - ba, pc - xcaaa, \text{class } 4 \rangle \quad (7.2301)$$

$$\langle a, b, c \mid cb, cac, pa - caaa, pb - ba, pc, \text{class } 4 \rangle \quad (7.2302)$$

$$\langle a, b, c \mid cb - caaa, cac, pa - caaa, pb - ba, pc, \text{class } 4 \rangle \quad (7.2303)$$

$$\langle a, b, c \mid cb, cac - caaa, pa - xcaaa, pb - ba, pc, \text{class } 4 \rangle (x \neq 0) \quad (7.2304)$$

$$\langle a, b, c \mid cb - caaa, cac - caaa, pa - xcaaa, pb - ba, pc, \text{class } 4 \rangle (x \neq 0) \quad (7.2305)$$

8.164 Descendants of 6.290

$2p^2 + p$ algebras

$$\langle a, b, c \mid cb - caa - xcaaa, cac, pa, pb - ba, pc - ycaaa, \text{class } 4 \rangle \quad (7.2306)$$

$$\langle a, b, c \mid cb - caa - xcaaa, cac, pa - caaa, pb - ba, pc, \text{class } 4 \rangle \quad (7.2307)$$

$$\langle a, b, c \mid cb - caa, cac - caaa, pa - xcaaa, pb - ba, pc - ycaaa, \text{class } 4 \rangle \quad (7.2308)$$

8.165 Descendants of 6.294

$p + 4 + 5 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid cb, baa, pa, pb - ca, pc - babb, \text{class } 4 \rangle \quad (7.2309)$$

$$\langle a, b, c \mid cb, baa, pa, pb - ca, pc - \omega babb, \text{class } 4 \rangle \quad (7.2310)$$

$$\langle a, b, c \mid cb, baa, pa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2311)$$

$$\langle a, b, c \mid cb, baa, pa - babb, pb - ca, pc, \text{class } 4 \rangle \quad (7.2312)$$

$$\langle a, b, c \mid cb, baa, pa - \omega babb, pb - ca, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2313)$$

$$\langle a, b, c \mid cb, baa, pa - \omega^2 babb, pb - ca, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2314)$$

$$\langle a, b, c \mid cb, baa, pa, pb - ca - babb, pc, \text{class } 4 \rangle \quad (7.2315)$$

$$\langle a, b, c \mid cb, baa, pa - babb, pb - ca - babb, pc, \text{class } 4 \rangle \quad (7.2316)$$

$$\langle a, b, c \mid cb, baa, pa - \omega babb, pb - ca - babb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2317)$$

$$\langle a, b, c \mid cb, baa, pa - \omega^2 babb, pb - ca - babb, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2318)$$

$$\langle a, b, c \mid cb, baa - babb, pa, pb - ca, pc - babb, \text{class } 4 \rangle \quad (7.2319)$$

$$\langle a, b, c \mid cb, baa - babb, pa, pb - ca, pc - \omega babb, \text{class } 4 \rangle \quad (7.2320)$$

$$\langle a, b, c \mid cb, baa - babb, pa, pb - ca, pc - x babb, \text{class } 4 \rangle \quad (x = \omega^i, i = 2, 3, 4, 5, p = 1 \pmod{3}) \quad (7.2321)$$

$$\langle a, b, c \mid cb, baa - babb, pa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2322)$$

$$\langle a, b, c \mid cb, baa - babb, pa, pb - ca - babb, pc, \text{class } 4 \rangle \quad (7.2323)$$

$$\langle a, b, c \mid cb, baa - babb, pa, pb - ca - \omega babb, pc, \text{class } 4 \rangle \quad (7.2324)$$

$$\langle a, b, c \mid cb, baa - babb, pa, pb - ca - \omega^2 babb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2325)$$

$$\langle a, b, c \mid cb, baa - babb, pa, pb - ca - \omega^3 babb, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2326)$$

$$\langle a, b, c \mid cb, baa - babb, pa - babb, pb - ca, pc, \text{class } 4 \rangle \quad (7.2327)$$

$$\langle a, b, c \mid cb, baa - babb, pa - \omega babb, pb - ca, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2328)$$

$$\langle a, b, c \mid cb, baa - babb, pa - \omega^2 babb, pb - ca, pc, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.2329)$$

$$\langle a, b, c \mid cb, baa - babb, pa - babb, pb - ca - xbabb, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.2330)$$

$$\langle a, b, c \mid cb, baa - babb, pa - \omega babb, pb - ca - xbabb, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2331)$$

$$\langle a, b, c \mid cb, baa - babb, pa - \omega^2 babb, pb - ca - xbabb, pc, \text{class } 4 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2332)$$

8.166 Descendants of 6.295

$(p + 5)/2$ algebras

$$\langle a, b, c \mid cb, baa, pa - bab - xbabb, pb - ca, pc, \text{class } 4 \rangle (x \sim -x) \quad (7.2333)$$

$$\langle a, b, c \mid cb, baa, pa - bab, pb - ca, pc - babb, \text{class } 4 \rangle \quad (7.2334)$$

$$\langle a, b, c \mid cb, baa, pa - bab, pb - ca, pc - \omega babb, \text{class } 4 \rangle \quad (7.2335)$$

8.167 Descendants of 6.296

$(p + 5)/2$ algebras

$$\langle a, b, c \mid cb, baa, pa - \omega bab - xbabb, pb - ca, pc, \text{class } 4 \rangle (x \sim -x) \quad (7.2336)$$

$$\langle a, b, c \mid cb, baa, pa - \omega bab, pb - ca, pc - babb, \text{class } 4 \rangle \quad (7.2337)$$

$$\langle a, b, c \mid cb, baa, pa - \omega bab, pb - ca, pc - \omega babb, \text{class } 4 \rangle \quad (7.2338)$$

8.168 Descendants of 6.297

$2p + p \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \mid cb, baa, pa, pb - ca - bab, pc - xbabb, \text{class } 4 \rangle \quad (7.2339)$$

$$\langle a, b, c \mid cb, baa, pa - babb, pb - ca - bab, pc - xbabb, \text{class } 4 \rangle \quad (7.2340)$$

$$\langle a, b, c \mid cb, baa, pa - \omega babb, pb - ca - bab, pc - xbabb, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2341)$$

$$\langle a, b, c \mid cb, baa, pa - \omega^2 babb, pb - ca - bab, pc - xbabb, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2342)$$

$$\langle a, b, c \mid cb, baa, pa - xbabb, pb - ca - bab - babb, pc + 2babb, \text{class } 4 \rangle \quad (7.2343)$$

8.169 Descendants of 6.298

$(3p - 1)/2$ algebras

$$\langle a, b, c \mid cb, baa, pa - bab, pb - ca - bab, pc - xbabb, \text{class } 4 \rangle \quad (7.2344)$$

$$\langle a, b, c \mid cb, baa, pa - bab - xbabb, pb - ca - bab, pc - 2babb, \text{class } 4 \rangle \quad (x \sim -x) \quad (7.2345)$$

8.170 Descendants of 6.299

$(3p - 1)/2$ algebras

$$\langle a, b, c \mid cb, baa, pa - \omega bab, pb - ca - bab, pc - xbabb, \text{class } 4 \rangle \quad (7.2346)$$

$$\langle a, b, c \mid cb, baa, pa - \omega bab - xbabb, pb - ca - bab, pc - 2babb, \text{class } 4 \rangle \quad (x \neq 0, x \sim -x) \quad (7.2347)$$

8.171 Descendants of 6.303

$p^2 + p + (p + 1) \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2348)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca - baaa, pc, \text{class } 4 \rangle \quad (7.2349)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca - \omega baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2350)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca - \omega^2 baaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2351)$$

$$\langle a, b, c \mid cb, bab - baa, pa - baaa, pb - ca - xbaaa, pc, \text{class } 4 \rangle \quad (7.2352)$$

$$\langle a, b, c \mid cb, bab - baa, pa - \omega baaa, pb - ca - xbaaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2353)$$

$$\langle a, b, c \mid cb, bab - baa, pa - \omega^2 baaa, pb - ca - xbaaa, pc, \text{class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.2354)$$

$$\langle a, b, c \mid cb, bab - baa - baaa, pa - xbaaa, pb - ca - ybaaa, pc, \text{class } 4 \rangle \quad (7.2355)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc - baaa, \text{class } 4 \rangle \quad (7.2356)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc - \omega baaa, \text{class } 4 \rangle \quad (7.2357)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc - \omega^2 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.2358)$$

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca, pc - \omega^3 baaa, \text{class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.2359)$$

$$\langle a, b, c \mid cb, bab - baa - baaa, pa, pb - ca, pc - xbaaa, \text{class } 4 \rangle \quad (x \neq 0) \quad (7.2360)$$

8.172 Descendants of 6.304

$p(p+1)/2$ algebras

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca - baa - ybaaa, pc - xbaaa, \text{class } 4 \rangle \quad (x \neq 0, (x, y) \sim (x, -y)) \quad (7.2361)$$

$$\langle a, b, c \mid cb, bab - baa, pa - xbaaa, pb - ca - baa, pc, \text{class } 4 \rangle \quad (x \sim -x) \quad (7.2362)$$

8.173 Descendants of 6.305

$p(p+1)/2$ algebras

$$\langle a, b, c \mid cb, bab - baa, pa, pb - ca - \omega baa - ybaaa, pc - xbaaa, \text{class } 4 \rangle \quad (x \neq 0, (x, y) \sim (x, -y)) \quad (7.2363)$$

$$\langle a, b, c \mid cb, bab - baa, pa - xbaaa, pb - ca - \omega baa, pc, \text{class } 4 \rangle \quad (x \sim -x) \quad (7.2364)$$

8.174 Descendants of 6.312

$4p + 2 + 2 \gcd(p-1, 3) + 4 \gcd(p-1, 4)$ algebras

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2365)$$

$$\langle a, b, c \mid cb, bab, pa - baaa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2366)$$

$$\langle a, b, c \mid cb - baaa, bab, pa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2367)$$

$$\langle a, b, c \mid cb - baaa, bab, pa - baaa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2368)$$

$$\langle a, b, c \mid cb - baaa, bab, pa - \omega baaa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2369)$$

$$\langle a, b, c \mid cb - baaa, bab, pa - xbaaa, pb - ca, pc, \text{class } 4 \rangle \quad (x = \omega^i, i = 2, 3, 4, 5, p = 1 \pmod{3}) \quad (7.2370)$$

$$\langle a, b, c \mid cb, bab - baaa, pa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2371)$$

$$\langle a, b, c \mid cb, bab - baaa, pa - baaa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2372)$$

$$\langle a, b, c \mid cb, bab - baaa, pa - \omega baaa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2373)$$

$$\langle a, b, c \mid cb, bab - baaa, pa - \omega^2baaa, pb - ca, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2374)$$

$$\langle a, b, c \mid cb, bab - baaa, pa - \omega^3baaa, pb - ca, pc, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2375)$$

$$\langle a, b, c \mid cb - baaa, bab - baaa, pa - xbaaa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2376)$$

$$\langle a, b, c \mid cb - \omega baaa, bab - baaa, pa - xbaaa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2377)$$

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc - baaa, \text{class } 4 \rangle \quad (7.2378)$$

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc - \omega baaa, \text{class } 4 \rangle \quad (7.2379)$$

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc - \omega^2baaa, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2380)$$

$$\langle a, b, c \mid cb, bab, pa, pb - ca, pc - \omega^3baaa, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2381)$$

$$\langle a, b, c \mid cb - baaa, bab, pa, pb - ca, pc - baaa, \text{class } 4 \rangle \quad (7.2382)$$

$$\langle a, b, c \mid cb - baaa, bab, pa, pb - ca, pc - \omega baaa, \text{class } 4 \rangle \quad (7.2383)$$

$$\langle a, b, c \mid cb - baaa, bab, pa, pb - ca, pc - \omega^2baaa, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2384)$$

$$\langle a, b, c \mid cb - baaa, bab, pa, pb - ca, pc - \omega^3baaa, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2385)$$

$$\langle a, b, c \mid cb, bab - baaa, pa, pb - ca, pc - baaa, \text{class } 4 \rangle \quad (7.2386)$$

$$\langle a, b, c \mid cb, bab - baaa, pa, pb - ca, pc - \omega baaa, \text{class } 4 \rangle \quad (7.2387)$$

$$\langle a, b, c \mid cb, bab - baaa, pa, pb - ca, pc - \omega^2baaa, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2388)$$

$$\langle a, b, c \mid cb, bab - baaa, pa, pb - ca, pc - \omega^3baaa, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.2389)$$

$$\langle a, b, c \mid cb - baaa, bab - baaa, pa, pb - ca, pc - xbaaa, \text{class } 4 \rangle (x \neq 0) \quad (7.2390)$$

$$\langle a, b, c \mid cb - \omega baaa, bab - baaa, pa, pb - ca, pc - xbaaa, \text{class } 4 \rangle (x \neq 0) \quad (7.2391)$$

8.175 Descendants of 6.313

$p + \gcd(p-1, 4) + \gcd(p-1, 5)$ algebras

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2392)$$

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc - baaa, \text{class } 4 \rangle \quad (7.2393)$$

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc - \omega baaa, \text{class } 4 \rangle \quad (7.2394)$$

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc - \omega^2 baaa, \text{class } 4 \rangle \ (p = 1 \bmod 4) \quad (7.2395)$$

$$\langle a, b, c \mid cb - baa, bab, pa, pb - ca, pc - \omega^3 baaa, \text{class } 4 \rangle \ (p = 1 \bmod 4) \quad (7.2396)$$

$$\langle a, b, c \mid cb - baa, bab, pa - baaa, pb - ca, pc, \text{class } 4 \rangle \quad (7.2397)$$

$$\langle a, b, c \mid cb - baa, bab, pa - xbaaa, pb - ca, pc, \text{class } 4 \rangle \ (x = \omega^i, i = 1, 2, 3, 4, p = 1 \bmod 5) \quad (7.2398)$$

$$\langle a, b, c \mid cb - baa, bab, pa - baaa, pb - ca, pc - xbaaa, \text{class } 4 \rangle \ (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.2399)$$

$$\langle a, b, c \mid cb - baa, bab, pa - ybaaa, pb - ca, pc - xbaaa, \text{class } 4 \rangle \ (x \neq 0, y = \omega^i, i = 1, 2, 3, 4, (x, y) \sim (ax, y) \text{ if } a^5 = 1, p = 1 \bmod 5) \quad (7.2400)$$

8.176 Descendants of 6.322

3 algebras

$$\langle a, b, c \mid ba, ca, cb, pb, pc, \text{class } 5 \rangle \quad (7.2401)$$

$$\langle a, b, c \mid ba - p^4 a, ca, cb, pb, pc, \text{class } 5 \rangle \quad (7.2402)$$

$$\langle a, b, c \mid ba, ca, cb - p^4 a, pb, pc, \text{class } 5 \rangle \quad (7.2403)$$

8.177 Descendants of 6.325

$19 + 5 \gcd(p-1, 3) + 6 \gcd(p-1, 4)$ algebras

$$\langle a, b, c \mid baaaa, bab, ca, cb, pa, pb, pc, \text{class } 5 \rangle \quad (7.2404)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb, pa - baaab, pb, pc, \text{class } 5 \rangle \quad (7.2405)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb, pa - \omega baaab, pb, pc, \text{class } 5 \rangle \quad (7.2406)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb, pa, pb - baaab, pc, \text{class } 5 \rangle \quad (7.2407)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb, pa - baaab, pb - baaab, pc, \text{class } 5 \rangle \quad (7.2408)$$

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- (7.2430)

$$\langle a, b, c \mid baaab, bab, ca, cb - baaaa, pa, pb - baaaa, pc, \text{ class } 5 \rangle \quad (7.2431)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaaa, pa, pb - \omega baaaa, pc, \text{ class } 5 \rangle \quad (7.2432)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaaa, pa, pb - \omega^2 baaaa, pc, \text{ class } 5 \rangle \quad (p = 1 \bmod 4) \quad (7.2433)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaaa, pa, pb - \omega^3 baaaa, pc, \text{ class } 5 \rangle \quad (p = 1 \bmod 4) \quad (7.2434)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaaa, pa, pb, pc - baaaa, \text{ class } 5 \rangle \quad (7.2435)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa, pb, pc, \text{ class } 5 \rangle \quad (7.2436)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa - baaaa, pb, pc, \text{ class } 5 \rangle \quad (7.2437)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa - \omega baaaa, pb, pc, \text{ class } 5 \rangle \quad (7.2438)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa - xbaaaa, pb, pc, \text{ class } 5 \rangle \quad (x = \omega^i, i = 2, 3, 4, 5, p = 1 \bmod 3) \quad (7.2439)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa, pb - baaaa, pc, \text{ class } 5 \rangle \quad (7.2440)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa, pb - \omega baaaa, pc, \text{ class } 5 \rangle \quad (7.2441)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa, pb - \omega^2 baaaa, pc, \text{ class } 5 \rangle \quad (p = 1 \bmod 4) \quad (7.2442)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa, pb - \omega^3 baaaa, pc, \text{ class } 5 \rangle \quad (p = 1 \bmod 4) \quad (7.2443)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb, pa, pb, pc - baaaa, \text{ class } 5 \rangle \quad (7.2444)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa, pb, pc, \text{ class } 5 \rangle \quad (7.2445)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa - baaaa, pb, pc, \text{ class } 5 \rangle \quad (7.2446)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa - \omega baaaa, pb, pc, \text{ class } 5 \rangle \quad (7.2447)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa - xbaaaa, pb, pc, \text{ class } 5 \rangle \quad (x = \omega^i, i = 2, 3, 4, 5, p = 1 \bmod 3) \quad (7.2448)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa, pb - baaaa, pc, \text{ class } 5 \rangle \quad (7.2449)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa, pb - \omega baaaa, pc, \text{ class } 5 \rangle \quad (7.2450)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa, pb - \omega^2 baaaa, pc, \text{ class } 5 \rangle \quad (p = 1 \bmod 4) \quad (7.2451)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa, pb - \omega^3 baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2452)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa, pb, pc - baaaa, \text{class } 5 \rangle \quad (7.2453)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa, pb, pc - \omega baaaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.2454)$$

$$\langle a, b, c \mid baaab, bab - baaaa, ca, cb - baaaa, pa, pb, pc - \omega^2 baaaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.2455)$$

8.178 Descendants of 6.326

$4p + 5 + 4 \gcd(p - 1, 3) + 2 \gcd(p - 1, 4) + 4 \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa, pb, pc, \text{class } 5 \rangle \quad (7.2456)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa - baaab, pb, pc, \text{class } 5 \rangle \quad (7.2457)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa - \omega baaab, pb, pc, \text{class } 5 \rangle \quad (7.2458)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa - xbaaab, pb, pc, \text{class } 5 \rangle (x = \omega^i, i = 2, 3, 4, 5, p = 1 \bmod 3) \quad (7.2459)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa, pb - baaab, pc, \text{class } 5 \rangle \quad (7.2460)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa, pb - xbaaab, pc, \text{class } 5 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \bmod 5) \quad (7.2461)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa - xbaaab, pb - baaab, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.2462)$$

In the following presentation we have $x \neq 0$, $y = \omega^i$, $i = 1, 2, 3, 4$, $(x, y) \sim (ax, y)$ if $a^5 = 1$, $p = 1 \bmod 5$:

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa - xbaaab, pb - ybaaab, pc, \text{class } 5 \rangle \quad (7.2463)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb, pa, pb, pc - baaab, \text{class } 5 \rangle \quad (7.2464)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa, pb, pc, \text{class } 5 \rangle \quad (7.2465)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa - baaab, pb, pc, \text{class } 5 \rangle \quad (7.2466)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa - \omega baaab, pb, pc, \text{class } 5 \rangle \quad (7.2467)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa - xbaaab, pb, pc, \text{class } 5 \rangle (x = \omega^i, i = 2, 3, 4, 5, p = 1 \bmod 3) \quad (7.2468)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa, pb - baaab, pc, \text{class } 5 \rangle \quad (7.2469)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa, pb - xbaaab, pc, \text{class } 5 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \bmod 5) \quad (7.2470)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa - xbaaab, pb - baaab, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.2471)$$

In the following presentation we have $x \neq 0$, $y = \omega^i$, $i = 1, 2, 3, 4$, $(x, y) \sim (ax, y)$ if $a^5 = 1$, $p = 1 \pmod{5}$:

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa - xbaaab, pb - ybaaab, pc, \text{class } 5 \rangle \quad (7.2472)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb, pa, pb, pc - baaab, \text{class } 5 \rangle \quad (7.2473)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa, pb, pc, \text{class } 5 \rangle \quad (7.2474)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa - baaaa, pb, pc, \text{class } 5 \rangle \quad (7.2475)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa - xbaaaa, pb, pc, \text{class } 5 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \pmod{5}) \quad (7.2476)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa, pb - baaaa, pc, \text{class } 5 \rangle \quad (7.2477)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa, pb - \omega baaaa, pc, \text{class } 5 \rangle \quad (7.2478)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa, pb - \omega^2 baaaa, pc, \text{class } 5 \rangle (p = 1 \pmod{4}) \quad (7.2479)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa, pb - \omega^3 baaaa, pc, \text{class } 5 \rangle (p = 1 \pmod{4}) \quad (7.2480)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa - xbaaaa, pb - baaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.2481)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa - xbaaaa, pb - \omega baaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.2482)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa - xbaaaa, pb - \omega^2 baaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.2483)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa - xbaaaa, pb - \omega^3 baaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \pmod{4}) \quad (7.2484)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb, pa, pb, pc - baaaa, \text{class } 5 \rangle \quad (7.2485)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa, pb, pc, \text{class } 5 \rangle \quad (7.2486)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa - baaaa, pb, pc, \text{class } 5 \rangle \quad (7.2487)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa - xbaaaa, pb, pc, \text{class } 5 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \pmod{5}) \quad (7.2488)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa, pb - baaaa, pc, \text{class } 5 \rangle \quad (7.2489)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa, pb - \omega baaaa, pc, \text{class } 5 \rangle \quad (7.2490)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa, pb - \omega^2 baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2491)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa, pb - \omega^3 baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2492)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa - xbaaaa, pb - baaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.2493)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa - xbaaaa, pb - \omega baaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.2494)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa - xbaaaa, pb - \omega^2 baaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.2495)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa - xbaaaa, pb - \omega^3 baaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.2496)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa, pb, pc - baaaa, \text{class } 5 \rangle \quad (7.2497)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaaa, pa, pb, pc - \omega baaaa, \text{class } 5 \rangle \quad (7.2498)$$

8.179 Descendants of 6.327

$3p + 12 + 2 \gcd(p - 1, 3) + 3 \gcd(p - 1, 4) + \gcd(p - 1, 7)$ algebras

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa, pb, pc, \text{class } 5 \rangle \quad (7.2499)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa - baaab, pb, pc, \text{class } 5 \rangle \quad (7.2500)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa - \omega baaab, pb, pc, \text{class } 5 \rangle \quad (7.2501)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa, pb - baaab, pc, \text{class } 5 \rangle \quad (7.2502)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa - baaab, pb - baaab, pc, \text{class } 5 \rangle \quad (7.2503)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa - \omega baaab, pb - baaab, pc, \text{class } 5 \rangle \quad (7.2504)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa - \omega^2 baaab, pb - baaab, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2505)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa - \omega^3 baaab, pb - baaab, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2506)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa, pb, pc - baaab, \text{class } 5 \rangle \quad (7.2507)$$

$$\langle a, b, c \mid baaaa, bab, ca, cb - baaa, pa, pb, pc - \omega baaab, \text{class } 5 \rangle \quad (7.2508)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa, pb, pc, \text{ class } 5 \rangle \quad (7.2509)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa - baaab, pb, pc, \text{ class } 5 \rangle \quad (7.2510)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa - \omega baaab, pb, pc, \text{ class } 5 \rangle \quad (7.2511)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa - xbaaab, pb, pc, \text{ class } 5 \rangle (x = \omega^i, i = 2, 3, 4, 5, p = 1 \pmod{3}) \quad (7.2512)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa, pb - baaab, pc, \text{ class } 5 \rangle \quad (7.2513)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa, pb - xbaaab, pc, \text{ class } 5 \rangle (x = \omega^i, i = 1, 2, 3, 4, 5, 6, p = 1 \pmod{7}) \quad (7.2514)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa - xbaaab, pb - baaab, pc, \text{ class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^7 = 1) \quad (7.2515)$$

In the following presentation we have $x \neq 0$, $y = \omega^i$, $i = 1, 2, 3, 4, 5, 6$, $((x, y) \sim (ax, y) \text{ if } a^7 = 1, p = 1 \pmod{7})$:

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa - xbaaab, pb - ybaaab, pc, \text{ class } 5 \rangle \quad (7.2516)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa, pb, pc - baaab, \text{ class } 5 \rangle \quad (7.2517)$$

$$\langle a, b, c \mid baaaa, bab, ca - baaab, cb - baaa, pa, pb, pc - \omega baaab, \text{ class } 5 \rangle \quad (7.2518)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb, pc, \text{ class } 5 \rangle \quad (7.2519)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb - baaaa, pc, \text{ class } 5 \rangle \quad (7.2520)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb - \omega baaaa, pc, \text{ class } 5 \rangle \quad (7.2521)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb - \omega^2 baaaa, pc, \text{ class } 5 \rangle (p = 1 \pmod{4}) \quad (7.2522)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb - \omega^3 baaaa, pc, \text{ class } 5 \rangle (p = 1 \pmod{4}) \quad (7.2523)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb, pc - baaaa, \text{ class } 5 \rangle \quad (7.2524)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb - baaaa, pc - baaaa, \text{ class } 5 \rangle \quad (7.2525)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb - \omega baaaa, pc - baaaa, \text{ class } 5 \rangle \quad (7.2526)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb - \omega^2 baaaa, pc - baaaa, \text{ class } 5 \rangle (p = 1 \pmod{4}) \quad (7.2527)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa, pb - \omega^3 baaaa, pc - baaaa, \text{ class } 5 \rangle (p = 1 \pmod{4}) \quad (7.2528)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa, pa - baaaa, pb, pc, \text{ class } 5 \rangle \quad (7.2529)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa - baaaa, pa, pb - xbaaaa, pc, \text{ class } 5 \rangle \quad (7.2530)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa - baaaa, pa, pb - xbaaaa, pc - baaaa, \text{ class } 5 \rangle \quad (7.2531)$$

$$\langle a, b, c \mid baaab, bab, ca, cb - baaa - baaaa, pa - baaaa, pb, pc, \text{ class } 5 \rangle \quad (7.2532)$$

8.180 Descendants of 6.328

$p^2 + 3p - 2 + (p + 3) \gcd(p - 1, 3) + 2 \gcd(p - 1, 4) + 2 \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa, pb, pc, \text{ class } 5 \rangle \quad (7.2533)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa - baaab, pb, pc, \text{ class } 5 \rangle \quad (7.2534)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa - \omega baaab, pb, pc, \text{ class } 5 \rangle \quad (7.2535)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa - xbaaab, pb, pc, \text{ class } 5 \rangle (x = \omega^i, i = 2, 3, 4, 5, p = 1 \pmod{3}) \quad (7.2536)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa, pb - baaab, pc, \text{ class } 5 \rangle \quad (7.2537)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa, pb - xbaaab, pc, \text{ class } 5 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \pmod{5}) \quad (7.2538)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa - xbaaab, pb - baaab, pc, \text{ class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.2539)$$

In the following presentation we have $x \neq 0$, $y = \omega^i$, $i = 1, 2, 3, 4$, $(x, y) \sim (ax, y)$ if $a^5 = 1$, $p = 1 \pmod{5}$:

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa - xbaaab, pb - ybaaab, pc, \text{ class } 5 \rangle \quad (7.2540)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa, pb, pc - baaab, \text{ class } 5 \rangle \quad (7.2541)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa, pb, pc - \omega baaab, \text{ class } 5 \rangle \quad (7.2542)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa, pb, pc - \omega^2 baaab, \text{ class } 5 \rangle (p = 1 \pmod{4}) \quad (7.2543)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca, cb - baaa, pa, pb, pc - \omega^3 baaab, \text{ class } 5 \rangle (p = 1 \pmod{4}) \quad (7.2544)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb - baaa, pa - xbaaab, pb, pc, \text{ class } 5 \rangle \quad (7.2545)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb - baaa, pa - xbaaab, pb - ybaaab, pc, \text{ class } 5 \rangle (y \neq 0, (x, y) \sim (x, ay) \text{ if } a^3 = 1) \quad (7.2546)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - baaab, cb - baaa, pa, pb, pc - xbaaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.2547)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - \omega baaab, cb - baaa, pa - xbaaab, pb, pc, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.2548)$$

In the next presentation we have $y \neq 0$, $(x, y) \sim (x, ay)$ if $a^3 = 1$, $p = 1 \bmod 3$:

$$\langle a, b, c \mid baaaa, bab - baaa, ca - \omega baaab, cb - baaa, pa - xbaaab, pb - ybaaab, pc, \text{class } 5 \rangle \quad (7.2549)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - \omega baaab, cb - baaa, pa, pb, pc - xbaaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2550)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - \omega^2 baaab, cb - baaa, pa - xbaaab, pb, pc, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.2551)$$

In the next presentation we have $y \neq 0$, $(x, y) \sim (x, ay)$ if $a^3 = 1$, $p = 1 \bmod 3$:

$$\langle a, b, c \mid baaaa, bab - baaa, ca - \omega^2 baaab, cb - baaa, pa - xbaaab, pb - ybaaab, pc, \text{class } 5 \rangle \quad (7.2552)$$

$$\langle a, b, c \mid baaaa, bab - baaa, ca - \omega^2 baaab, cb - baaa, pa, pb, pc - xbaaab, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2553)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb, pc, \text{class } 5 \rangle \quad (7.2554)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb - baaaa, pc, \text{class } 5 \rangle \quad (7.2555)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb - \omega baaaa, pc, \text{class } 5 \rangle \quad (7.2556)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb - \omega^2 baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2557)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb - \omega^3 baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2558)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa - baaaa, pb, pc, \text{class } 5 \rangle \quad (7.2559)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa - xbaaaa, pb, pc, \text{class } 5 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \bmod 5) \quad (7.2560)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa - baaaa, pb - xbaaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^5 = 1) \quad (7.2561)$$

In the next presentation we have $x \neq 0$, $y = \omega^i$, $i = 1, 2, 3, 4$, $(x, y) \sim (ax, y)$ if $a^5 = 1$, $p = 1 \bmod 5$:

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa - ybaaaa, pb - xbaaaa, pc, \text{class } 5 \rangle \quad (7.2562)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb, pc - baaaa, \text{class } 5 \rangle \quad (7.2563)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb, pc - \omega baaaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.2564)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb, pc - \omega^2baaaa, \text{class } 5 \rangle (p = 1 \bmod 3) \quad (7.2565)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb - xbaaaa, pc - baaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.2566)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb - xbaaaa, pc - \omega baaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2567)$$

$$\langle a, b, c \mid baaab, bab - baaa, ca, cb - baaa, pa, pb - xbaaaa, pc - \omega^2baaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.2568)$$

8.181 Descendants of 6.362

$p^2 + 7p + 3 + 2 \gcd(p - 1, 3) + 3 \gcd(p - 1, 4) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, pc, \text{class } 5 \rangle \quad (7.2569)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - baaaa, pc, \text{class } 5 \rangle \quad (7.2570)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega baaaa, pc, \text{class } 5 \rangle \quad (7.2571)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega^2baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2572)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega^3baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2573)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, pc - baaaa, \text{class } 5 \rangle \quad (7.2574)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - baaaa, pc - baaaa, \text{class } 5 \rangle \quad (7.2575)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega baaaa, pc - baaaa, \text{class } 5 \rangle \quad (7.2576)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega^2baaaa, pc - baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2577)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa, pb - \omega^3baaaa, pc - baaaa, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2578)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa - baaaa, pb, pc, \text{class } 5 \rangle \quad (7.2579)$$

$$\langle a, b, c \mid bab, ca, cb - baa, pa - baaaa, pb, pc - baaaa, \text{class } 5 \rangle \quad (7.2580)$$

$$\langle a, b, c \mid bab, ca, cb - baa - baaaa, pa, pb - xbaaaa, pc, \text{class } 5 \rangle \quad (7.2581)$$

$$\langle a, b, c \mid bab, ca, cb - baa - baaaa, pa, pb - xbaaaa, pc - baaaa, \text{class } 5 \rangle \quad (7.2582)$$

$$\langle a, b, c \mid bab, ca, cb - baa - baaaa, pa - baaaa, pb, pc, \text{class } 5 \rangle \quad (7.2583)$$

$$\langle a, b, c \mid bab, ca, cb - baa - baaaa, pa - xbaaaa, pb, pc - baaaa, \text{class } 5 \rangle (x \neq 0, x \sim -x) \quad (7.2584)$$

$$\langle a, b, c \mid bab, ca, cb - baa - \omega baaaa, pa, pb - xbaaaa, pc, \text{class } 5 \rangle \quad (7.2585)$$

$$\langle a, b, c \mid bab, ca, cb - baa - \omega baaaa, pa, pb - xbaaaa, pc - baaaa, \text{class } 5 \rangle \quad (7.2586)$$

$$\langle a, b, c \mid bab, ca, cb - baa - \omega baaaa, pa - baaaa, pb, pc, \text{class } 5 \rangle \quad (7.2587)$$

$$\langle a, b, c \mid bab, ca, cb - baa - \omega baaaa, pa - xbaaaa, pb, pc - baaaa, \text{class } 5 \rangle (x \neq 0, x \sim -x) \quad (7.2588)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb, pc, \text{class } 5 \rangle \quad (7.2589)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb, pc - baaaa, \text{class } 5 \rangle \quad (7.2590)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb, pc - xbaaaa, \text{class } 5 \rangle (x = \omega^i, i = 1, 2, 3, 4, p = 1 \bmod 5) \quad (7.2591)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb - baaaa, pc, \text{class } 5 \rangle \quad (7.2592)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb - \omega baaaa, pc, \text{class } 5 \rangle \quad (7.2593)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb - \omega^2 baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2594)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb - \omega^3 baaaa, pc, \text{class } 5 \rangle (p = 1 \bmod 4) \quad (7.2595)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb - baaaa, pc - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1) \quad (7.2596)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb - \omega baaaa, pc - xbaaaa, \text{class } 5 \rangle ((x \neq 0, x \sim ax \text{ if } a^4 = 1)) \quad (7.2597)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb - \omega^2 baaaa, pc - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.2598)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa, pb - \omega^3 baaaa, pc - xbaaaa, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^4 = 1, p = 1 \bmod 4) \quad (7.2599)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - baaaa, pb, pc, \text{class } 5 \rangle \quad (7.2600)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - \omega baaaa, pb, pc, \text{class } 5 \rangle \quad (7.2601)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - xbaaaa, pb, pc, \text{class } 5 \rangle (x = \omega^i, i = 2, 3, 4, 5, p = 1 \bmod 3) \quad (7.2602)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - baaaa, pb - xbaaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.2603)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - \omega baaaa, pb - xbaaaa, pc, \text{class } 5 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.2604)$$

In the next presentation we have $x \neq 0$, $y = \omega^i$, $i = 2, 3, 4, 5$, $(x, y) \sim (ax, y)$ if $a^3 = 1$, $p = 1 \pmod{3}$:

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - ybaaaa, pb - xbaaaa, pc, \text{class } 5 \rangle \quad (7.2605)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - baaaa, pb - ybaaaa, pc - xbaaaa, \text{class } 5 \rangle (x \neq 0, (x, y) \sim (ax, a^2y) \text{ if } a^6 = 1) \quad (7.2606)$$

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - \omega baaaa, pb - ybaaaa, pc - xbaaaa, \text{class } 5 \rangle (x \neq 0, (x, y) \sim (ax, a^2y) \text{ if } a^6 = 1) \quad (7.2607)$$

In the next presentation we have $x \neq 0$, $y = \omega^i$, $i = 2, 3, 4, 5$, $(x, y, z) \sim (ax, y, a^2z)$ if $a^6 = 1$, $p = 1 \pmod{3}$:

$$\langle a, b, c \mid bab - baaaa, ca, cb - baa, pa - ybaaaa, pb - zbaaaa, pc - xbaaaa, \text{class } 5 \rangle \quad (7.2608)$$

8.182 Descendants of 6.85

3 algebras

$$\langle a, b, c \mid ba, ca, cb, p^2b, p^2c, \text{class } 3 \rangle \quad (7.2609)$$

$$\langle a, b, c \mid ba, ca - p^2a, cb, p^2b, p^2c, \text{class } 3 \rangle \quad (7.2610)$$

$$\langle a, b, c \mid ba, ca, cb - p^2a, p^2b, p^2c, \text{class } 3 \rangle \quad (7.2611)$$

8.183 Descendants of 6.86

$p + 17$ algebras

$$\langle a, b, c \mid baa, bab, p^2a, p^2b, ca, cb, pc, \text{class } 3 \rangle \quad (7.2612)$$

$$\langle a, b, c \mid baa, bab, pba, p^2a, ca, cb, pc, \text{class } 3 \rangle \quad (7.2613)$$

$$\langle a, b, c \mid baa, bab, pba, p^2a, ca, cb - p^2b, pc, \text{class } 3 \rangle \quad (7.2614)$$

$$\langle a, b, c \mid baa, bab, pba, p^2a, ca - p^2b, cb, pc, \text{class } 3 \rangle \quad (7.2615)$$

$$\langle a, b, c \mid baa, bab, p^2a, p^2b - pba, ca, cb, pc, \text{class } 3 \rangle \quad (7.2616)$$

$$\langle a, b, c \mid baa, pba, p^2a, p^2b, ca, cb, pc, \text{class } 3 \rangle \quad (7.2617)$$

$$\langle a, b, c \mid baa, pba, p^2a, p^2b, ca, cb, pc - bab, \text{class } 3 \rangle \quad (7.2618)$$

$$\langle a, b, c \mid baa, pba, p^2a, p^2b, ca - bab, cb, pc, \text{class } 3 \rangle \quad (7.2619)$$

$$\langle a, b, c \mid baa, pba, p^2a, p^2b, ca - bab, cb, pc - bab, \text{class } 3 \rangle \quad (7.2620)$$

$$\langle a, b, c \mid baa, pba, p^2a, p^2b - bab, ca, cb, pc, \text{ class } 3 \rangle \quad (7.2621)$$

$$\langle a, b, c \mid baa, pba, p^2a, p^2b - bab, ca - bab, cb, pc, \text{ class } 3 \rangle \quad (7.2622)$$

$$\langle a, b, c \mid baa, pba, p^2a - bab, p^2b, ca, cb, pc, \text{ class } 3 \rangle \quad (7.2623)$$

$$\langle a, b, c \mid baa, pba, p^2a - bab, p^2b, ca - bab, cb, pc, \text{ class } 3 \rangle \quad (7.2624)$$

$$\langle a, b, c \mid baa, pba, p^2a - \omega bab, p^2b, ca, cb, pc, \text{ class } 3 \rangle \quad (7.2625)$$

$$\langle a, b, c \mid baa, pba, p^2a - \omega bab, p^2b, ca - bab, cb, pc, \text{ class } 3 \rangle \quad (7.2626)$$

$$\langle a, b, c \mid baa, pba - bab, p^2a - xbab, p^2b, ca, cb, pc, \text{ class } 3 \rangle \quad (7.2627)$$

$$\langle a, b, c \mid baa, pba - bab, p^2a + bab, p^2b, ca, cb, pc - bab, \text{ class } 3 \rangle \quad (7.2628)$$

$$\langle a, b, c \mid baa, pba - bab, p^2a, p^2b - bab, ca, cb, pc, \text{ class } 3 \rangle \quad (7.2629)$$

8.184 Descendants of 6.87

5 algebras

$$\langle a, b, c \mid p^2a, p^2b, ca, cb, pc - ba, \text{ class } 3 \rangle \quad (7.2630)$$

$$\langle a, b, c \mid p^2a, pba, ca, cb, pc - ba, \text{ class } 3 \rangle \quad (7.2631)$$

$$\langle a, b, c \mid p^2a, pba, ca, cb - p^2b, pc - ba, \text{ class } 3 \rangle \quad (7.2632)$$

$$\langle a, b, c \mid p^2a, pba, ca - p^2b, cb, pc - ba, \text{ class } 3 \rangle \quad (7.2633)$$

$$\langle a, b, c \mid p^2a, pba, ca - \omega * p^2b, cb, pc - ba, \text{ class } 3 \rangle \quad (7.2634)$$

8.185 Descendants of 6.88

26 algebras

$$\langle a, b, c \mid baa, bab, p^2b, ca, cb, pa, \text{ class } 3 \rangle \quad (7.2635)$$

$$\langle a, b, c \mid baa, bab, p^2b, ca, cb - p^2c, pa, \text{ class } 3 \rangle \quad (7.2636)$$

$$\langle a, b, c \mid baa, bab, p^2b, ca - p^2c, cb, pa, \text{ class } 3 \rangle \quad (7.2637)$$

$$\langle a, b, c \mid baa, bab, p^2c, ca, cb, pa, \text{ class } 3 \rangle \quad (7.2638)$$

$$\langle a, b, c \mid baa, bab, p^2c, ca, cb - p^2b, pa, \text{ class } 3 \rangle \quad (7.2639)$$

$$\langle a, b, c \mid baa, bab, p^2c, ca - p^2b, cb, pa, \text{class } 3 \rangle \quad (7.2640)$$

$$\langle a, b, c \mid baa, p^2b, p^2c, ca, cb, pa, \text{class } 3 \rangle \quad (7.2641)$$

$$\langle a, b, c \mid baa, p^2b, p^2c, ca - bab, cb, pa, \text{class } 3 \rangle \quad (7.2642)$$

$$\langle a, b, c \mid baa, p^2b, p^2c, ca, cb, pa - bab, \text{class } 3 \rangle \quad (7.2643)$$

$$\langle a, b, c \mid baa, p^2b, p^2c, ca - bab, cb, pa - bab, \text{class } 3 \rangle \quad (7.2644)$$

$$\langle a, b, c \mid baa, p^2b, p^2c, ca, cb, pa - \omega bab, \text{class } 3 \rangle \quad (7.2645)$$

$$\langle a, b, c \mid baa, p^2b, p^2c, ca - bab, cb, pa - \omega bab, \text{class } 3 \rangle \quad (7.2646)$$

$$\langle a, b, c \mid baa, p^2b, p^2c - bab, ca, cb, pa, \text{class } 3 \rangle \quad (7.2647)$$

$$\langle a, b, c \mid baa, p^2b, p^2c - bab, ca - bab, cb, pa, \text{class } 3 \rangle \quad (7.2648)$$

$$\langle a, b, c \mid baa, p^2b - bab, p^2c, ca, cb, pa, \text{class } 3 \rangle \quad (7.2649)$$

$$\langle a, b, c \mid baa, p^2b - bab, p^2c, ca - bab, cb, pa, \text{class } 3 \rangle \quad (7.2650)$$

$$\langle a, b, c \mid bab, p^2b, p^2c, ca, cb, pa, \text{class } 3 \rangle \quad (7.2651)$$

$$\langle a, b, c \mid bab, p^2b, p^2c, ca, cb, pa - baa, \text{class } 3 \rangle \quad (7.2652)$$

$$\langle a, b, c \mid bab, p^2b, p^2c, ca, cb - baa, pa, \text{class } 3 \rangle \quad (7.2653)$$

$$\langle a, b, c \mid bab, p^2b, p^2c, ca, cb - baa, pa - baa, \text{class } 3 \rangle \quad (7.2654)$$

$$\langle a, b, c \mid bab, p^2b, p^2c - baa, ca, cb, pa, \text{class } 3 \rangle \quad (7.2655)$$

$$\langle a, b, c \mid bab, p^2b, p^2c - baa, ca, cb - baa, pa, \text{class } 3 \rangle \quad (7.2656)$$

$$\langle a, b, c \mid bab, p^2b - baa, p^2c, ca, cb, pa, \text{class } 3 \rangle \quad (7.2657)$$

$$\langle a, b, c \mid bab, p^2b - baa, p^2c, ca, cb - baa, pa, \text{class } 3 \rangle \quad (7.2658)$$

$$\langle a, b, c \mid bab, p^2b - \omega baa, p^2c, ca, cb, pa, \text{class } 3 \rangle \quad (7.2659)$$

$$\langle a, b, c \mid bab, p^2b - \omega baa, p^2c, ca, cb - baa, pa, \text{class } 3 \rangle \quad (7.2660)$$

8.186 Descendants of 6.89

$p + 6$ algebras

$$\langle a, b, c \mid p^2b, p^2c, ca, cb, pa - ba, \text{class } 3 \rangle \quad (7.2661)$$

$$\langle a, b, c \mid bab, p^2c, ca, cb, pa - ba, \text{class } 3 \rangle \quad (7.2662)$$

$$\langle a, b, c \mid bab, p^2c, ca, cb - p^2b, pa - ba, \text{class } 3 \rangle \quad (7.2663)$$

$$\langle a, b, c \mid bab, p^2c, ca - p^2b, cb, pa - ba, \text{class } 3 \rangle \quad (7.2664)$$

$$\langle a, b, c \mid bab, p^2b, ca, cb - xp^2c, pa - ba, \text{class } 3 \rangle \quad (7.2665)$$

$$\langle a, b, c \mid bab, p^2b, ca - p^2c, cb, pa - ba, \text{class } 3 \rangle \quad (7.2666)$$

$$\langle a, b, c \mid p^2b, p^2c - bab, ca, cb, pa - ba, \text{class } 3 \rangle \quad (7.2667)$$

8.187 Descendants of 6.90

30 algebras

$$\langle a, b, c \mid baa, bab, bac, caa, cac, cb, pb, pc, \text{class } 3 \rangle \quad (7.2668)$$

$$\langle a, b, c \mid baa, bab, bac, caa, cac, cb - p^2a, pb, pc, \text{class } 3 \rangle \quad (7.2669)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a, cb, pb, pc, \text{class } 3 \rangle \quad (7.2670)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a, cb - caa, pb, pc, \text{class } 3 \rangle \quad (7.2671)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a, cb, pb, pc - caa, \text{class } 3 \rangle \quad (7.2672)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a, cb - caa, pb, pc - caa, \text{class } 3 \rangle \quad (7.2673)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a, cb, pb, pc - \omega caa, \text{class } 3 \rangle \quad (7.2674)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a, cb - caa, pb, pc - \omega caa, \text{class } 3 \rangle \quad (7.2675)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a, cb, pb - caa, pc, \text{class } 3 \rangle \quad (7.2676)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a, cb - caa, pb - caa, pc, \text{class } 3 \rangle \quad (7.2677)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a - caa, cb, pb, pc, \text{class } 3 \rangle \quad (7.2678)$$

$$\langle a, b, c \mid baa, bab, bac, cac, p^2a - caa, cb - caa, pb, pc, \text{class } 3 \rangle \quad (7.2679)$$

- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2a, pb, pc, \text{class } 3 \rangle$ (7.2680)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2a, pb, pc - cac, \text{class } 3 \rangle$ (7.2681)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2a, pb - cac, pc, \text{class } 3 \rangle$ (7.2682)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2a - cac, pb, pc, \text{class } 3 \rangle$ (7.2683)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2a - \omega cac, pb, pc, \text{class } 3 \rangle$ (7.2684)
- $\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2a, pb, pc, \text{class } 3 \rangle$ (7.2685)
- $\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2a, pb, pc - baa, \text{class } 3 \rangle$ (7.2686)
- $\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2a, pb - baa, pc, \text{class } 3 \rangle$ (7.2687)
- $\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2a, pb - \omega baa, pc, \text{class } 3 \rangle$ (7.2688)
- $\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2a - baa, pb, pc, \text{class } 3 \rangle$ (7.2689)
- $\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2a - \omega baa, pb, pc, \text{class } 3 \rangle$ (7.2690)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2a, pb, pc, \text{class } 3 \rangle$ (7.2691)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2a, pb, pc - bab, \text{class } 3 \rangle$ (7.2692)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2a, pb - bab, pc - bab, \text{class } 3 \rangle$ (7.2693)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2a - bab, pb, pc, \text{class } 3 \rangle$ (7.2694)
- $\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2a, pb, pc, \text{class } 3 \rangle$ (7.2695)
- $\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2a, pb, pc - bab, \text{class } 3 \rangle$ (7.2696)
- $\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2a - bab, pb, pc, \text{class } 3 \rangle$ (7.2697)

8.188 Descendants of 6.91

$2p + 88 + \gcd(p - 1, 4)$ algebras

- $\langle a, b, c \mid baa, bab, bac, caa, cac, cb, pa, pc, \text{class } 3 \rangle$ (7.2698)
- $\langle a, b, c \mid baa, bab, bac, caa, cac, cb - p^2b, pa, pc, \text{class } 3 \rangle$ (7.2699)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2700)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2b, pa - cac, pc, \text{class } 3 \rangle$ (7.2701)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2b, pa - \omega cac, pc, \text{class } 3 \rangle$ (7.2702)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2b, pa, pc - cac, \text{class } 3 \rangle$ (7.2703)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2b, pa - cac, pc - cac, \text{class } 3 \rangle$ (7.2704)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2b, pa - \omega cac, pc - cac, \text{class } 3 \rangle$ (7.2705)
- $\langle a, b, c \mid baa, bab, bac, caa, cb, p^2b - cac, pa, pc, \text{class } 3 \rangle$ (7.2706)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2707)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2b, pa - bab, pc, \text{class } 3 \rangle$ (7.2708)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2b, pa - \omega bab, pc, \text{class } 3 \rangle$ (7.2709)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2b, pa, pc - bab, \text{class } 3 \rangle$ (7.2710)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2b, pa - bab, pc - bab, \text{class } 3 \rangle$ (7.2711)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2b, pa - \omega bab, pc - bab, \text{class } 3 \rangle$ (7.2712)
- $\langle a, b, c \mid baa, bac, caa, cac + bab, cb, p^2b - bab, pa, pc, \text{class } 3 \rangle$ (7.2713)
- $\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2714)
- $\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2b, pa - bab, pc, \text{class } 3 \rangle$ (7.2715)
- $\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2b, pa - \omega bab, pc, \text{class } 3 \rangle$ (7.2716)
- $\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2b, pa, pc - bab, \text{class } 3 \rangle$ (7.2717)
- $\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2b, pa - bab, pc - bab, \text{class } 3 \rangle$ (7.2718)

$$\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2b, pa - \omega bab, pc - bab, \text{class } 3 \rangle \quad (7.2719)$$

$$\langle a, b, c \mid baa, bac, caa, cac + \omega bab, cb, p^2b - bab, pa, pc, \text{class } 3 \rangle \quad (7.2720)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2b, pa, pc, \text{class } 3 \rangle \quad (7.2721)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2b, pa - baa, pc, \text{class } 3 \rangle \quad (7.2722)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2b, pa - \omega baa, pc, \text{class } 3 \rangle \quad (7.2723)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2b, pa, pc - baa, \text{class } 3 \rangle \quad (7.2724)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2b, pa - baa, pc - baa, \text{class } 3 \rangle \quad (7.2725)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2b, pa - \omega baa, pc - baa, \text{class } 3 \rangle \quad (7.2726)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2b - baa, pa, pc, \text{class } 3 \rangle \quad (7.2727)$$

$$\langle a, b, c \mid bab, bac, caa, cac - baa, cb, p^2b - \omega baa, pa, pc, \text{class } 3 \rangle \quad (7.2728)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + baa, cb, p^2b, pa, pc, \text{class } 3 \rangle \quad (7.2729)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + baa, cb, p^2b, pa, pc - baa, \text{class } 3 \rangle \quad (7.2730)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + baa, cb, p^2b, pa, pc - \omega baa, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.2731)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + baa, cb, p^2b, pa - baa, pc - xbaa, \text{class } 3 \rangle \quad (x \sim -x) \quad (7.2732)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + baa, cb, p^2b, pa - \omega baa, pc - xbaa, \text{class } 3 \rangle \quad (x \sim -x) \quad (7.2733)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + baa, cb, p^2b - baa, pa, pc, \text{class } 3 \rangle \quad (7.2734)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + baa, cb, p^2b - \omega baa, pa, pc, \text{class } 3 \rangle \quad (7.2735)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + \omega baa, cb, p^2b, pa, pc, \text{class } 3 \rangle \quad (7.2736)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + \omega baa, cb, p^2b, pa, pc - baa, \text{class } 3 \rangle \quad (7.2737)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + \omega baa, cb, p^2b, pa, pc - \omega baa, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.2738)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + \omega baa, cb, p^2b, pa - baa, pc - xbaa, \text{class } 3 \rangle \quad (x \sim -x) \quad (7.2739)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac + \omega baa, cb, p^2b, pa - \omega baa, pc - xbaa, \text{class } 3 \rangle \quad (x \sim -x) \quad (7.2740)$$

- $\langle a, b, c \mid bab - baa, bac, caa, cac + \omega baa, cb, p^2b - baa, pa, pc, \text{class } 3 \rangle$ (7.2741)
- $\langle a, b, c \mid bab - baa, bac, caa, cac + \omega baa, cb, p^2b - \omega baa, pa, pc, \text{class } 3 \rangle$ (7.2742)
- $\langle a, b, c \mid baa, bab, caa, cac, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2743)
- $\langle a, b, c \mid baa, bab, caa, cac, cb, p^2b, pa, pc - bac, \text{class } 3 \rangle$ (7.2744)
- $\langle a, b, c \mid baa, bab, caa, cac, cb, p^2b, pa - bac, pc, \text{class } 3 \rangle$ (7.2745)
- $\langle a, b, c \mid baa, bab, caa, cac, cb, p^2b, pa - bac, pc - bac, \text{class } 3 \rangle$ (7.2746)
- $\langle a, b, c \mid baa, bab, caa, cac, cb, p^2b - bac, pa, pc, \text{class } 3 \rangle$ (7.2747)
- $\langle a, b, c \mid baa, bab, caa - bac, cac, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2748)
- $\langle a, b, c \mid baa, bab, caa - bac, cac, cb, p^2b, pa, pc - bac, \text{class } 3 \rangle$ (7.2749)
- $\langle a, b, c \mid baa, bab, caa - bac, cac, cb, p^2b, pa, pc - \omega bac, \text{class } 3 \rangle$ (7.2750)
- $\langle a, b, c \mid baa, bab, caa - bac, cac, cb, p^2b, pa - bac, pc, \text{class } 3 \rangle$ (7.2751)
- $\langle a, b, c \mid baa, bab, caa - bac, cac, cb, p^2b, pa - bac, pc - bac, \text{class } 3 \rangle$ (7.2752)
- $\langle a, b, c \mid baa, bab, caa - bac, cac, cb, p^2b, pa - bac, pc - \omega bac, \text{class } 3 \rangle$ (7.2753)
- $\langle a, b, c \mid baa, bab, caa - bac, cac, cb, p^2b - bac, pa, pc, \text{class } 3 \rangle$ (7.2754)
- $\langle a, b, c \mid baa, bab, bac, cac, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2755)
- $\langle a, b, c \mid baa, bab, bac, cac, cb, p^2b, pa, pc - caa, \text{class } 3 \rangle$ (7.2756)
- $\langle a, b, c \mid baa, bab, bac, cac, cb, p^2b, pa, pc - \omega caa, \text{class } 3 \rangle$ (7.2757)
- $\langle a, b, c \mid baa, bab, bac, cac, cb - caa, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2758)
- $\langle a, b, c \mid baa, bab, bac, cac, cb - caa, p^2b, pa, pc - caa, \text{class } 3 \rangle$ (7.2759)
- $\langle a, b, c \mid baa, bab, bac, cac, cb - caa, p^2b, pa, pc - \omega caa, \text{class } 3 \rangle$ (7.2760)
- $\langle a, b, c \mid baa, bab, bac, cac, cb, p^2b, pa - caa, pc, \text{class } 3 \rangle$ (7.2761)
- $\langle a, b, c \mid baa, bab, bac, cac, cb - caa, p^2b, pa - caa, pc, \text{class } 3 \rangle$ (7.2762)

- $\langle a, b, c \mid baa, bab, bac, cac, cb, p^2b - caa, pa, pc, \text{class } 3 \rangle$ (7.2763)
- $\langle a, b, c \mid baa, bab, bac, cac, cb - caa, p^2b - caa, pa, pc, \text{class } 3 \rangle$ (7.2764)
- $\langle a, b, c \mid baa, bac, caa - bab, cac, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2765)
- $\langle a, b, c \mid baa, bac, caa - bab, cac, cb, p^2b, pa, pc - bab, \text{class } 3 \rangle$ (7.2766)
- $\langle a, b, c \mid baa, bac, caa - bab, cac, cb, p^2b, pa, pc - \omega bab, \text{class } 3 \rangle$ (7.2767)
- $\langle a, b, c \mid baa, bac, caa - bab, cac, cb, p^2b, pa - bab, pc, \text{class } 3 \rangle$ (7.2768)
- $\langle a, b, c \mid baa, bac, caa - bab, cac, cb, p^2b, pa - \omega bab, pc, \text{class } 3 \rangle$ (7.2769)
- $\langle a, b, c \mid baa, bac, caa - bab, cac, cb, p^2b - bab, pa, pc, \text{class } 3 \rangle$ (7.2770)
- $\langle a, b, c \mid baa, bac, caa, cac, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2771)
- $\langle a, b, c \mid baa, bac, caa, cac, cb, p^2b, pa, pc - bab, \text{class } 3 \rangle$ (7.2772)
- $\langle a, b, c \mid baa, bac, caa, cac, cb, p^2b, pa - bab, pc, \text{class } 3 \rangle$ (7.2773)
- $\langle a, b, c \mid baa, bac, caa, cac, cb, p^2b, pa - \omega bab, pc, \text{class } 3 \rangle$ (7.2774)
- $\langle a, b, c \mid baa, bac, caa, cac, cb, p^2b - bab, pa, pc, \text{class } 3 \rangle$ (7.2775)
- $\langle a, b, c \mid bab, bac, caa, cac, cb, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2776)
- $\langle a, b, c \mid bab, bac, caa, cac, cb, p^2b, pa, pc - baa, \text{class } 3 \rangle$ (7.2777)
- $\langle a, b, c \mid bab, bac, caa, cac, cb, p^2b, pa - baa, pc, \text{class } 3 \rangle$ (7.2778)
- $\langle a, b, c \mid bab, bac, caa, cac, cb, p^2b - baa, pa, pc, \text{class } 3 \rangle$ (7.2779)
- $\langle a, b, c \mid bab, bac, caa, cac, cb, p^2b - \omega baa, pa, pc, \text{class } 3 \rangle$ (7.2780)
- $\langle a, b, c \mid bab, bac, caa, cac, cb - baa, p^2b, pa, pc, \text{class } 3 \rangle$ (7.2781)
- $\langle a, b, c \mid bab, bac, caa, cac, cb - baa, p^2b, pa, pc - baa, \text{class } 3 \rangle$ (7.2782)
- $\langle a, b, c \mid bab, bac, caa, cac, cb - baa, p^2b, pa - baa, pc, \text{class } 3 \rangle$ (7.2783)
- $\langle a, b, c \mid bab, bac, caa, cac, cb - baa, p^2b - baa, pa, pc, \text{class } 3 \rangle$ (7.2784)

$$\langle a, b, c \mid bab, bac, caa, cac, cb - baa, p^2b - \omega baa, pa, pc, \text{class } 3 \rangle \quad (7.2785)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac, cb, p^2b, pa, pc, \text{class } 3 \rangle \quad (7.2786)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac, cb, p^2b, pa - baa, pc, \text{class } 3 \rangle \quad (7.2787)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac, cb, p^2b, pa - \omega baa, pc, \text{class } 3 \rangle \quad (7.2788)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac, cb, p^2b, pa, pc - baa, \text{class } 3 \rangle \quad (7.2789)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac, cb, p^2b - baa, pa, pc, \text{class } 3 \rangle \quad (7.2790)$$

$$\langle a, b, c \mid bab - baa, bac, caa, cac, cb, p^2b - \omega baa, pa, pc, \text{class } 3 \rangle \quad (7.2791)$$

8.189 Descendants of 6.92

$2p + 13$ algebras

$$\langle a, b, c \mid baa, caa, cac, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.2792)$$

$$\langle a, b, c \mid baa, caa, cac, cb - p^2a, pb - ba, pc, \text{class } 3 \rangle \quad (7.2793)$$

$$\langle a, b, c \mid baa, caa, p^2a, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.2794)$$

$$\langle a, b, c \mid baa, caa, p^2a, cb, pb - ba, pc - cac, \text{class } 3 \rangle \quad (7.2795)$$

$$\langle a, b, c \mid baa, caa, p^2a, cb, pb - ba - cac, pc, \text{class } 3 \rangle \quad (7.2796)$$

$$\langle a, b, c \mid baa, caa, p^2a, cb, pb - ba - cac, pc - cac, \text{class } 3 \rangle \quad (7.2797)$$

$$\langle a, b, c \mid baa, caa, p^2a - cac, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.2798)$$

$$\langle a, b, c \mid baa, caa, p^2a - \omega cac, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.2799)$$

$$\langle a, b, c \mid baa, cac, p^2a, cb, pb - ba, pc - xcaa, \text{class } 3 \rangle \quad (7.2800)$$

$$\langle a, b, c \mid baa, cac, p^2a, cb - caa, pb - ba, pc - xcaa, \text{class } 3 \rangle \quad (7.2801)$$

$$\langle a, b, c \mid baa, cac, p^2a - caa, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.2802)$$

$$\langle a, b, c \mid baa, cac, p^2a - caa, cb - caa, pb - ba, pc, \text{class } 3 \rangle \quad (7.2803)$$

$$\langle a, b, c \mid caa, cac, p^2a, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.2804)$$

$$\langle a, b, c \mid caa, cac - baa, p^2a, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.2805)$$

$$\langle a, b, c \mid caa - baa, cac, p^2a, cb, pb - ba, pc, \text{class } 3 \rangle \quad (7.2806)$$

8.190 Descendants of 6.93

$p + 15 + 2 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

- $$\langle a, b, c \mid baa, bab, caa, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2807)$$
- $$\langle a, b, c \mid baa, bab, caa, cb - p^2a, pb - ca, pc, \text{class } 3 \rangle \quad (7.2808)$$
- $$\langle a, b, c \mid baa, bab, caa, cb - \omega p^2a, pb - ca, pc, \text{class } 3 \rangle \quad (7.2809)$$
- $$\langle a, b, c \mid baa, bab, p^2a, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2810)$$
- $$\langle a, b, c \mid baa, bab, p^2a - caa, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2811)$$
- $$\langle a, b, c \mid baa, caa, p^2a, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2812)$$
- $$\langle a, b, c \mid baa, caa, p^2a, cb, pb - ca, pc - bab, \text{class } 3 \rangle \quad (7.2813)$$
- $$\langle a, b, c \mid baa, caa, p^2a, cb, pb - ca - bab, pc, \text{class } 3 \rangle \quad (7.2814)$$
- $$\langle a, b, c \mid baa, caa, p^2a - bab, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2815)$$
- $$\langle a, b, c \mid baa, caa, p^2a - \omega bab, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2816)$$
- $$\langle a, b, c \mid baa, caa - bab, p^2a - xbab, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2817)$$
- $$\langle a, b, c \mid baa, caa - bab, p^2a + bab, cb, pb - ca, pc - bab, \text{class } 3 \rangle \quad (7.2818)$$
- $$\langle a, b, c \mid baa, caa - bab, p^2a + bab, cb, pb - ca, pc - \omega bab, \text{class } 3 \rangle \quad (7.2819)$$
- $$\langle a, b, c \mid bab, caa, p^2a, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2820)$$
- $$\langle a, b, c \mid bab, caa, p^2a, cb, pb - ca, pc - baa, \text{class } 3 \rangle \quad (7.2821)$$
- $$\langle a, b, c \mid bab, caa, p^2a, cb, pb - ca, pc - \omega baa, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.2822)$$
- $$\langle a, b, c \mid bab, caa, p^2a, cb, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.2823)$$
- $$\langle a, b, c \mid bab, caa, p^2a, cb - baa, pb - ca, pc, \text{class } 3 \rangle \quad (7.2824)$$
- $$\langle a, b, c \mid bab, caa, p^2a, cb - baa, pb - ca, pc - baa, \text{class } 3 \rangle \quad (7.2825)$$
- $$\langle a, b, c \mid bab, caa, p^2a, cb - baa, pb - ca, pc - \omega baa, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.2826)$$
- $$\langle a, b, c \mid bab, caa, p^2a, cb - baa, pb - ca, pc - \omega^2 baa, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.2827)$$

$$\langle a, b, c \mid bab, caa, p^2a - baa, cb, pb - ca, pc, \text{class } 3 \rangle \quad (7.2828)$$

$$\langle a, b, c \mid bab, caa, p^2a - baa, cb - baa, pb - ca, pc, \text{class } 3 \rangle \quad (7.2829)$$

$$\langle a, b, c \mid bab, caa, p^2a - baa, cb - \omega baa, pb - ca, pc, \text{class } 3 \rangle \quad (7.2830)$$

$$\langle a, b, c \mid bab, caa, p^2a - baa, cb - \omega^2 baa, pb - ca, pc, \text{class } 3 \rangle \quad (p = 1 \pmod{4}) \quad (7.2831)$$

$$\langle a, b, c \mid bab, caa, p^2a - baa, cb - \omega^3 baa, pb - ca, pc, \text{class } 3 \rangle \quad (p = 1 \pmod{4}) \quad (7.2832)$$

8.191 Descendants of 6.94

$2p + 15 + 3 \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \mid caa, cac, cb, pa, pc - ba, \text{class } 3 \rangle \quad (7.2833)$$

$$\langle a, b, c \mid caa, cac, cb - p^2b, pa, pc - ba, \text{class } 3 \rangle \quad (7.2834)$$

$$\langle a, b, c \mid caa, p^2b, cb, pa, pc - ba, \text{class } 3 \rangle \quad (7.2835)$$

$$\langle a, b, c \mid caa, p^2b, cb, pa - cac, pc - ba, \text{class } 3 \rangle \quad (7.2836)$$

$$\langle a, b, c \mid caa, p^2b, cb, pa - \omega cac, pc - ba, \text{class } 3 \rangle \quad (7.2837)$$

$$\langle a, b, c \mid caa, p^2b, cb, pa, pc - ba - cac, \text{class } 3 \rangle \quad (7.2838)$$

$$\langle a, b, c \mid caa, p^2b, cb, pa - cac, pc - ba - cac, \text{class } 3 \rangle \quad (7.2839)$$

$$\langle a, b, c \mid caa, p^2b, cb, pa - \omega cac, pc - ba - cac, \text{class } 3 \rangle \quad (7.2840)$$

$$\langle a, b, c \mid caa, p^2b - cac, cb, pa, pc - ba, \text{class } 3 \rangle \quad (7.2841)$$

$$\langle a, b, c \mid cac, p^2b, cb, pa, pc - ba, \text{class } 3 \rangle \quad (7.2842)$$

$$\langle a, b, c \mid cac, p^2b, cb, pa - caa, pc - ba, \text{class } 3 \rangle \quad (7.2843)$$

$$\langle a, b, c \mid cac, p^2b, cb - caa, pa, pc - ba, \text{class } 3 \rangle \quad (7.2844)$$

$$\langle a, b, c \mid cac, p^2b, cb - caa, pa - caa, pc - ba, \text{class } 3 \rangle \quad (7.2845)$$

$$\langle a, b, c \mid cac, p^2b, cb - caa, pa - \omega caa, pc - ba, \text{class } 3 \rangle \quad (7.2846)$$

$$\langle a, b, c \mid cac, p^2b, cb - caa, pa - \omega^2 caa, pc - ba, \text{class } 3 \rangle \quad (p = 1 \pmod{4}) \quad (7.2847)$$

$$\langle a, b, c \mid cac, p^2b, cb - caa, pa - \omega^3caa, pc - ba, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.2848)$$

$$\langle a, b, c \mid cac, p^2b - caa, cb, pa, pc - ba, \text{class } 3 \rangle \quad (7.2849)$$

$$\langle a, b, c \mid cac, p^2b - \omega caa, cb, pa, pc - ba, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.2850)$$

$$\langle a, b, c \mid cac, p^2b - \omega^2caa, cb, pa, pc - ba, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.2851)$$

$$\langle a, b, c \mid cac, p^2b - caa, cb - caa, pa, pc - ba, \text{class } 3 \rangle \quad (7.2852)$$

$$\langle a, b, c \mid cac, p^2b - \omega caa, cb - caa, pa, pc - ba, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.2853)$$

$$\langle a, b, c \mid cac, p^2b - \omega^2caa, cb - caa, pa, pc - ba, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.2854)$$

$$\langle a, b, c \mid cac - caa, p^2b, cb, pa, pc - ba, \text{class } 3 \rangle \quad (7.2855)$$

$$\langle a, b, c \mid cac - caa, p^2b, cb, pa, pc - ba - caa, \text{class } 3 \rangle \quad (7.2856)$$

$$\langle a, b, c \mid cac - caa, p^2b, cb, pa, pc - ba - \omega caa, \text{class } 3 \rangle \quad (7.2857)$$

$$\langle a, b, c \mid cac - caa, p^2b, cb, pa - caa, pc - ba - xcaa, \text{class } 3 \rangle \quad (7.2858)$$

$$\langle a, b, c \mid cac - caa, p^2b, cb, pa - \omega caa, pc - ba - xcaa, \text{class } 3 \rangle \quad (7.2859)$$

$$\langle a, b, c \mid cac - caa, p^2b - caa, cb, pa, pc - ba, \text{class } 3 \rangle \quad (7.2860)$$

$$\langle a, b, c \mid cac - caa, p^2b - \omega caa, cb, pa, pc - ba, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.2861)$$

$$\langle a, b, c \mid cac - caa, p^2b - \omega^2caa, cb, pa, pc - ba, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.2862)$$

8.192 Descendants of 6.95

$5p + 10$ algebras

$$\langle a, b, c \mid baa, bab, cb, pa, pc - ca, \text{class } 3 \rangle \quad (7.2863)$$

$$\langle a, b, c \mid baa, bab, cb - p^2b, pa, pc - ca, \text{class } 3 \rangle \quad (7.2864)$$

$$\langle a, b, c \mid baa, p^2b, cb, pa, pc - ca, \text{class } 3 \rangle \quad (7.2865)$$

$$\langle a, b, c \mid baa, p^2b, cb, pa - bab, pc - ca, \text{class } 3 \rangle \quad (7.2866)$$

$$\langle a, b, c \mid baa, p^2b, cb, pa - \omega bab, pc - ca, \text{class } 3 \rangle \quad (7.2867)$$

$$\langle a, b, c \mid baa, p^2b, cb, pa, pc - ca - bab, \text{class } 3 \rangle \quad (7.2868)$$

$$\langle a, b, c \mid baa, p^2b, cb, pa - bab, pc - ca - bab, \text{class } 3 \rangle \quad (7.2869)$$

$$\langle a, b, c \mid baa, p^2b, cb, pa - \omega bab, pc - ca - bab, \text{class } 3 \rangle \quad (7.2870)$$

$$\langle a, b, c \mid baa, p^2b - bab, cb, pa, pc - ca, \text{class } 3 \rangle \quad (7.2871)$$

$$\langle a, b, c \mid bab, p^2b - xbaa, cb, pa, pc - ca, \text{class } 3 \rangle \quad (7.2872)$$

$$\langle a, b, c \mid bab, p^2b - xbaa, cb - baa, pa, pc - ca, \text{class } 3 \rangle \quad (7.2873)$$

$$\langle a, b, c \mid bab, p^2b, cb, pa - baa, pc - ca, \text{class } 3 \rangle \quad (7.2874)$$

$$\langle a, b, c \mid bab, p^2b, cb - baa, pa - baa, pc - ca, \text{class } 3 \rangle \quad (7.2875)$$

$$\langle a, b, c \mid bab - baa, p^2b - xbaa, cb, pa, pc - ca, \text{class } 3 \rangle (x \neq 0) \quad (7.2876)$$

$$\langle a, b, c \mid bab - baa, p^2b, cb, pa - xbaa, pc - ca, \text{class } 3 \rangle \quad (7.2877)$$

$$\langle a, b, c \mid bab - baa, p^2b, cb, pa - xbaa, pc - ca - baa, \text{class } 3 \rangle \quad (7.2878)$$

8.193 Descendants of 6.96

$2p + 26$ algebras

$$\langle a, b, c \mid bab, caa, cac, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2879)$$

$$\langle a, b, c \mid bab, caa, cac, cb - p^2b, pa - ba, pc, \text{class } 3 \rangle \quad (7.2880)$$

$$\langle a, b, c \mid bab, caa, p^2b, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2881)$$

$$\langle a, b, c \mid bab, caa, p^2b, cb, pa - ba - cac, pc, \text{class } 3 \rangle \quad (7.2882)$$

$$\langle a, b, c \mid bab, caa, p^2b, cb, pa - ba - \omega cac, pc, \text{class } 3 \rangle \quad (7.2883)$$

$$\langle a, b, c \mid bab, caa, p^2b, cb, pa - ba, pc - cac, \text{class } 3 \rangle \quad (7.2884)$$

$$\langle a, b, c \mid bab, caa, p^2b, cb, pa - ba - cac, pc - cac, \text{class } 3 \rangle \quad (7.2885)$$

$$\langle a, b, c \mid bab, caa, p^2b, cb, pa - ba - \omega cac, pc - cac, \text{class } 3 \rangle \quad (7.2886)$$

$$\langle a, b, c \mid bab, caa, p^2b - cac, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2887)$$

$$\langle a, b, c \mid bab, cac, p^2b, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2888)$$

$$\langle a, b, c \mid bab, cac, p^2b, cb, pa - ba, pc - caa, \text{class } 3 \rangle \quad (7.2889)$$

$$\langle a, b, c \mid bab, cac, p^2b, cb, pa - ba, pc - \omega caa, \text{class } 3 \rangle \quad (7.2890)$$

$$\langle a, b, c \mid bab, cac, p^2b, cb - caa, pa - ba, pc - xcaa, \text{class } 3 \rangle \quad (7.2891)$$

$$\langle a, b, c \mid bab, cac, p^2b, cb - \omega caa, pa - ba, pc - xcaa, \text{class } 3 \rangle \quad (7.2892)$$

$$\langle a, b, c \mid bab, cac, p^2b - caa, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2893)$$

$$\langle a, b, c \mid bab, cac, p^2b - caa, cb - caa, pa - ba, pc, \text{class } 3 \rangle \quad (7.2894)$$

$$\langle a, b, c \mid bab, cac, p^2b - caa, cb - \omega caa, pa - ba, pc, \text{class } 3 \rangle \quad (7.2895)$$

$$\langle a, b, c \mid caa, cac, p^2b, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2896)$$

$$\langle a, b, c \mid caa, cac, p^2b, cb, pa - ba, pc - bab, \text{class } 3 \rangle \quad (7.2897)$$

$$\langle a, b, c \mid caa, cac, p^2b - bab, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2898)$$

$$\langle a, b, c \mid caa, cac + bab, p^2b, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2899)$$

$$\langle a, b, c \mid caa, cac + bab, p^2b - bab, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2900)$$

$$\langle a, b, c \mid caa, cac + \omega bab, p^2b, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2901)$$

$$\langle a, b, c \mid caa, cac + \omega bab, p^2b - bab, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2902)$$

$$\langle a, b, c \mid cac, caa - bab, p^2b, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2903)$$

$$\langle a, b, c \mid cac, caa - bab, p^2b, cb, pa - ba, pc - bab, \text{class } 3 \rangle \quad (7.2904)$$

$$\langle a, b, c \mid cac, caa - bab, p^2b, cb, pa - ba, pc - \omega bab, \text{class } 3 \rangle \quad (7.2905)$$

$$\langle a, b, c \mid cac, caa - bab, p^2b - bab, cb, pa - ba, pc, \text{class } 3 \rangle \quad (7.2906)$$

8.194 Descendants of 6.97

$3p + 18 + \gcd(p - 1, 3)$ algebras

- $\langle a, b, c \mid baa, bab, bac, cb - xp^2b, pa - ca, pc, \text{class } 3 \rangle$ (7.2907)
- $\langle a, b, c \mid baa, bab, p^2b, cb, pa - ca, pc, \text{class } 3 \rangle$ (7.2908)
- $\langle a, b, c \mid baa, bab, p^2b - bac, cb, pa - ca, pc, \text{class } 3 \rangle$ (7.2909)
- $\langle a, b, c \mid baa, bac, p^2b, cb, pa - ca, pc, \text{class } 3 \rangle$ (7.2910)
- $\langle a, b, c \mid baa, bac, p^2b, cb, pa - ca - bab, pc, \text{class } 3 \rangle$ (7.2911)
- $\langle a, b, c \mid baa, bac, p^2b, cb, pa - ca - \omega bab, pc, \text{class } 3 \rangle$ (7.2912)
- $\langle a, b, c \mid baa, bac, p^2b, cb, pa - ca, pc - bab, \text{class } 3 \rangle$ (7.2913)
- $\langle a, b, c \mid baa, bac, p^2b - bab, cb, pa - ca, pc, \text{class } 3 \rangle$ (7.2914)
- $\langle a, b, c \mid bab, bac, p^2b, cb, pa - ca, pc, \text{class } 3 \rangle$ (7.2915)
- $\langle a, b, c \mid bab, bac, p^2b, cb - baa, pa - ca, pc, \text{class } 3 \rangle$ (7.2916)
- $\langle a, b, c \mid bab, bac, p^2b, cb - \omega baa, pa - ca, pc, \text{class } 3 \rangle$ (7.2917)
- $\langle a, b, c \mid bab, bac, p^2b, cb, pa - ca, pc - baa, \text{class } 3 \rangle$ (7.2918)
- $\langle a, b, c \mid bab, bac, p^2b, cb - baa, pa - ca, pc - baa, \text{class } 3 \rangle$ (7.2919)
- $\langle a, b, c \mid bab, bac, p^2b, cb - \omega baa, pa - ca, pc - baa, \text{class } 3 \rangle$ (7.2920)
- $\langle a, b, c \mid bab, bac, p^2b - baa, cb - xbaa, pa - ca, pc, \text{class } 3 \rangle$ (7.2921)
- $\langle a, b, c \mid bab, bac, p^2b - \omega baa, cb - xbaa, pa - ca, pc, \text{class } 3 \rangle$ (7.2922)
- $\langle a, b, c \mid bab - baa, bac, p^2b, cb, pa - ca, pc, \text{class } 3 \rangle$ (7.2923)
- $\langle a, b, c \mid bab - baa, bac, p^2b, cb, pa - ca - baa, pc, \text{class } 3 \rangle$ (7.2924)
- $\langle a, b, c \mid bab - baa, bac, p^2b, cb, pa - ca - \omega baa, pc, \text{class } 3 \rangle$ (7.2925)
- $\langle a, b, c \mid bab - baa, bac, p^2b, cb, pa - ca, pc - baa, \text{class } 3 \rangle$ (7.2926)
- $\langle a, b, c \mid bab - baa, bac, p^2b, cb, pa - ca, pc - \omega baa, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.2927)

$$\langle a, b, c \mid bab - baa, bac, p^2b, cb, pa - ca, pc - \omega^2baa, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.2928)$$

$$\langle a, b, c \mid bab - baa, bac, p^2b - baa, cb, pa - ca, pc, \text{class } 3 \rangle \quad (7.2929)$$

$$\langle a, b, c \mid bab - baa, bac, p^2b - \omega baa, cb, pa - ca, pc, \text{class } 3 \rangle \quad (7.2930)$$

8.195 Descendants of 6.98

$(5p + 1)/2$ algebras

$$\langle a, b, c \mid baa, caa, cb, pb - ba, pc - ca, \text{class } 3 \rangle \quad (7.2931)$$

$$\langle a, b, c \mid baa, caa, cb - p^2a, pb - ba, pc - ca, \text{class } 3 \rangle \quad (7.2932)$$

$$\langle a, b, c \mid baa, p^2a, cb, pb - ba, pc - ca, \text{class } 3 \rangle \quad (7.2933)$$

$$\langle a, b, c \mid baa, p^2a, cb, pb - ba - caa, pc - ca, \text{class } 3 \rangle \quad (7.2934)$$

$$\langle a, b, c \mid baa, caa, cb, pb - ba, pc - xca, \text{class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.2935)$$

$$\langle a, b, c \mid baa, caa, cb - p^2a, pb - ba, pc - xca, \text{class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.2936)$$

$$\langle a, b, c \mid baa, p^2a, cb, pb - ba, pc - xca, \text{class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.2937)$$

$$\langle a, b, c \mid caa, p^2a, cb, pb - ba, pc - xca, \text{class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.2938)$$

$$\langle a, b, c \mid caa - baa, p^2a, cb, pb - ba, pc - xca, \text{class } 3 \rangle \quad (x = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.2939)$$

$$\langle a, b, c \mid baa, caa, cb, pb - ba, pc + ca, \text{class } 3 \rangle \quad (7.2940)$$

$$\langle a, b, c \mid baa, caa, cb - p^2a, pb - ba, pc + ca, \text{class } 3 \rangle \quad (7.2941)$$

$$\langle a, b, c \mid baa, p^2a, cb, pb - ba, pc + ca, \text{class } 3 \rangle \quad (7.2942)$$

$$\langle a, b, c \mid caa - baa, p^2a, cb, pb - ba, pc + ca, \text{class } 3 \rangle \quad (7.2943)$$

8.196 Descendants of 6.99

$p + 4$ algebras

$$\langle a, b, c \mid baa, caa, cb, pb - ba - ca, pc - ca, \text{class } 3 \rangle \quad (7.2944)$$

$$\langle a, b, c \mid baa, caa, cb - p^2a, pb - ba - ca, pc - ca, \text{class } 3 \rangle \quad (7.2945)$$

$$\langle a, b, c \mid baa, caa, cb - \omega p^2a, pb - ba - ca, pc - ca, \text{class } 3 \rangle \quad (7.2946)$$

$$\langle a, b, c \mid baa, p^2a, cb, pb - ba - ca, pc - ca, \text{class } 3 \rangle \quad (7.2947)$$

$$\langle a, b, c \mid caa, p^2a, cb, pb - ba - ca, pc - ca - xbaa, \text{class } 3 \rangle \quad (7.2948)$$

8.197 Descendants of 4.1

1361 algebras when $p = 3$, $p^5 + 2p^4 + 7p^3 + 25p^2 + 88p + 270 + (p + 4) \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras when $p > 3$

$$\langle a, b, c, d \mid ba, ca, da, cb, db, dc, pd, \text{class } 2 \rangle \quad (7.2949)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pa, pb, \text{class } 2 \rangle \quad (7.2950)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pa - ba, pb, \text{class } 2 \rangle \quad (7.2951)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pb, pc - ba, \text{class } 2 \rangle \quad (7.2952)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pb, pc, \text{class } 2 \rangle \quad (7.2953)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pb - ba, pc, \text{class } 2 \rangle \quad (7.2954)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pc, pd, \text{class } 2 \rangle \quad (7.2955)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc, pc - ba, pd, \text{class } 2 \rangle \quad (7.2956)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pa, pc, \text{class } 2 \rangle \quad (7.2957)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pa - ba, pc, \text{class } 2 \rangle \quad (7.2958)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pa, pb, \text{class } 2 \rangle \quad (7.2959)$$

$$\langle a, b, c, d \mid ca, da, cb, db, dc - ba, pa - ba, pb, \text{class } 2 \rangle \quad (7.2960)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pb, pc, pd, \text{class } 2 \rangle \quad (7.2961)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa, pc, pd, \text{class } 2 \rangle \quad (7.2962)$$

- $\langle a, b, c, d \mid cb, da, db, dc, pb - ba, pc, pd, \text{class } 2 \rangle$ (7.2963)
- $\langle a, b, c, d \mid cb, da, db, dc, pb - ca, pc, pd, \text{class } 2 \rangle$ (7.2964)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pc - ba, pd, \text{class } 2 \rangle$ (7.2965)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pc - ca, pd, \text{class } 2 \rangle$ (7.2966)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pc, pd, \text{class } 2 \rangle$ (7.2967)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ca, pc, pd, \text{class } 2 \rangle$ (7.2968)
- $\langle a, b, c, d \mid cb, da, db, dc, pb - ba, pc - xca, pd, \text{class } 2 \rangle$ ($x \neq 0, x \sim x^{-1}$) (7.2969)
- $\langle a, b, c, d \mid cb, da, db, dc, pb - ba - ca, pc - ca, pd, \text{class } 2 \rangle$ (7.2970)
- $\langle a, b, c, d \mid cb, da, db, dc, pb - \omega ca, pc - ba, pd, \text{class } 2 \rangle$ (7.2971)
- $\langle a, b, c, d \mid cb, da, db, dc, pb - xca, pc - ba - ca, pd, \text{class } 2 \rangle$ ($1 + 4x$ not a square) (7.2972)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ca, pc - ba, pd, \text{class } 2 \rangle$ (7.2973)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pc - ca, pd, \text{class } 2 \rangle$ (7.2974)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb, pc, \text{class } 2 \rangle$ (7.2975)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb, pc, \text{class } 2 \rangle$ (7.2976)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ba, pc, \text{class } 2 \rangle$ (7.2977)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ca, pb - ba, pc, \text{class } 2 \rangle$ (7.2978)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ca, pc, \text{class } 2 \rangle$ (7.2979)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb - ca, pc, \text{class } 2 \rangle$ (7.2980)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ba, pc - xca, \text{class } 2 \rangle$ ($x \neq 0, x \sim x^{-1}$) (7.2981)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ba - ca, pc - ca, \text{class } 2 \rangle$ (7.2982)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb - \omega ca, pc - ba, \text{class } 2 \rangle$ (7.2983)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb - xca, pc - ba - ca, \text{class } 2 \rangle$ ($1 + 4x$ not a square) (7.2984)

- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb, pd - ca, \text{class } 2 \rangle$ (7.2985)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pb - ba, pd - ca, \text{class } 2 \rangle$ (7.2986)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb, pd - ca, \text{class } 2 \rangle$ (7.2987)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pc, pd - ca, \text{class } 2 \rangle$ (7.2988)
- $\langle a, b, c, d \mid cb, da, db, dc, pa, pc - ba, pd - ca, \text{class } 2 \rangle$ (7.2989)
- $\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pc, pd - ca, \text{class } 2 \rangle$ (7.2990)
- $\langle a, b, c, d \mid cb, da, db, dc, pb, pc, pd - ca, \text{class } 2 \rangle$ (7.2991)
- $\langle a, b, c, d \mid cb, da, db, dc, pb, pc - ba, pd - ca, \text{class } 2 \rangle$ (7.2992)
- $\langle a, b, c, d \mid cb, da, db, dc, pb - ba, pc, pd - ca, \text{class } 2 \rangle$ (7.2993)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb, pc, \text{class } 2 \rangle$ (7.2994)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb, pc - ba, \text{class } 2 \rangle$ (7.2995)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb, pc - dc, \text{class } 2 \rangle$ (7.2996)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb, pc - ba - dc, \text{class } 2 \rangle$ (7.2997)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - ba, pc, \text{class } 2 \rangle$ (7.2998)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - ba, pc - ba, \text{class } 2 \rangle$ (7.2999)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - ba, pc - dc, \text{class } 2 \rangle$ (7.3000)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - ba, pc - ba - dc, \text{class } 2 \rangle$ (7.3001)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - dc, pc, \text{class } 2 \rangle$ (7.3002)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - dc, pc - ba, \text{class } 2 \rangle$ (7.3003)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - dc, pc - dc, \text{class } 2 \rangle$ (7.3004)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - dc, pc - ba - dc, \text{class } 2 \rangle$ (7.3005)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - ba - dc, pc, \text{class } 2 \rangle$ (7.3006)

- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - ba - dc, pc - ba, \text{class } 2 \rangle$ (7.3007)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pb - ba - dc, pc - xba - dc, \text{class } 2 \rangle$ (7.3008)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - ba, pb - dc, pc, \text{class } 2 \rangle$ (7.3009)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - ba, pb - dc, pc - ba, \text{class } 2 \rangle$ (7.3010)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - ba, pb - dc, pc - dc, \text{class } 2 \rangle$ (7.3011)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - ba, pb - dc, pc - ba - dc, \text{class } 2 \rangle$ (7.3012)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc, pd - pb, \text{class } 2 \rangle$ (7.3013)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc, pd + ba - pb, \text{class } 2 \rangle$ (7.3014)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc, pd + ba + dc - pb, \text{class } 2 \rangle$ (7.3015)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc - ba, pd - pb, \text{class } 2 \rangle$ (7.3016)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc - ba, pd + dc - pb, \text{class } 2 \rangle$ (7.3017)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc - dc, pd - pb, \text{class } 2 \rangle$ (7.3018)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc - dc, pd + ba - pb, \text{class } 2 \rangle$ (7.3019)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc - ba - dc, pd - pb, \text{class } 2 \rangle$ (7.3020)
- $\langle a, b, c, d \mid ca, da, cb, db, pa, pc - ba - dc, pd + ba - pb, \text{class } 2 \rangle$ (7.3021)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - ba, pc - ba, pd - pb, \text{class } 2 \rangle$ (7.3022)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - ba, pc - ba, pd + dc - pb, \text{class } 2 \rangle$ (7.3023)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - ba, pc - ba - xdc, pd - pb, \text{class } 2 \rangle (x \neq 0)$ (7.3024)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - ba, pc - xdc, pd - pb, \text{class } 2 \rangle (x \neq 0, x \sim x^{-1})$ (7.3025)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - dc, pc - ba, pd - pb, \text{class } 2 \rangle$ (7.3026)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - dc, pc - \omega ba, pd - pb, \text{class } 2 \rangle$ (7.3027)
- $\langle a, b, c, d \mid ca, da, cb, db, pa - dc, pc - xba - dc, pd - pb, \text{class } 2 \rangle (x \neq 0)$ (7.3028)

$$\langle a, b, c, d \mid ca, da, cb, db, pa - ba - dc, pc - xba - ydc, pd - pb, \text{class } 2 \rangle (x, y \neq 0, (x, y) \sim (xy^{-2}, y^{-1})) \quad (7.3029)$$

$$\langle a, b, c, d \mid ca, da, cb, db, pa - ba - dc, pc - xba - xdc, pd + ba - pb, \text{class } 2 \rangle (x \neq 0, x \sim x^{-1}) \quad (7.3030)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb, pc, \text{class } 2 \rangle \quad (7.3031)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa - ba, pb, pc, \text{class } 2 \rangle \quad (7.3032)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa - ca, pb, pc, \text{class } 2 \rangle \quad (7.3033)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb - ba, pc, \text{class } 2 \rangle \quad (7.3034)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb - ca, pc, \text{class } 2 \rangle \quad (7.3035)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa - ba, pb - ca, pc, \text{class } 2 \rangle \quad (7.3036)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa - ca, pb - ba, pc, \text{class } 2 \rangle \quad (7.3037)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb, pc - ba, \text{class } 2 \rangle \quad (7.3038)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa - ca, pb, pc - ba, \text{class } 2 \rangle \quad (7.3039)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb - ba, pc - ba, \text{class } 2 \rangle \quad (7.3040)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa - ca, pb - ba, pc - ba, \text{class } 2 \rangle \quad (7.3041)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb - ba - xca, pc - ba, \text{class } 2 \rangle (x \neq 0) \quad (7.3042)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb - ca, pc - ba, \text{class } 2 \rangle \quad (7.3043)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb - \omega ca, pc - ba, \text{class } 2 \rangle \quad (7.3044)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb, pc - ca, \text{class } 2 \rangle \quad (7.3045)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa - ba, pb, pc - ca, \text{class } 2 \rangle \quad (7.3046)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb - xba, pc - ca, \text{class } 2 \rangle (x \neq 0) \quad (7.3047)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pa, pb - ba - ca, pc - ca, \text{class } 2 \rangle \quad (7.3048)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb, pc, pd, \text{class } 2 \rangle \quad (7.3049)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb - ba, pc, pd, \text{class } 2 \rangle \quad (7.3050)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb - ca, pc, pd, \text{class } 2 \rangle \quad (7.3051)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb, pc - ba, pd, \text{class } 2 \rangle \quad (7.3052)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb - ca, pc - ba, pd, \text{class } 2 \rangle \quad (7.3053)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb - \omega ca, pc - ba, pd, \text{class } 2 \rangle \quad (7.3054)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb - xba, pc - ca, pd, \text{class } 2 \rangle \quad (7.3055)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb, pc, pd - ba, \text{class } 2 \rangle \quad (7.3056)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb - ca, pc, pd - ba, \text{class } 2 \rangle \quad (7.3057)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb, pc, pd - ca, \text{class } 2 \rangle \quad (7.3058)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb - ba, pc, pd - ca, \text{class } 2 \rangle \quad (7.3059)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb, pc - ba, pd - ca, \text{class } 2 \rangle \quad (7.3060)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc, pb, pc - ca, pd - ba, \text{class } 2 \rangle \quad (7.3061)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc - \omega ba, pa, pb, pc, \text{class } 2 \rangle \quad (7.3062)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc - \omega ba, pa - ba, pb, pc, \text{class } 2 \rangle \quad (7.3063)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc - \omega ba, pa, pb - ba - xca, pc, \text{class } 2 \rangle (x \sim -x) \quad (7.3064)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc - \omega ba, pa - ca, pb - ba - xca, pc, \text{class } 2 \rangle (x \sim -x) \quad (7.3065)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc - \omega ba, pa, pb - ca, pc, \text{class } 2 \rangle \quad (7.3066)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc - \omega ba, pa - ba, pb - ca, pc, \text{class } 2 \rangle \quad (7.3067)$$

$$\langle a, b, c, d \mid da, cb, db - ca, dc - \omega ba, pa, pb - xba - yca, pc - zba - tca, \text{class } 2 \rangle \quad (7.3068)$$

Here, in presentation 7.3068, we put the parameters x, y, z, t in a matrix $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$. (We require A to be non-singular.) We have an action on the matrices A defined by

$$A \rightarrow \alpha^{-1} \begin{pmatrix} \mu & \nu \\ \pm\omega\nu & \pm\mu \end{pmatrix} A \begin{pmatrix} \mu & \nu \\ \pm\omega\nu & \pm\mu \end{pmatrix}^{-1}.$$

Two matrices give isomorphic algebras if they lie in the same orbit under this action. There are $(p+1)^2/2$ orbits.

$$\langle a, b, c, d \mid da, db, dc, pa, pb, pc, pd, \text{ class } 2 \rangle \quad (7.3069)$$

$$\langle a, b, c, d \mid da, db, dc, pa - cb, pb, pc, pd, \text{ class } 2 \rangle \quad (7.3070)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb, pc, pd, \text{ class } 2 \rangle \quad (7.3071)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ca, pb - xcb, pc, pd, \text{ class } 2 \rangle (x \neq 0, x \sim x^{-1}) \quad (7.3072)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ca - cb, pb - cb, pc, pd, \text{ class } 2 \rangle \quad (7.3073)$$

$$\langle a, b, c, d \mid da, db, dc, pa - \omega cb, pb - ca, pc, pd, \text{ class } 2 \rangle \quad (7.3074)$$

$$\langle a, b, c, d \mid da, db, dc, pa - xcb, pb - ca - cb, pc, pd, \text{ class } 2 \rangle (1 + 4x \text{ not a square}) \quad (7.3075)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb - ca, pc, pd, \text{ class } 2 \rangle \quad (7.3076)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb - cb, pc, pd, \text{ class } 2 \rangle \quad (7.3077)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb - cb, pc - xba - ca, pd, \text{ class } 2 \rangle \quad (7.3078)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb - ca, pc - cb, pd, \text{ class } 2 \rangle \quad (7.3079)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb - ca, pc + cb, pd, \text{ class } 2 \rangle \quad (7.3080)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb - ca, pc - \omega ba + cb, pd, \text{ class } 2 \rangle \quad (7.3081)$$

$$\langle a, b, c, d \mid da, db, dc, pa, pb, pc, pd - cb, \text{ class } 2 \rangle \quad (7.3082)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb, pc, pd - cb, \text{ class } 2 \rangle \quad (7.3083)$$

$$\langle a, b, c, d \mid da, db, dc, pa, pb - ba, pc, pd - cb, \text{ class } 2 \rangle \quad (7.3084)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ca, pb - ba, pc, pd - cb, \text{ class } 2 \rangle \quad (7.3085)$$

$$\langle a, b, c, d \mid da, db, dc, pa, pb - ca, pc, pd - cb, \text{ class } 2 \rangle \quad (7.3086)$$

$$\langle a, b, c, d \mid da, db, dc, pa - ba, pb - ca, pc, pd - cb, \text{ class } 2 \rangle \quad (7.3087)$$

$$\langle a, b, c, d \mid da, db, dc, pa, pb - ba, pc - xca, pd - cb, \text{ class } 2 \rangle (x \neq 0, x \sim x^{-1}) \quad (7.3088)$$

$$\langle a, b, c, d \mid da, db, dc, pa, pb - ba - ca, pc - ca, pd - cb, \text{ class } 2 \rangle \quad (7.3089)$$

- $\langle a, b, c, d \mid da, db, dc, pa, pb - \omega ca, pc - ba, pd - cb, \text{class } 2 \rangle$ (7.3090)
- $\langle a, b, c, d \mid da, db, dc, pa, pb - xca, pc - ba - ca, pd - cb, \text{class } 2 \rangle$ ($1 + 4x$ not a square) (7.3091)
- $\langle a, b, c, d \mid ca, da, db, pa, pb, pc - cb, pd, \text{class } 2 \rangle$ (7.3092)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - dc, pc - cb, pd, \text{class } 2 \rangle$ (7.3093)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - ba, pc - cb, pd, \text{class } 2 \rangle$ (7.3094)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - ba - dc, pc - cb, pd, \text{class } 2 \rangle$ (7.3095)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - cb, pc - cb, pd, \text{class } 2 \rangle$ (7.3096)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - cb, pc - cb - dc, pd, \text{class } 2 \rangle$ (7.3097)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - cb, pc - ba - cb - dc, pd, \text{class } 2 \rangle$ (7.3098)
- $\langle a, b, c, d \mid ca, da, db, pa, pb, pc, pd, \text{class } 2 \rangle$ (7.3099)
- $\langle a, b, c, d \mid ca, da, db, pa, pb, pc - dc, pd, \text{class } 2 \rangle$ (7.3100)
- $\langle a, b, c, d \mid ca, da, db, pa, pb, pc - ba, pd, \text{class } 2 \rangle$ (7.3101)
- $\langle a, b, c, d \mid ca, da, db, pa, pb, pc - ba - dc, pd, \text{class } 2 \rangle$ (7.3102)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - dc, pc - dc, pd, \text{class } 2 \rangle$ (7.3103)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - dc, pc - ba, pd, \text{class } 2 \rangle$ (7.3104)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - dc, pc - ba - dc, pd, \text{class } 2 \rangle$ (7.3105)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - ba, pc - dc, pd, \text{class } 2 \rangle$ (7.3106)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - ba, pc - ba - dc, pd, \text{class } 2 \rangle$ (7.3107)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - ba - dc, pc - ba - xdc, pd, \text{class } 2 \rangle$ ($x \neq 0$) (7.3108)
- $\langle a, b, c, d \mid ca, da, db, pa, pb - zcb, pc - xba - ycb, pd - dc, \text{class } 2 \rangle$ ($x = 0, 1, y = 0, 1, z \neq 0, -1$) (7.3109)
- $\langle a, b, c, d \mid ca, da, db, pa, pb, pc, pd - dc, \text{class } 2 \rangle$ (7.3110)
- $\langle a, b, c, d \mid ca, da, db, pa, pb, pc - ba, pd - dc, \text{class } 2 \rangle$ (7.3111)

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ba, pc, pd - dc, \text{class } 2 \rangle \quad (7.3112)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ba, pc - ba, pd - dc, \text{class } 2 \rangle \quad (7.3113)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb, pc - cb, pd - dc, \text{class } 2 \rangle \quad (7.3114)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ba, pc - cb, pd - dc, \text{class } 2 \rangle \quad (7.3115)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb, pc, pd - dc, \text{class } 2 \rangle \quad (7.3116)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb, pc - cb, pd - dc, \text{class } 2 \rangle \quad (7.3117)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb, pc - ba, pd - dc, \text{class } 2 \rangle \quad (7.3118)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb, pc - ba - cb, pd - dc, \text{class } 2 \rangle \quad (7.3119)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb - dc, pc, pd - dc, \text{class } 2 \rangle \quad (7.3120)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb - dc, pc - cb, pd - dc, \text{class } 2 \rangle \quad (7.3121)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb - dc, pc - ba, pd - dc, \text{class } 2 \rangle \quad (7.3122)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb - dc, pc - ba - cb, pd - dc, \text{class } 2 \rangle \quad (7.3123)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ba - cb - xdc, pc - ba - ydc, pd - cb, \text{class } 2 \rangle \quad (7.3124)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - cb - zdc, pc - xba - ydc, pd - cb, \text{class } 2 \rangle (x = 0, 1, y = 0, 1) \quad (7.3125)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - zdc, pc - xba - ydc, pd - cb, \text{class } 2 \rangle (x = 0, 1, y = 0, 1, z = 0, 1, \omega) \quad (7.3126)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ba - xdc, pc - ydc, pd - cb, \text{class } 2 \rangle (x = 0, 1, \omega, y = 0, 1) \quad (7.3127)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ba - xdc, pc - ba - ydc, pd - cb, \text{class } 2 \rangle (x = 1, \omega) \quad (7.3128)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ba, pc - ba - xdc, pd - cb, \text{class } 2 \rangle (x = 0, 1, \omega) \quad (7.3129)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ba - cb - ydc, pc - xdc, pd - cb, \text{class } 2 \rangle (x = 0, 1) \quad (7.3130)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - cb, pc, pd - ba, \text{class } 2 \rangle \quad (7.3131)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - cb, pc - dc, pd - ba, \text{class } 2 \rangle \quad (7.3132)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - cb, pc - cb, pd - ba, \text{class } 2 \rangle \quad (7.3133)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - cb, pc - cb - dc, pd - ba, \text{class } 2 \rangle \quad (7.3134)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ydc, pc - zcb - xdc, pd - ba, \text{class } 2 \rangle (x = 0, 1, y = 0, 1, z = 0, 1) \quad (7.3135)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - ycb, pc - xdc, pd - ba - dc, \text{class } 2 \rangle (x = 0, 1) \quad (7.3136)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - xcb - dc, pc - ydc, pd - ba - dc, \text{class } 2 \rangle (x = 0, -1, y = 0, 1) \quad (7.3137)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb - xcb - ydc, pc - cb, pd - ba - dc, \text{class } 2 \rangle (y = 0, 1) \quad (7.3138)$$

$$\langle a, b, c, d \mid ca, da, db, pa, pb + cb - xdc, pc - cb - dc, pd - ba - dc, \text{class } 2 \rangle \quad (7.3139)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - xba - ycb, pc - cb, pd - dc, \text{class } 2 \rangle (x = 0, 1) \quad (7.3140)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - ycb, pc - xba, pd - dc, \text{class } 2 \rangle (x = 0, 1) \quad (7.3141)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - ba, pc - xba, pd - dc, \text{class } 2 \rangle (x = 0, 1) \quad (7.3142)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - xba - cb, pc - yba - dc, pd - cb, \text{class } 2 \rangle \quad (7.3143)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - ba - cb, pc - xba, pd - cb, \text{class } 2 \rangle \quad (7.3144)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - cb, pc - xba, pd - cb, \text{class } 2 \rangle (x = 0, 1, \omega) \quad (7.3145)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - ba, pc - xba - dc, pd - cb, \text{class } 2 \rangle \quad (7.3146)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb, pc - xba - dc, pd - cb, \text{class } 2 \rangle (x = 0, 1) \quad (7.3147)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - xba, pc - yba, pd - cb, \text{class } 2 \rangle (x = 0, 1, y = 0, 1) \quad (7.3148)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - ba, pc - \omega ba, pd - cb, \text{class } 2 \rangle (p = 1 \bmod 3) \quad (7.3149)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - ba, pc - \omega^2 ba, pd - cb, \text{class } 2 \rangle (p = 1 \bmod 3) \quad (7.3150)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb, pc, pd - ba, \text{class } 2 \rangle \quad (7.3151)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - cb, pc - cb - dc, pd - ba, \text{class } 2 \rangle \quad (7.3152)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - cb, pc - xcb, pd - ba, \text{class } 2 \rangle \quad (7.3153)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - xcb, pc - ycb, pd - ba - dc, \text{class } 2 \rangle \quad (7.3154)$$

$$\langle a, b, c, d \mid ca, da, db, pa - dc, pb - xcb, pc - (1 + x)^{-1} cb - dc, pd - ba - dc, \text{class } 2 \rangle (x \neq -1) \quad (7.3155)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ba - xdc, pc - zba - ycb, pd - dc, \text{class } 2 \rangle (x = 0, 1) \quad (7.3156)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - xdc, pc - yba - cb, pd - dc, \text{class } 2 \rangle (x = 0, 1) \quad (7.3157)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ydc, pc - xba, pd - dc, \text{class } 2 \rangle (x = 0, 1, \omega, y = 0, 1) \quad (7.3158)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb, pc, pd - cb, \text{class } 2 \rangle \quad (7.3159)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb, pc - ba, pd - cb, \text{class } 2 \rangle \quad (7.3160)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb, pc - \omega ba, pd - cb, \text{class } 2 \rangle \quad (7.3161)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - dc, pc - ba, pd - cb, \text{class } 2 \rangle \quad (7.3162)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - dc, pc - \omega ba, pd - cb, \text{class } 2 \rangle \quad (7.3163)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - \omega dc, pc - \omega ba, pd - cb, \text{class } 2 \rangle \quad (7.3164)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - xdc, pc - yba - dc, pd - cb, \text{class } 2 \rangle (x = 1, \omega) \quad (7.3165)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb, pc - xba - dc, pd - cb, \text{class } 2 \rangle (x = 0, 1, \omega) \quad (7.3166)$$

In presentations 7.3167 and 7.3168 below we give a complete set of representatives for the isomorphism classes of algebras

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - xba - ydc, pc - zba - tdc, pd - cb, \text{class } 2 \rangle$$

where $x, t \neq 0$, $x \neq \pm t$. Here (x, y, z, t) and (t, z, y, x) give isomorphic algebras, as do (x, y, z, t) and (abx, a^2y, b^2z, abt) for all $a, b \neq 0$. So we can take $x = 1$ and $t = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}$. With x, t fixed we can take $y = 1, \omega$ and z arbitrary, or take $y = 0$ and $z = 0, 1, \omega$.

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ba - ydc, pc - zba - tdc, pd - cb, \text{class } 2 \rangle (y = 1, \omega, t = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.3167)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ba, pc - zba - tdc, pd - cb, \text{class } 2 \rangle (z = 0, 1, \omega, t = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.3168)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ba - dc, pc - xba - ydc, pd - cb, \text{class } 2 \rangle (x \neq 0, y = 1, -1) \quad (7.3169)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ba - \omega dc, pc - \omega x^2 ba - ydc, pd - cb, \text{class } 2 \rangle (x \neq 0, y = 1, -1, (x, y) \sim (-x, y)) \quad (7.3170)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ba, pc - xba - ydc, pd - cb, \text{class } 2 \rangle (x = 0, 1, \omega, y = 1, -1) \quad (7.3171)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ba - ydc, pc - zba - xdc, pd - cb - dc, \text{class } 2 \rangle \quad (7.3172)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ydc, pc - xba - dc, pd - cb - dc, \text{class } 2 \rangle \quad (7.3173)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ydc, pc - xba, pd - cb - dc, \text{class } 2 \rangle (x = 0, 1, \omega) \quad (7.3174)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - ba - ydc, pc - zcb - xdc, pd - ba - dc, \text{class } 2 \rangle \quad (7.3175)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - dc, pc - xcb - ydc, pd - ba - dc, \text{class } 2 \rangle \quad (7.3176)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb, pc - cb - xdc, pd - ba - dc, \text{class } 2 \rangle \quad (7.3177)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb, pc - xdc, pd - ba - dc, \text{class } 2 \rangle (x = 0, 1, \omega) \quad (7.3178)$$

$$\langle a, b, c, d \mid ca, da, db, pa - cb, pb - xba - ydc, pc - zba - tdc, pd - ba - cb - dc, \text{class } 2 \rangle ((x, y, z, t) \sim (t + 1, z + 1, y - 1, x - 1)) \quad (7.3179)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba, pb - xcb, pc - ycb, pd - dc, \text{class } 2 \rangle ((x, y) \sim (y, x)) \quad (7.3180)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba, pb - xcb, pc - ba + cb, pd - dc, \text{class } 2 \rangle \quad (7.3181)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba, pb + cb - dc, pc - ba + cb, pd - dc, \text{class } 2 \rangle \quad (7.3182)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba, pb - xcb, pc - ycb, pd - ba - dc, \text{class } 2 \rangle \quad (7.3183)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba, pb + cb - dc, pc - xcb, pd - ba - dc, \text{class } 2 \rangle \quad (7.3184)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba, pb - xcb, pc + cb - dc, pd - ba - dc, \text{class } 2 \rangle (x \neq -1) \quad (7.3185)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba - dc, pb - xcb, pc - zcb, pd - ba - ydc, \text{class } 2 \rangle (y \neq 0, (x, y, z) \sim (yz, y, \frac{x}{y})) \quad (7.3186)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba - dc, pb - xcb, pc - ycb - dc, pd - ba - dc, \text{class } 2 \rangle (x, y \neq 0, (x, y) \sim (y, x)) \quad (7.3187)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba - dc, pb - dc, pc - xcb, pd - ba - dc, \text{class } 2 \rangle (x \neq 0) \quad (7.3188)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba - dc, pb, pc - dc, pd - ba - dc, \text{class } 2 \rangle \quad (7.3189)$$

$$\langle a, b, c, d \mid ca, da, db, pa - ba - dc, pb - dc, pc - xdc, pd - ba - dc, \text{class } 2 \rangle (x \neq 0, x \sim x^{-1}) \quad (7.3190)$$

In the presentation below we have $y \neq 0, 1, (x + y)(1 + z) = 1, (x, y, z) \sim (yz, y, \frac{x}{y})$:

$$\langle a, b, c, d \mid ca, da, db, pa - ba - dc, pb - xcb, pc - zcb - dc, pd - ba - ydc, \text{class } 2 \rangle \quad (7.3191)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - ba, pc - ca, pd - xda, \text{class } 2 \rangle (x \neq 0) \quad (7.3192)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - ba - ca, pc - ca, pd - da, \text{class } 2 \rangle \quad (7.3193)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - xda, pc - ba - yda, pd - ca - da, \text{class } 2 \rangle (x \neq 0) \quad (7.3194)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - xda, pc - ba - yda, pd - ca, \text{class } 2 \rangle (x \neq 0, y = 1, \omega, (x, y) \sim (-x, y)) \quad (7.3195)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - da, pc - ba, pd - ca, \text{class } 2 \rangle \quad (7.3196)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - \omega da, pc - ba, pd - ca, \text{class } 2 \rangle (p = 1 \pmod{3}) \quad (7.3197)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - \omega^2 da, pc - ba, pd - ca, \text{class } 2 \rangle (p = 1 \pmod{3}) \quad (7.3198)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - ca, pc - da, pd, \text{class } 2 \rangle \quad (7.3199)$$

$$\langle a, b, c, d \mid cb, db, dc, pa - ba, pb - ca, pc - da, pd, \text{class } 2 \rangle \quad (7.3200)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - ba, pc - da, pd, \text{class } 2 \rangle \quad (7.3201)$$

$$\langle a, b, c, d \mid cb, db, dc, pa - ca, pb - ba, pc - da, pd, \text{class } 2 \rangle \quad (7.3202)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - ba, pc - ca, pd, \text{class } 2 \rangle \quad (7.3203)$$

$$\langle a, b, c, d \mid cb, db, dc, pa - da, pb - ba, pc - ca, pd, \text{class } 2 \rangle \quad (7.3204)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - xca, pc - ba - ca, pd, \text{class } 2 \rangle (x \neq 0) \quad (7.3205)$$

$$\langle a, b, c, d \mid cb, db, dc, pa - da, pb - xca, pc - ba - ca, pd, \text{class } 2 \rangle (x \neq 0) \quad (7.3206)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - ca, pc - ba, pd, \text{class } 2 \rangle \quad (7.3207)$$

$$\langle a, b, c, d \mid cb, db, dc, pa - da, pb - ca, pc - ba, pd, \text{class } 2 \rangle \quad (7.3208)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - \omega ca, pc - ba, pd, \text{class } 2 \rangle \quad (7.3209)$$

$$\langle a, b, c, d \mid cb, db, dc, pa - da, pb - \omega ca, pc - ba, pd, \text{class } 2 \rangle \quad (7.3210)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - ba, pc, pd, \text{class } 2 \rangle \quad (7.3211)$$

$$\langle a, b, c, d \mid cb, db, dc, pa - ca, pb - ba, pc, pd, \text{class } 2 \rangle \quad (7.3212)$$

$$\langle a, b, c, d \mid cb, db, dc, pa, pb - ca, pc, pd, \text{class } 2 \rangle \quad (7.3213)$$

$$\langle a, b, c, d \mid cb, db, dc, pa - ba, pb - ca, pc, pd, \text{class } 2 \rangle \quad (7.3214)$$

- $\langle a, b, c, d \mid cb, db, dc, pa - da, pb - ca, pc, pd, \text{class } 2 \rangle$ (7.3215)
- $\langle a, b, c, d \mid cb, db, dc, pa, pb, pc, pd, \text{class } 2 \rangle$ (7.3216)
- $\langle a, b, c, d \mid cb, db, dc, pa - ba, pb, pc, pd, \text{class } 2 \rangle$ (7.3217)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc, pd, \text{class } 2 \rangle$ (7.3218)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - ba, pc, pd, \text{class } 2 \rangle$ (7.3219)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - cb, pc, pd, \text{class } 2 \rangle$ (7.3220)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - cb, pc - ba, pd, \text{class } 2 \rangle$ (7.3221)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - xdb, pc - ba, pd, \text{class } 2 \rangle$ (7.3222)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc - cb, pd, \text{class } 2 \rangle$ (7.3223)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - ba, pc - cb, pd, \text{class } 2 \rangle$ (7.3224)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - db, pc - cb, pd, \text{class } 2 \rangle$ (7.3225)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc - db, pd, \text{class } 2 \rangle$ (7.3226)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - ba, pc - db, pd, \text{class } 2 \rangle$ (7.3227)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - \omega ba, pc - db, pd, \text{class } 2 \rangle$ (7.3228)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - cb, pc - db, pd, \text{class } 2 \rangle$ (7.3229)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - xdb, pc - ba, pd - cb, \text{class } 2 \rangle$ (7.3230)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc - ba, pd - db, \text{class } 2 \rangle$ (7.3231)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - cb, pc - ba, pd - db, \text{class } 2 \rangle$ (7.3232)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc - cb, pd - db, \text{class } 2 \rangle$ (7.3233)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - ba, pc - cb, pd - db, \text{class } 2 \rangle$ (7.3234)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc - cb - db, pd - db, \text{class } 2 \rangle$ (7.3235)
- $\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - ba, pc - cb - db, pd - db, \text{class } 2 \rangle$ (7.3236)

$$\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - \omega ba, pc - cb - db, pd - db, \text{class } 2 \rangle \quad (7.3237)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc - xdb, pd - cb - db, \text{class } 2 \rangle \left(x \neq 0, -\frac{1}{4}\right) \quad (7.3238)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - ba, pc - xdb, pd - cb - db, \text{class } 2 \rangle \left(x \neq 0, -\frac{1}{4}\right) \quad (7.3239)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc - db, pd - cb, \text{class } 2 \rangle \quad (7.3240)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - ba, pc - db, pd - cb, \text{class } 2 \rangle \quad (7.3241)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa, pb, pc - \omega db, pd - cb, \text{class } 2 \rangle \quad (7.3242)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa, pb - ba, pc - \omega db, pd - cb, \text{class } 2 \rangle \quad (7.3243)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc, pd, \text{class } 2 \rangle \quad (7.3244)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb - cb, pc, pd, \text{class } 2 \rangle \quad (7.3245)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc - xcb, pd, \text{class } 2 \rangle \left(x \neq 0\right) \quad (7.3246)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb - db, pc - xcb, pd, \text{class } 2 \rangle \left(x \neq 0\right) \quad (7.3247)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc, pd - cb, \text{class } 2 \rangle \quad (7.3248)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb - db, pc, pd - cb, \text{class } 2 \rangle \quad (7.3249)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc - xcb, pd - xdb, \text{class } 2 \rangle \left(x \neq 0\right) \quad (7.3250)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc - ba + cb, pd + db, \text{class } 2 \rangle \quad (7.3251)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc - xcb, pd - ydb, \text{class } 2 \rangle \left(x, y \neq 0, x \neq y, (x, y) \sim (y, x)\right) \quad (7.3252)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc - xcb, pd - ba + db, \text{class } 2 \rangle \left(x \neq 0, -1\right) \quad (7.3253)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc - xcb, pd - cb - xdb, \text{class } 2 \rangle \left(x \neq 0\right) \quad (7.3254)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc - ba + cb, pd - cb + db, \text{class } 2 \rangle \quad (7.3255)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - ba, pb, pc - xdb, pd - cb - ydb, \text{class } 2 \rangle \left(x \neq 0, 4x + y^2 \text{ not a square}\right) \quad (7.3256)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - cb, pb, pc, pd, \text{class } 2 \rangle \quad (7.3257)$$

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$$\langle a, b, c, d \mid ca, da, dc - ba, pa - cb, pb - xdb, pc - db, pd - \omega ba, \text{ class 2} \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.3280)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - cb, pb - xdb, pc - db, pd - \omega^2 ba, \text{ class 2} \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.3281)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - cb, pb - xba - ydb, pc - db, pd - ba, \text{ class 2} \rangle (x \neq 0, (x, y) \sim (ax, a^2y) \text{ if } a^3 = 1) \quad (7.3282)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - cb, pb - xba - ydb, pc - db, pd - \omega ba, \text{ class 2} \rangle (x \neq 0, (x, y) \sim (ax, a^2y) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.3283)$$

$$\langle a, b, c, d \mid ca, da, dc - ba, pa - cb, pb - xba - ydb, pc - db, pd - \omega^2 ba, \text{ class 2} \rangle (x \neq 0, (x, y) \sim (ax, a^2y) \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.3284)$$

$$\langle a, b, c, d \mid cb, da, db - ca, pa - jba - kca - mdc, pb - nba - rca - sdc, pc - tba - uca - vdc, pd - xba - yca - zdc, \text{ class 2} \rangle \quad (7.3285)$$

In presentation 7.3285 we have 12 parameters giving the values of pa, pb, pc, pd . We write

$$\begin{pmatrix} pa \\ pb \\ pc \\ pd \end{pmatrix} = A \begin{pmatrix} ba \\ ca \\ dc \end{pmatrix}$$

with A a 4×3 matrix. Since ba, ca, dc all have order p , we can think of the matrix A as having entries in $\text{GF}(p)$. Let L be the Lie ring corresponding to the matrix $A = 0$, and let G be the automorphism group of L . The automorphism group G induces a group H of matrices in $\text{GL}(4, p)$ giving the action of G on L/L^2 , and H consists of the non-singular matrices of the form

$$\begin{pmatrix} \alpha\lambda & \beta\lambda & \beta\mu & -\alpha\mu \\ \gamma\lambda & \delta\lambda & \delta\mu & -\gamma\mu \\ \gamma\nu & \delta\nu & \delta\xi & -\gamma\xi \\ -\alpha\nu & -\beta\nu & -\beta\xi & \alpha\xi \end{pmatrix}.$$

If we let V be the $\text{GF}(p)$ subspace of L spanned by a, b, c, d , then the matrices in H are precisely the matrices in $\text{GL}(4, p)$ which define non-singular linear transformations from V into V which preserve the relations $cb = da = db - ca = 0$. There is a homomorphism from $\text{GL}(2, p) \times \text{GL}(2, p) \rightarrow H$ given by

$$\left(\begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \begin{pmatrix} \lambda & \mu \\ \nu & \xi \end{pmatrix} \right) \mapsto \begin{pmatrix} \alpha\lambda & \beta\lambda & \beta\mu & -\alpha\mu \\ \gamma\lambda & \delta\lambda & \delta\mu & -\gamma\mu \\ \gamma\nu & \delta\nu & \delta\xi & -\gamma\xi \\ -\alpha\nu & -\beta\nu & -\beta\xi & \alpha\xi \end{pmatrix},$$

and the kernel of this homomorphism is $\{(kI, k^{-1}I) \mid k \in \text{GF}(p) \setminus \{0\}\}$. So H has order $(p^2 - 1)^2(p^2 - p)^2/(p - 1)$. There is an action of H on the space of 4×3 matrices over $\text{GF}(p)$ given by

$$A \mapsto (\alpha\delta - \beta\gamma)^{-1} \begin{pmatrix} \alpha\lambda & \beta\lambda & \beta\mu & -\alpha\mu \\ \gamma\lambda & \delta\lambda & \delta\mu & -\gamma\mu \\ \gamma\nu & \delta\nu & \delta\xi & -\gamma\xi \\ -\alpha\nu & -\beta\nu & -\beta\xi & \alpha\xi \end{pmatrix} A \begin{pmatrix} \lambda^2 & 2\lambda\mu & \mu^2 \\ \lambda\nu & \lambda\xi + \mu\nu & \mu\xi \\ \nu^2 & 2\nu\xi & \xi^2 \end{pmatrix}^{-1},$$

and two matrices give isomorphic groups if and only if they lie in the same orbit under the action of H . So, up to isomorphism, the number of Lie rings 7.3285 is the number of orbits of 4×3 matrices under the action of H . If M is the

Lie ring corresponding to a particular matrix A , then we have a homomorphism from the automorphism group of M into H , where the image of $\theta \in \text{Aut}(M)$ is the linear transformation induced by θ on M/M^2 . The image of $\text{Aut}(M)$ is the stabilizer in H of the matrix A .

If we take $\mu = 0$ in the matrices in H , then we obtain a subgroup K of index $p + 1$ in H . The subspace spanned by a, b is invariant under K , and it turns out that pa, pb has 11 orbits under the action of K . So the problem of computing orbits of 12 parameters reduces to 11 problems with 6 parameters. There are 550 orbits when $p = 3$ and

$$\begin{aligned} p^5 + p^4 + 4p^3 + 6p^2 + 18p + 19 & \text{ if } p \equiv 1 \pmod{3}, \\ p^5 + p^4 + 4p^3 + 6p^2 + 16p + 17 & \text{ if } p \equiv 2 \pmod{3} \end{aligned}$$

orbits when $p > 3$. This implies that we need presentations with at least 5 parameters to describe this situation, so 6 parameters is not at all bad! Nevertheless computing the orbits for a specific prime p require serious computational effort.

$$\langle a, b, c, d \mid da, db - \omega ca, dc - ba, pa - jba - kca - mcb, pb - nba - rca - scb, pc - tba - uca - vcb, pd - xba - yca - zcb, \text{ class 2} \rangle \quad (7.3286)$$

In presentation 7.3286 we again have 12 parameters giving the values of pa, pb, pc, pd . We let

$$\begin{pmatrix} pa \\ pb \\ pc \\ pd \end{pmatrix} = A \begin{pmatrix} ba \\ ca \\ cb \end{pmatrix}$$

where A is a 4×3 matrix. The isomorphism classes given by the matrices correspond to their orbits under transformations of the form

$$A \mapsto \begin{pmatrix} \alpha & 0 & 0 & \delta \\ \pm\lambda & \pm\gamma & \pm\omega\beta & \pm\mu \\ \nu & \beta & \gamma & \xi \\ \pm\omega\delta & 0 & 0 & \pm\alpha \end{pmatrix} AB^{-1},$$

where

$$B = \begin{pmatrix} \pm(\alpha\gamma - \omega\beta\delta) & \pm(\omega\alpha\beta - \omega\gamma\delta) & 0 \\ \alpha\beta - \gamma\delta & \alpha\gamma - \omega\beta\delta & 0 \\ \pm(\beta\lambda - \gamma\nu + \omega\beta\xi - \gamma\mu) & \pm(\gamma\lambda - \omega\beta\mu + \omega\gamma\xi - \omega\beta\nu) & \pm(\gamma^2 - \omega\beta^2) \end{pmatrix}.$$

We note that $\langle a, d \rangle + L^2$ is a characteristic subalgebra, and first investigate the orbits of pa, pd . We consider three separate cases: $pa = pd = 0$, pa and pd span a one dimensional subspace, and pa, pd are linearly independent. It turns out that there are $p + 4$ orbits of pa, pd . We pick a set of orbit representatives for pa, pd , and work out a set of orbit representatives for pc, pd in each case. There are $\frac{9}{2}p + \frac{13}{2} + 3p^2 + \frac{1}{2}p^4 + \frac{1}{2}p^3$ orbits of matrices A .

8.198 Descendants of 5.3

$p^4 + 5p^3 + 19p^2 + 64p + 140 + (p + 6) \gcd(p - 1, 3) + (p + 7) \gcd(p - 1, 4) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c, d \mid ca, cb, da, db, dc, pa, pb, pc - baa, pd - bab, \text{ class 3} \rangle \quad (7.3287)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc, pa, pb, pc - bab, pd, \text{ class 3} \rangle \quad (7.3288)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc, pa - baa, pb, pc - bab, pd, \text{ class 3} \rangle \quad (7.3289)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc, pa, pb - baa, pc - bab, pd, \text{ class 3} \rangle \quad (7.3290)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc, pa, pb - \omega baa, pc - bab, pd, \text{ class 3} \rangle \quad (7.3291)$$

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- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3314)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - \omega bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3315)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb - xbaa, pc - bab, pd, \text{class } 3 \rangle$ (7.3316)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - baa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3317)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3318)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - bab, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3319)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - \omega bab, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3320)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - \omega^2 bab, pb, pc - baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.3321)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - \omega^3 bab, pb, pc - baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.3322)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - xbab, pb - bab, pc - baa, pd, \text{class } 3 \rangle$ ($x \sim ax$ if $a^3 = 1$) (7.3323)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - xbab, pb - \omega bab, pc - baa, pd, \text{class } 3 \rangle$ ($x \sim ax$ if $a^3 = 1, p = 1 \pmod{3}$) (7.3324)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - xbab, pb - \omega^2 bab, pc - baa, pd, \text{class } 3 \rangle$ ($x \sim ax$ if $a^3 = 1, p = 1 \pmod{3}$) (7.3325)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb, pc - bab, pd - baa, \text{class } 3 \rangle$ (7.3326)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb, pc, pd - baa, \text{class } 3 \rangle$ (7.3327)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - bab, pb, pc, pd - baa, \text{class } 3 \rangle$ (7.3328)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - \omega bab, pb, pc, pd - baa, \text{class } 3 \rangle$ (7.3329)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb - bab, pc, pd - baa, \text{class } 3 \rangle$ (7.3330)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - bab, pb - bab, pc, pd - baa, \text{class } 3 \rangle$ (7.3331)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - \omega bab, pb - bab, pc, pd - baa, \text{class } 3 \rangle$ (7.3332)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb, pc - baa, pd - bab, \text{class } 3 \rangle$ (7.3333)
- $\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb - baa, pc, pd - bab, \text{class } 3 \rangle$ (7.3334)

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb - \omega baa, pc, pd - bab, \text{class } 3 \rangle \quad (7.3335)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle \quad (7.3336)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc, pa - baa, pb, pc, pd - bab, \text{class } 3 \rangle \quad (7.3337)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.3338)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa, pb, pc, pd, \text{class } 3 \rangle \quad (7.3339)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3340)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3341)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3342)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3343)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - bab, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3344)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3345)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - \omega bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle \ (p = 1 \bmod 4) \quad (7.3346)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - \omega bab, pb - baa, pc, pd, \text{class } 3 \rangle \ (p = 3 \bmod 4) \quad (7.3347)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - xbab, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3348)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - xbab, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3349)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa, pb - xbab, pc, pd, \text{class } 3 \rangle \ (x \neq 0, x \sim x^{-1}) \quad (7.3350)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - bab, pb - xbab, pc, pd, \text{class } 3 \rangle \ (x \neq 0) \quad (7.3351)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - \omega bab, pb - xbab, pc, pd, \text{class } 3 \rangle \ (x \neq 0) \quad (7.3352)$$

In presentations 7.3353 and 7.3354 below we give a complete set of representatives for the isomorphism classes of algebras

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - zbaa - tbab, pb - xbaa - ybab, pc, pd, \text{class } 3 \rangle$$

where $x, y, z, t \neq 0$, $y \neq \pm z$. Here (x, y, z, t) and $(-t, z, y, -x)$ give isomorphic algebras, as do (x, y, z, t) and (a^2x, aby, abz, b^2t) for all $a, b \neq 0$. So we can take $z = 1$ and $y = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}$. With y, z fixed we can take $t = 1, \omega$.

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - bab, pb - xbaa - ybab, pc, pd, \text{class } 3 \rangle \ (x \neq 0, y = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.3353)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - \omega bab, pb - xbaa - ybab, pc, pd, \text{class } 3 \rangle (x \neq 0, y = \omega, \omega^2, \dots, \omega^{\frac{p-3}{2}}) \quad (7.3354)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - bab, pb - x^2baa + bab, pc, pd, \text{class } 3 \rangle (x \neq 0, x \sim -x) \quad (7.3355)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - bab, pb - x^2baa - bab, pc, pd, \text{class } 3 \rangle (x \neq 0, x \sim -x) \quad (7.3356)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - bab, pb - x^2\omega baa + bab, pc, pd, \text{class } 3 \rangle (x \neq 0, x \sim -x) \quad (7.3357)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - bab, pb - x^2\omega baa - bab, pc, pd, \text{class } 3 \rangle (x \neq 0, x \sim -x) \quad (7.3358)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - \omega bab, pb - x^2\omega baa + bab, pc, pd, \text{class } 3 \rangle (x \neq 0, x \sim -x, p = 1 \pmod{4}) \quad (7.3359)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - \omega bab, pb - x^2\omega baa - bab, pc, pd, \text{class } 3 \rangle (x \neq 0, x \sim -x, p = 1 \pmod{4}) \quad (7.3360)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - \omega bab, pb - x^2baa + bab, pc, pd, \text{class } 3 \rangle (x \neq 0, x \sim -x, p = 3 \pmod{4}) \quad (7.3361)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa - \omega bab, pb - x^2baa - bab, pc, pd, \text{class } 3 \rangle (x \neq 0, x \sim -x, p = 3 \pmod{4}) \quad (7.3362)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.3363)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - bab, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.3364)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - \omega bab, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.3365)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - xbab, pb - bab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3366)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - xbab, pb - ybab, pc - baa - bab, pd, \text{class } 3 \rangle ((x, y) \sim (y, x)) \quad (7.3367)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa, pb, pc - bab, pd - baa, \text{class } 3 \rangle \quad (7.3368)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.3369)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - bab, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.3370)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - \omega bab, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.3371)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa, pb - bab, pc, pd - baa, \text{class } 3 \rangle \quad (7.3372)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - bab, pb - bab, pc, pd - baa, \text{ class } 3 \rangle \quad (7.3373)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - \omega bab, pb - bab, pc, pd - baa, \text{ class } 3 \rangle \quad (7.3374)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa, pb, pc - baa, pd - baa - bab, \text{ class } 3 \rangle \quad (7.3375)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa, pb, pc, pd - baa - bab, \text{ class } 3 \rangle \quad (7.3376)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa, pb, pc, pd - baa - bab, \text{ class } 3 \rangle \quad (7.3377)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - \omega baa, pb, pc, pd - baa - bab, \text{ class } 3 \rangle \quad (7.3378)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa, pb - \omega x^2 baa, pc, pd - baa - bab, \text{ class } 3 \rangle (x \neq 0, x \sim -x) \quad (7.3379)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - baa, pb - x^2 baa, pc, pd - baa - bab, \text{ class } 3 \rangle (x \neq 0, x \sim -x \sim x^{-1}) \quad (7.3380)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, pa - \omega baa, pb - \omega x^2 baa, pc, pd - baa - bab, \text{ class } 3 \rangle (x \neq 0, x \sim -x \sim x^{-1}) \quad (7.3381)$$

$$\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa, pb, pc, pd, \text{ class } 3 \rangle \quad (7.3382)$$

$$\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa - xbaa - bab, pb, pc, pd, \text{ class } 3 \rangle (x \sim -x) \quad (7.3383)$$

$$\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa - xbaa - \omega bab, pb, pc, pd, \text{ class } 3 \rangle (x \sim -x) \quad (7.3384)$$

$$\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa - baa, pb, pc, pd, \text{ class } 3 \rangle \quad (7.3385)$$

$$\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa - \omega baa, pb, pc, pd, \text{ class } 3 \rangle (p = 1 \pmod{4}) \quad (7.3386)$$

$$\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa - xbaa - ybab, pb - zbaa - tbab, pc, pd, \text{ class } 3 \rangle \quad (7.3387)$$

In presentation 7.3387, if we write the parameters x, y, z, t in a matrix $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$ (which is assumed to be non-singular) then two matrices give isomorphic algebras if and only if they are in the same orbit under the action

$$A \rightarrow \frac{1}{\det P} PAP^{-1},$$

where P lies in the group of non-singular matrices of the form

$$\begin{pmatrix} \alpha & \beta \\ \pm\omega\beta & \pm\alpha \end{pmatrix}.$$

$$\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa - xbaa, pb - ybaa, pc - bab, pd, \text{ class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.3388)$$

- $\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa, pb, pc - bab, pd - baa, \text{class } 3 \rangle$ (7.3389)
- $\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa - bab, pb - xbab, pc, pd - baa, \text{class } 3 \rangle (x \sim -x)$ (7.3390)
- $\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa - \omega bab, pb - xbab, pc, pd - baa, \text{class } 3 \rangle (x \sim -x)$ (7.3391)
- $\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa, pb, pc, pd - baa, \text{class } 3 \rangle$ (7.3392)
- $\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa, pb - bab, pc, pd - baa, \text{class } 3 \rangle$ (7.3393)
- $\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db, dc, pa, pb - \omega bab, pc, pd - baa, \text{class } 3 \rangle (p = 1 \bmod 4)$ (7.3394)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3395)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3396)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3397)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3398)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - baa, pb - xbab, pc, pd, \text{class } 3 \rangle$ (7.3399)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - baa, pb - baa - bab, pc, pd, \text{class } 3 \rangle$ (7.3400)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - baa, pb - \omega baa - bab, pc, pd, \text{class } 3 \rangle$ (7.3401)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3402)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - bab, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3403)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3404)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - bab, pb - xbaa - bab, pc, pd, \text{class } 3 \rangle$ (7.3405)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3406)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3407)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3408)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb - xbaa - bab, pc, pd, \text{class } 3 \rangle$ (7.3409)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc, pd - baa, \text{class } 3 \rangle$ (7.3410)

- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - bab, pc, pd - baa, \text{class } 3 \rangle$ (7.3411)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - bab, pb, pc, pd - baa, \text{class } 3 \rangle$ (7.3412)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb, pc, pd - baa, \text{class } 3 \rangle$ (7.3413)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3414)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - baa, pc, pd - bab, \text{class } 3 \rangle$ (7.3415)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - \omega baa, pc, pd - bab, \text{class } 3 \rangle$ (7.3416)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - baa, pb - xbaa, pc, pd - bab, \text{class } 3 \rangle$ (7.3417)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc - baa, pd - xbab, \text{class } 3 \rangle$ ($x \neq 0$) (7.3418)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc - baa, pd - baa - bab, \text{class } 3 \rangle$ (7.3419)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - xbab, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3420)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - xbab, pb - bab, pc - baa, pd, \text{class } 3 \rangle$ (7.3421)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc - bab, pd - xbaa - bab, \text{class } 3 \rangle$ ($x \neq 0$) (7.3422)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc - bab, pd - baa, \text{class } 3 \rangle$ (7.3423)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc - bab, pd - \omega baa, \text{class } 3 \rangle$ (7.3424)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3425)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - baa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3426)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega baa, pb, pc - bab, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3427)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega^2 baa, pb, pc - bab, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3428)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - baa, pc - bab, pd, \text{class } 3 \rangle$ (7.3429)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - \omega baa, pc - bab, pd, \text{class } 3 \rangle$ (7.3430)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - \omega^2 baa, pc - bab, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.3431)
- $\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa, pb - \omega^3 baa, pc - bab, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.3432)

$$\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - baa, pb - xbaa, pc - bab, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.3433)$$

$$\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega baa, pb - xbaa, pc - bab, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.3434)$$

$$\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - \omega^2 baa, pb - xbaa, pc - bab, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.3435)$$

$$\langle a, b, c, d \mid ca, cb - baa, da, db - bab, dc, pa - xbaa, pb - ybaa, pc - bab, pd - bab, \text{class } 3 \rangle \quad (7.3436)$$

$$\langle a, b, c, d \mid ca - bab, cb - baa, da, db - bab, dc, pa - rbaa - sbab, pb - tbaa - ubab, pc - vbaa - xbab, pd - ybaa - zbab, \text{class } 3 \rangle \quad (7.3437)$$

In presentation 7.3437 we write

$$\begin{pmatrix} pa \\ pb \\ pc \\ pd \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

where A is 4×2 matrix. Two matrices A give isomorphic algebras if they lie in the same orbit under the action

$$A \rightarrow \frac{1}{\alpha^2 + \beta^2} \begin{pmatrix} \alpha & -\beta & \gamma & \delta \\ \pm\beta & \pm\alpha & \pm\lambda & \pm\mu \\ 0 & 0 & \alpha^2 - \beta^2 & -4\alpha\beta \\ 0 & 0 & \pm\alpha\beta & \pm(\alpha^2 - \beta^2) \end{pmatrix} A \begin{pmatrix} \pm\alpha & \mp\beta \\ \beta & \alpha \end{pmatrix}^{-1}.$$

$$\langle a, b, c, d \mid ca - bab, cb - \omega baa, da, db - bab, dc, pa - rbaa - sbab, pb - tbaa - ubab, pc - vbaa - xbab, pd - ybaa - zbab, \text{class } 3 \rangle \quad (7.3438)$$

In presentation 7.3438 we write

$$\begin{pmatrix} pa \\ pb \\ pc \\ pd \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

where A is 4×2 matrix. Two matrices A give isomorphic algebras if they lie in the same orbit under the action

$$A \rightarrow \frac{1}{\alpha^2 + \omega\beta^2} \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \mp\omega\beta & \pm\alpha & \pm\lambda & \pm\mu \\ 0 & 0 & \alpha^2 - \omega\beta^2 & 4\omega\alpha\beta \\ 0 & 0 & \mp\alpha\beta & \pm(\alpha^2 - \omega\beta^2) \end{pmatrix} A \begin{pmatrix} \pm\alpha & \pm\beta \\ -\omega\beta & \alpha \end{pmatrix}^{-1}.$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.3439)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3440)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3441)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.3442)$$

- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - xbab, pc, pd, \text{class } 3 \rangle$ (7.3443)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - baa - bab, pc, pd, \text{class } 3 \rangle$ (7.3444)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - \omega baa - bab, pc, pd, \text{class } 3 \rangle$ (7.3445)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3446)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3447)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3448)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - xbaa - bab, pc, pd, \text{class } 3 \rangle$ (7.3449)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3450)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3451)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3452)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - xbaa - bab, pc, pd, \text{class } 3 \rangle$ (7.3453)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3454)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - baa, pc - baa, pd, \text{class } 3 \rangle$ (7.3455)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - \omega baa, pc - baa, pd, \text{class } 3 \rangle$ (7.3456)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - bab, pc - baa, pd, \text{class } 3 \rangle$ (7.3457)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - xbab, pc - baa, pd, \text{class } 3 \rangle$ (7.3458)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - baa - bab, pc - baa, pd, \text{class } 3 \rangle$ (7.3459)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - \omega baa - bab, pc - baa, pd, \text{class } 3 \rangle$ (7.3460)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3461)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - baa, pc - baa, pd, \text{class } 3 \rangle$ (7.3462)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - \omega baa, pc - baa, pd, \text{class } 3 \rangle$ (7.3463)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - xbaa - bab, pc - baa, pd, \text{class } 3 \rangle$ (7.3464)

- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3465)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - baa, pc - baa, pd, \text{class } 3 \rangle$ (7.3466)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - \omega baa, pc - baa, pd, \text{class } 3 \rangle$ (7.3467)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - xbaa - bab, pc - baa, pd, \text{class } 3 \rangle$ (7.3468)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa - xbab, pb - baa - ybab, pc - bab, pd, \text{class } 3 \rangle$ (7.3469)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa - xbab, pb - \omega baa - ybab, pc - bab, pd, \text{class } 3 \rangle$ (7.3470)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - xbab, pc - bab, pd, \text{class } 3 \rangle$ (7.3471)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa - bab, pb - xbab, pc - bab, pd, \text{class } 3 \rangle$ (7.3472)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa - \omega bab, pb - xbab, pc - bab, pd, \text{class } 3 \rangle$ (7.3473)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - xbab, pb - baa - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.3474)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - xbab, pb - \omega baa - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.3475)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - baa, pc - bab, pd, \text{class } 3 \rangle$ (7.3476)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - baa, pc - bab, pd, \text{class } 3 \rangle$ (7.3477)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - baa, pc - bab, pd, \text{class } 3 \rangle$ (7.3478)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - \omega baa, pc - bab, pd, \text{class } 3 \rangle$ (7.3479)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - \omega baa, pc - bab, pd, \text{class } 3 \rangle$ (7.3480)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - \omega baa, pc - bab, pd, \text{class } 3 \rangle$ (7.3481)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3482)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3483)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3484)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.3485)
- $\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.3486)

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - bab, pc - bab, pd, \text{class } 3 \rangle \quad (7.3487)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3488)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - baa, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3489)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - \omega baa, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3490)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - \omega^2 baa, pc - baa, pd - bab, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.3491)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - \omega^3 baa, pc - baa, pd - bab, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.3492)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa, pb - bab, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3493)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - xbab, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3494)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - baa, pb - xbaa - bab, pc - baa, pd - bab, \text{class } 3 \rangle (x \neq 0) \quad (7.3495)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - bab, pb - xbaa - ybab, pc - baa, pd - bab, \text{class } 3 \rangle (x, y) \sim (x, -y) \quad (7.3496)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, pa - \omega bab, pb - xbaa - ybab, pc - baa, pd - bab, \text{class } 3 \rangle ((x, y) \sim (x, -y)) \quad (7.3497)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.3498)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3499)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3500)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.3501)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa, pb - xbab, pc, pd, \text{class } 3 \rangle \quad (7.3502)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa, pb - baa - bab, pc, pd, \text{class } 3 \rangle \quad (7.3503)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa, pb - \omega baa - bab, pc, pd, \text{class } 3 \rangle \quad (7.3504)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3505)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - bab, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3506)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3507)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - bab, pb - xbaa - bab, pc, pd, \text{class } 3 \rangle \quad (7.3508)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3509)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega bab, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3510)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega bab, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3511)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega bab, pb - xbaa - bab, pc, pd, \text{class } 3 \rangle \quad (7.3512)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.3513)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - baa, pc - baa, pd, \text{class } 3 \rangle \quad (7.3514)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega baa, pc - baa, pd, \text{class } 3 \rangle \quad (7.3515)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - bab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3516)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega bab, pc - baa, pd, \text{class } 3 \rangle \ (p = 1 \pmod{3}) \quad (7.3517)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega^2 bab, pc - baa, pd, \text{class } 3 \rangle \ (p = 1 \pmod{3}) \quad (7.3518)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa, pb - xbab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3519)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega baa, pb - xbab, pc - baa, pd, \text{class } 3 \rangle \ (p = 1 \pmod{3}) \quad (7.3520)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega^2 baa, pb - xbab, pc - baa, pd, \text{class } 3 \rangle \ (p = 1 \pmod{3}) \quad (7.3521)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa, pb - xbaa - bab, pc - baa, pd, \text{class } 3 \rangle \ (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.3522)$$

In the next two presentations we have $x \neq 0$, $x \sim ax$ if $a^3 = 1$, $p = 1 \pmod{3}$:

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega baa, pb - xbaa - \omega bab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3523)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega^2 baa, pb - xbaa - \omega^2 bab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3524)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - bab, pb - xbaa - ybab, pc - baa, pd, \text{class } 3 \rangle \ ((x, y) \sim (x, -y), p = 3 \pmod{4}) \quad (7.3525)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega bab, pb - xbaa - ybab, pc - baa, pd, \text{class } 3 \rangle \ ((x, y) \sim (x, -y), p = 3 \pmod{4}) \quad (7.3526)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - bab, pb - xbaa, pc - baa, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.3527)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega bab, pb - xbaa, pc - baa, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.3528)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega^2 bab, pb - xbaa, pc - baa, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.3529)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega^3 bab, pb - xbaa, pc - baa, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.3530)$$

In the next four presentations we have $y \neq 0$, $(x, y) \sim (a^2x, ay)$ if $a^4 = 1$, $p = 1 \bmod 4$:

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - bab, pb - xbaa - ybab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3531)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega bab, pb - xbaa - ybab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3532)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega^2 bab, pb - xbaa - ybab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3533)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega^3 bab, pb - xbaa - ybab, pc - baa, pd, \text{class } 3 \rangle \quad (7.3534)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa - xbab, pb - ybaa - zbab, pc - bab, pd, \text{class } 3 \rangle \quad (7.3535)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - bab, pc - bab, pd, \text{class } 3 \rangle \quad (7.3536)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa, pc - bab, pd, \text{class } 3 \rangle \quad (7.3537)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - bab, pb - xbaa, pc - bab, pd, \text{class } 3 \rangle \quad (7.3538)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega bab, pb - xbaa, pc - bab, pd, \text{class } 3 \rangle \quad (7.3539)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.3540)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - baa, pc, pd - baa, \text{class } 3 \rangle \quad (7.3541)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - \omega baa, pc, pd - baa, \text{class } 3 \rangle \quad (7.3542)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - bab, pc, pd - baa, \text{class } 3 \rangle (x \neq -2) \quad (7.3543)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb, pc, pd - \omega baa, \text{class } 3 \rangle \quad (7.3544)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - baa, pc, pd - \omega baa, \text{class } 3 \rangle \quad (7.3545)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - \omega baa, pc, pd - \omega baa, \text{class } 3 \rangle \quad (7.3546)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - bab, pc, pd - \omega baa, \text{class } 3 \rangle (x \neq -2\omega) \quad (7.3547)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa + 2bab, pb - xbaa, pc, pd - baa, \text{class } 3 \rangle \quad (7.3548)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa + 2\omega bab, pb - xbaa, pc, pd - \omega baa, \text{class } 3 \rangle \quad (7.3549)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - zbab, pc - bab, pd - baa, \text{class } 3 \rangle (x \neq -2, (x, y, z) \sim (x, y, -z)) \quad (7.3550)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - zbab, pc - bab, pd - \omega baa, \text{class } 3 \rangle (x \neq -2\omega, (x, y, z) \sim (x, y, -z)) \quad (7.3551)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa + 2bab, pb - ybaa, pc - bab, pd - baa, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.3552)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa + 2\omega bab, pb - ybaa, pc - bab, pd - \omega baa, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.3553)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa - bab, pb - ybaa - zbab, pc, pd - bab, \text{class } 3 \rangle \quad (7.3554)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa - \omega bab, pb - ybaa - zbab, pc, pd - bab, \text{class } 3 \rangle \quad (7.3555)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa, pb - xbaa - ybab, pc, pd - bab, \text{class } 3 \rangle ((x, y) \sim (-x, y), p = 1 \bmod 4) \quad (7.3556)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega baa, pb - xbaa - ybab, pc, pd - bab, \text{class } 3 \rangle ((x, y) \sim (-x, y), p = 1 \bmod 4) \quad (7.3557)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega^2 baa, pb - xbaa - ybab, pc, pd - bab, \text{class } 3 \rangle ((x, y) \sim (-x, y), p = 1 \bmod 4) \quad (7.3558)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega^3 baa, pb - xbaa - ybab, pc, pd - bab, \text{class } 3 \rangle ((x, y) \sim (-x, y), p = 1 \bmod 4) \quad (7.3559)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - baa, pb - xbaa - ybab, pc, pd - bab, \text{class } 3 \rangle (p = 3 \bmod 4) \quad (7.3560)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - \omega baa, pb - xbaa - ybab, pc, pd - bab, \text{class } 3 \rangle (p = 3 \bmod 4) \quad (7.3561)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - bab, pc, pd - bab, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.3562)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - \omega bab, pc, pd - bab, \text{class } 3 \rangle (x \sim -x, p = 1 \pmod{4}) \quad (7.3563)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - \omega^2 bab, pc, pd - bab, \text{class } 3 \rangle (x \sim -x, p = 1 \pmod{4}) \quad (7.3564)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - \omega^3 bab, pc, pd - bab, \text{class } 3 \rangle (x \sim -x, p = 1 \pmod{4}) \quad (7.3565)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - bab, pc, pd - bab, \text{class } 3 \rangle (p = 3 \pmod{4}) \quad (7.3566)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - \omega bab, pc, pd - bab, \text{class } 3 \rangle (p = 3 \pmod{4}) \quad (7.3567)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb, pc, pd - bab, \text{class } 3 \rangle \quad (7.3568)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - baa, pc, pd - bab, \text{class } 3 \rangle \quad (7.3569)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega baa, pc, pd - bab, \text{class } 3 \rangle \quad (7.3570)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega^2 baa, pc, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.3571)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega^3 baa, pc, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.3572)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega^4 baa, pc, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.3573)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - \omega^5 baa, pc, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.3574)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa - ybab, pb - zbaa - tbab, pc - baa, pd - bab, \text{class } 3 \rangle (p \neq 1 \pmod{5}) \quad (7.3575)$$

In the next 5 presentations we have $x \neq 0$, $(x, y, z, t) \sim (ax, a^3y, a^4z, at)$ if $a^5 = 1$, $p = 1 \pmod{5}$:

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa - ybab, pb - zbaa - tbab, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3576)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa - ybab, pb - zbaa - tbab, pc - \omega baa, pd - bab, \text{class } 3 \rangle \quad (7.3577)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa - ybab, pb - zbaa - tbab, pc - \omega^2 baa, pd - bab, \text{class } 3 \rangle \quad (7.3578)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa - ybab, pb - zbaa - tbab, pc - \omega^3 baa, pd - bab, \text{class } 3 \rangle \quad (7.3579)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbaa - ybab, pb - zbaa - tbab, pc - \omega^4 baa, pd - bab, \text{class } 3 \rangle \quad (7.3580)$$

In the next 5 presentations we have $x \neq 0$, $(x, y, z) \sim (a^3x, a^4y, az)$ if $a^5 = 1$, $p = 1 \pmod{5}$:

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - zbab, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3581)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - zbab, pc - \omega baa, pd - bab, \text{class } 3 \rangle \quad (7.3582)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - zbab, pc - \omega^2 baa, pd - bab, \text{class } 3 \rangle \quad (7.3583)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - zbab, pc - \omega^3 baa, pd - bab, \text{class } 3 \rangle \quad (7.3584)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa - xbab, pb - ybaa - zbab, pc - \omega^4 baa, pd - bab, \text{class } 3 \rangle \quad (7.3585)$$

In the next 5 presentations we have $x \neq 0$, $(x, y) \sim (a^4x, ay)$ if $a^5 = 1$, $p = 1 \pmod{5}$:

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - ybab, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3586)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - ybab, pc - \omega baa, pd - bab, \text{class } 3 \rangle \quad (7.3587)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - ybab, pc - \omega^2 baa, pd - bab, \text{class } 3 \rangle \quad (7.3588)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - ybab, pc - \omega^3 baa, pd - bab, \text{class } 3 \rangle \quad (7.3589)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbaa - ybab, pc - \omega^4 baa, pd - bab, \text{class } 3 \rangle \quad (7.3590)$$

And in the next 5 presentations we have $x \neq 0$, $x \sim ax$ if $a^5 = 1$, $p = 1 \pmod{5}$:

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbab, pc - baa, pd - bab, \text{class } 3 \rangle \quad (7.3591)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbab, pc - \omega baa, pd - bab, \text{class } 3 \rangle \quad (7.3592)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbab, pc - \omega^2 baa, pd - bab, \text{class } 3 \rangle \quad (7.3593)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbab, pc - \omega^3 baa, pd - bab, \text{class } 3 \rangle \quad (7.3594)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb - xbab, pc - \omega^4 baa, pd - bab, \text{class } 3 \rangle \quad (7.3595)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb, pc - baa, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{5}) \quad (7.3596)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb, pc - \omega baa, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{5}) \quad (7.3597)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb, pc - \omega^2 baa, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{5}) \quad (7.3598)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb, pc - \omega^3 baa, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{5}) \quad (7.3599)$$

$$\langle a, b, c, d \mid ca - bab, cb, da, db, dc - baa, pa, pb, pc - \omega^4 baa, pd - bab, \text{class } 3 \rangle (p = 1 \pmod{5}) \quad (7.3600)$$

8.199 Descendants of 6.10

8 algebras

- $$\langle a, b, c, d \mid ca, cb, da, db, dc, pa, pb, pc - ba, \text{class } 3 \rangle \quad (7.3601)$$
- $$\langle a, b, c, d \mid ca, cb, da, db - p^2d, dc, pa, pb, pc - ba, \text{class } 3 \rangle \quad (7.3602)$$
- $$\langle a, b, c, d \mid ca, cb, da, db, dc - p^2d, pa, pb, pc - ba, \text{class } 3 \rangle \quad (7.3603)$$
- $$\langle a, b, c, d \mid ca, cb, da, db - p^2d, dc - p^2d, pa, pb, pc - ba, \text{class } 3 \rangle \quad (7.3604)$$
- $$\langle a, b, c, d \mid ca - p^2d, cb, da, db, dc, pa, pb, pc - ba, \text{class } 3 \rangle \quad (7.3605)$$
- $$\langle a, b, c, d \mid ca - p^2d, cb, da, db - p^2d, dc, pa, pb, pc - ba, \text{class } 3 \rangle \quad (7.3606)$$
- $$\langle a, b, c, d \mid ca - p^2d, cb, da, db, dc - p^2d, pa, pb, pc - ba, \text{class } 3 \rangle \quad (7.3607)$$
- $$\langle a, b, c, d \mid ca - p^2d, cb, da, db - p^2d, dc - p^2d, pa, pb, pc - ba, \text{class } 3 \rangle \quad (7.3608)$$

8.200 Descendants of 6.11

39 algebras

- $$\langle a, b, c, d \mid baa, bab, ca, cb, da, db, dc, pb, pc, pd, \text{class } 3 \rangle \quad (7.3609)$$
- $$\langle a, b, c, d \mid baa, bab, ca - p^2a, cb, da, db, dc, pb, pc, pd, \text{class } 3 \rangle \quad (7.3610)$$
- $$\langle a, b, c, d \mid baa, bab, ca, cb - p^2a, da, db, dc, pb, pc, pd, \text{class } 3 \rangle \quad (7.3611)$$
- $$\langle a, b, c, d \mid baa, bab, ca, cb - p^2a, da - p^2a, db, dc, pb, pc, pd, \text{class } 3 \rangle \quad (7.3612)$$
- $$\langle a, b, c, d \mid baa, bab, ca, cb, da, db, dc - p^2a, pb, pc, pd, \text{class } 3 \rangle \quad (7.3613)$$
- $$\langle a, b, c, d \mid baa, ca, cb, da, db, dc - bab, p^2a - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3614)$$
- $$\langle a, b, c, d \mid baa, ca, cb, da, db, dc, p^2a - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3615)$$
- $$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dc, p^2a - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3616)$$
- $$\langle a, b, c, d \mid baa, ca, cb, da, db, dc - bab, p^2a - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3617)$$
- $$\langle a, b, c, d \mid baa, ca, cb, da, db, dc, p^2a - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3618)$$
- $$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dc, p^2a - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.3619)$$

- $\langle a, b, c, d \mid baa, ca, cb, da, db, dc - bab, p^2a, pb, pc, pd, \text{class } 3 \rangle$ (7.3620)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dc - bab, p^2a, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3621)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dc - bab, p^2a, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3622)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dc, p^2a, pb, pc, pd, \text{class } 3 \rangle$ (7.3623)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dc, p^2a, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3624)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dc, p^2a, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3625)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dc, p^2a, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3626)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dc, p^2a, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3627)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dc, p^2a, pb, pc, pd, \text{class } 3 \rangle$ (7.3628)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dc, p^2a, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3629)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc, p^2a - baa, pb, pc, pd, \text{class } 3 \rangle$ (7.3630)
- $\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, p^2a - baa, pb, pc, pd, \text{class } 3 \rangle$ (7.3631)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, p^2a - baa, pb, pc, pd, \text{class } 3 \rangle$ (7.3632)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, p^2a, pb, pc, pd, \text{class } 3 \rangle$ (7.3633)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, p^2a, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3634)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, p^2a, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3635)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, p^2a, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3636)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, p^2a, pb - baa, pc - baa, pd, \text{class } 3 \rangle$ (7.3637)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, p^2a, pb - \omega baa, pc - baa, pd, \text{class } 3 \rangle$ (7.3638)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc, p^2a, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3639)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc, p^2a, pb, pc, pd, \text{class } 3 \rangle$ (7.3640)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc, p^2a, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3641)

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, p^2a, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3642)$$

$$\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, p^2a, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.3643)$$

$$\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, p^2a, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.3644)$$

$$\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, p^2a, pb, pc, pd, \text{class } 3 \rangle \quad (7.3645)$$

$$\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, p^2a, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3646)$$

$$\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, p^2a, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3647)$$

8.201 Descendants of 6.12

8 algebras

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dc, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3648)$$

$$\langle a, b, c, d \mid baa, ca - p^2a, cb, da, db, dc, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3649)$$

$$\langle a, b, c, d \mid baa, ca, cb - p^2a, da - p^2a, db, dc, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3650)$$

$$\langle a, b, c, d \mid baa, ca, cb - p^2a, da, db, dc, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3651)$$

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dc - p^2a, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3652)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc, p^2a, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3653)$$

$$\langle a, b, c, d \mid ca - baa, cb, da, db, dc, p^2a, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3654)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - baa, p^2a, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3655)$$

8.202 Descendants of 6.13

11 algebras

$$\langle a, b, c, d \mid ca, cb, da, db, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3656)$$

$$\langle a, b, c, d \mid ca - p^2a, cb, da, db, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3657)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da, db, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3658)$$

$$\langle a, b, c, d \mid ca, cb - \omega p^2a, da, db, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3659)$$

$$\langle a, b, c, d \mid ca, cb, da - p^2a, db, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3660)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da - p^2a, db, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3661)$$

$$\langle a, b, c, d \mid ca, cb - \omega p^2a, da - p^2a, db, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3662)$$

$$\langle a, b, c, d \mid ca, cb, da, db - p^2a, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3663)$$

$$\langle a, b, c, d \mid ca - p^2a, cb, da, db - p^2a, dc, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3664)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - p^2a, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3665)$$

$$\langle a, b, c, d \mid ca, cb, da, db - p^2a, dc - p^2a, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3666)$$

8.203 Descendants of 6.14

39 algebras

$$\langle a, b, c, d \mid baa, bab, ca, cb, da, db, dc, pa, pb, pd, \text{class } 3 \rangle \quad (7.3667)$$

$$\langle a, b, c, d \mid baa, bab, ca - p^2c, cb, da, db, dc, pa, pb, pd, \text{class } 3 \rangle \quad (7.3668)$$

$$\langle a, b, c, d \mid baa, bab, ca, cb, da - p^2c, db, dc, pa, pb, pd, \text{class } 3 \rangle \quad (7.3669)$$

$$\langle a, b, c, d \mid baa, bab, ca, cb - p^2c, da - p^2c, db, dc, pa, pb, pd, \text{class } 3 \rangle \quad (7.3670)$$

$$\langle a, b, c, d \mid baa, bab, ca, cb, da, db, dc - p^2c, pa, pb, pd, \text{class } 3 \rangle \quad (7.3671)$$

$$\langle a, b, c, d \mid baa, bab, ca, cb, da - p^2c, db, dc - p^2c, pa, pb, pd, \text{class } 3 \rangle \quad (7.3672)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb, p^2c, pd - baa, \text{class } 3 \rangle \quad (7.3673)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb - baa, p^2c, pd, \text{class } 3 \rangle \quad (7.3674)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb - \omega baa, p^2c, pd, \text{class } 3 \rangle \quad (7.3675)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb, p^2c, pd, \text{class } 3 \rangle \quad (7.3676)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa - baa, pb, p^2c, pd, \text{class } 3 \rangle \quad (7.3677)$$

$$\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, pa, pb, p^2c, pd - baa, \text{class } 3 \rangle \quad (7.3678)$$

$$\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, pa, pb - baa, p^2c, pd, \text{class } 3 \rangle \quad (7.3679)$$

$$\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, pa, pb - \omega baa, p^2c, pd, \text{class } 3 \rangle \quad (7.3680)$$

- $\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, pa, pb, p^2c, pd, \text{class } 3 \rangle$ (7.3681)
- $\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, pa - baa, pb, p^2c, pd, \text{class } 3 \rangle$ (7.3682)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc, pa, pb, p^2c, pd - baa, \text{class } 3 \rangle$ (7.3683)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc, pa, pb - baa, p^2c, pd, \text{class } 3 \rangle$ (7.3684)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc, pa, pb - \omega baa, p^2c, pd, \text{class } 3 \rangle$ (7.3685)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc, pa, pb, p^2c, pd, \text{class } 3 \rangle$ (7.3686)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc, pa - baa, pb, p^2c, pd, \text{class } 3 \rangle$ (7.3687)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb, p^2c, pd - baa, \text{class } 3 \rangle$ (7.3688)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb - baa, p^2c, pd - baa, \text{class } 3 \rangle$ (7.3689)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb - \omega baa, p^2c, pd - baa, \text{class } 3 \rangle$ (7.3690)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb - baa, p^2c, pd, \text{class } 3 \rangle$ (7.3691)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb - \omega baa, p^2c, pd, \text{class } 3 \rangle$ (7.3692)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb, p^2c, pd, \text{class } 3 \rangle$ (7.3693)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa - baa, pb, p^2c, pd, \text{class } 3 \rangle$ (7.3694)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc - baa, pa, pb, p^2c, pd - baa, \text{class } 3 \rangle$ (7.3695)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc - baa, pa, pb - baa, p^2c, pd, \text{class } 3 \rangle$ (7.3696)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc - baa, pa, pb - \omega baa, p^2c, pd, \text{class } 3 \rangle$ (7.3697)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc - baa, pa, pb, p^2c, pd, \text{class } 3 \rangle$ (7.3698)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc - baa, pa - baa, pb, p^2c, pd, \text{class } 3 \rangle$ (7.3699)
- $\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb, p^2c - baa, pd, \text{class } 3 \rangle$ (7.3700)
- $\langle a, b, c, d \mid bab, ca, cb - baa, da, db, dc, pa, pb, p^2c - baa, pd, \text{class } 3 \rangle$ (7.3701)
- $\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc, pa, pb, p^2c - baa, pd, \text{class } 3 \rangle$ (7.3702)

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baa, pa, pb, p^2c - baa, pd, \text{class } 3 \rangle \quad (7.3703)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db - baa, dc - baa, pa, pb, p^2c - baa, pd, \text{class } 3 \rangle \quad (7.3704)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db - \omega baa, dc - baa, pa, pb, p^2c - baa, pd, \text{class } 3 \rangle \quad (7.3705)$$

8.204 Descendants of 6.15

$2p + 6$ algebras

$$\langle a, b, c, d \mid ca, cb - xp^2c, da, db, dc, pa - ba, pb, pd, \text{class } 3 \rangle \quad (7.3706)$$

$$\langle a, b, c, d \mid ca - p^2c, cb, da, db, dc, pa - ba, pb, pd, \text{class } 3 \rangle \quad (7.3707)$$

$$\langle a, b, c, d \mid ca, cb - xp^2c, da - p^2c, db, dc, pa - ba, pb, pd, \text{class } 3 \rangle \quad (7.3708)$$

$$\langle a, b, c, d \mid ca, cb, da, db - p^2c, dc, pa - ba, pb, pd, \text{class } 3 \rangle \quad (7.3709)$$

$$\langle a, b, c, d \mid ca - p^2c, cb, da, db - p^2c, dc, pa - ba, pb, pd, \text{class } 3 \rangle \quad (7.3710)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - p^2c, pa - ba, pb, pd, \text{class } 3 \rangle \quad (7.3711)$$

$$\langle a, b, c, d \mid ca, cb, da - p^2c, db, dc - p^2c, pa - ba, pb, pd, \text{class } 3 \rangle \quad (7.3712)$$

$$\langle a, b, c, d \mid ca, cb, da, db - p^2c, dc - p^2c, pa - ba, pb, pd, \text{class } 3 \rangle \quad (7.3713)$$

8.205 Descendants of 6.16

6 algebras

$$\langle a, b, c, d \mid ca, cb, da, db, dc - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.3714)$$

$$\langle a, b, c, d \mid ca - p^2a, cb, da, db, dc - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.3715)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - ba - p^2a, pb, pc, pd, \text{class } 3 \rangle \quad (7.3716)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da, db, dc - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.3717)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da - p^2a, db, dc - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.3718)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da - \omega p^2a, db, dc - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.3719)$$

8.206 Descendants of 6.17

$p + 7$ algebras

$$\langle a, b, c, d \mid ca, cb, da, db, dc - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3720)$$

$$\langle a, b, c, d \mid ca - p^2a, cb, da, db, dc - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3721)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - ba - p^2a, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3722)$$

$$\langle a, b, c, d \mid ca - p^2a, cb, da, db, dc - ba - p^2a, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3723)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da, db, dc - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3724)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da - p^2a, db, dc - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3725)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da - \omega p^2a, db, dc - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3726)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da - xp^2a, db, dc - ba - p^2a, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3727)$$

8.207 Descendants of 6.18

$3p + 5$ algebras

$$\langle a, b, c, d \mid ca, cb, da - xp^2a, db, dc - ba, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3728)$$

$$\langle a, b, c, d \mid ca - p^2a, cb, da, db, dc - ba, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3729)$$

$$\langle a, b, c, d \mid ca, cb, da, db, dc - ba - p^2a, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3730)$$

$$\langle a, b, c, d \mid ca, cb, da, db - p^2a, dc - ba, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3731)$$

$$\langle a, b, c, d \mid ca - p^2a, cb, da, db - p^2a, dc - ba, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3732)$$

$$\langle a, b, c, d \mid ca - \omega p^2a, cb, da, db - p^2a, dc - ba, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3733)$$

$$\langle a, b, c, d \mid ca, cb - p^2a, da - xp^2a, db, dc - ba, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3734)$$

$$\langle a, b, c, d \mid ca, cb - \omega p^2a, da - xp^2a, db, dc - ba, pb, pc - ba, pd, \text{class } 3 \rangle \quad (7.3735)$$

8.208 Descendants of 6.19

97 algebras

- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3736)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3737)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3738)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3739)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc, pa - bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3740)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3741)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc, pa - \omega bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3742)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3743)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3744)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3745)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa - bab, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3746)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa - \omega bab, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3747)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3748)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3749)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3750)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa - bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3751)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3752)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da, db, dc - bab, pa - \omega bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3753)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da - bab, db, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3754)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da - bab, db, dc, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3755)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da - bab, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3756)

- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da - bab, db, dc, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3757)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da - bab, db, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3758)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da - bab, db, dc, pa - bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3759)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da - bab, db, dc, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3760)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac, da - bab, db, dc, pa - \omega bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3761)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3762)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3763)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3764)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3765)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3766)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3767)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb, pc - \omega bab, pd, \text{class } 3 \rangle$ (7.3768)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3769)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.3770)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb, pc - \omega bab, pd, \text{class } 3 \rangle$ (7.3771)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3772)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3773)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3774)
- $\langle a, b, c, d \mid cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3775)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da, db, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3776)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3777)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da, db, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3778)

- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da, db, dc, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3779)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da, db, dc, pa - bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3780)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da, db, dc, pa - \omega bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3781)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da, db, dc, pa, pb - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.3782)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da, db, dc, pa - bab, pb - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.3783)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da, db, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3784)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3785)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da, db, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3786)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da, db, dc, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3787)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da, db, dc, pa - bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3788)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da, db, dc, pa - \omega bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3789)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da - bab, db, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3790)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da - bab, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3791)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da - bab, db, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3792)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da - bab, db, dc, pa, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3793)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da - bab, db, dc, pa - bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3794)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da - bab, db, dc, pa - \omega bab, pb - bab, pc, pd, \text{class } 3 \rangle$ (7.3795)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da - bab, db, dc, pa, pb - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.3796)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + bab, da - bab, db, dc, pa - bab, pb - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.3797)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da - bab, db, dc, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.3798)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da - bab, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3799)
- $\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da - bab, db, dc, pa - bab, pb, pc, pd, \text{class } 3 \rangle$ (7.3800)

$$\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da - bab, db, dc, pa, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.3801)$$

$$\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da - bab, db, dc, pa - bab, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.3802)$$

$$\langle a, b, c, d \mid cb, baa, bac, caa, cac + \omega bab, da - bab, db, dc, pa - \omega bab, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.3803)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc, pa, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.3804)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.3805)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc, pa - baa, pb, pc, pd, \text{class } 3 \rangle \quad (7.3806)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc, pa, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3807)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc, pa, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3808)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc, pa, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.3809)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db - baa, dc, pa, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.3810)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db - baa, dc, pa, pb, pc - baa, pd - baa, \text{class } 3 \rangle \quad (7.3811)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db - baa, dc, pa, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.3812)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db - baa, dc, pa, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.3813)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db - baa, dc, pa, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.3814)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db - baa, dc, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.3815)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db - baa, dc, pa - baa, pb, pc, pd, \text{class } 3 \rangle \quad (7.3816)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.3817)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb - baa, pc, pd - baa, \text{class } 3 \rangle \quad (7.3818)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb - \omega baa, pc, pd - baa, \text{class } 3 \rangle \quad (7.3819)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.3820)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb - baa, pc - baa, pd, \text{class } 3 \rangle \quad (7.3821)$$

$$\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb - \omega baa, pc - baa, pd, \text{class } 3 \rangle \quad (7.3822)$$

- $\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3823)
- $\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3824)
- $\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3825)
- $\langle a, b, c, d \mid cb, bab, bac, caa, cac, da, db, dc - baa, pa - baa, pb, pc, pd, \text{class } 3 \rangle$ (7.3826)
- $\langle a, b, c, d \mid cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb, pc, pd - baa, \text{class } 3 \rangle$ (7.3827)
- $\langle a, b, c, d \mid cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.3828)
- $\langle a, b, c, d \mid cb - baa, bab, bac, caa, cac, da, db, dc, pa - baa, pb, pc, pd, \text{class } 3 \rangle$ (7.3829)
- $\langle a, b, c, d \mid cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb - baa, pc, pd, \text{class } 3 \rangle$ (7.3830)
- $\langle a, b, c, d \mid cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3831)
- $\langle a, b, c, d \mid cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb, pc - baa, pd, \text{class } 3 \rangle$ (7.3832)

8.209 Descendants of 6.20

$6p + 35$ algebras

- $\langle a, b, c, d \mid cb, da, db, dc, caa, pa - ba, pb, pc, pd - cac, \text{class } 3 \rangle$ (7.3833)
- $\langle a, b, c, d \mid cb, da, db, dc, caa, pa - ba, pb - cac, pc, pd, \text{class } 3 \rangle$ (7.3834)
- $\langle a, b, c, d \mid cb, da, db, dc, caa, pa - ba, pb, pc, pd, \text{class } 3 \rangle$ (7.3835)
- $\langle a, b, c, d \mid cb, da, db, dc, caa, pa - ba, pb, pc - cac, pd, \text{class } 3 \rangle$ (7.3836)
- $\langle a, b, c, d \mid cb, da, db, dc, caa, pa - ba - cac, pb, pc, pd, \text{class } 3 \rangle$ (7.3837)
- $\langle a, b, c, d \mid cb, da, db, dc, caa, pa - ba - cac, pb, pc - cac, pd, \text{class } 3 \rangle$ (7.3838)
- $\langle a, b, c, d \mid cb, da, db, dc, caa, pa - ba - \omega cac, pb, pc, pd, \text{class } 3 \rangle$ (7.3839)
- $\langle a, b, c, d \mid cb, da, db, dc, caa, pa - ba - \omega cac, pb, pc - cac, pd, \text{class } 3 \rangle$ (7.3840)
- $\langle a, b, c, d \mid cb, da - cac, db, dc, caa, pa - ba, pb, pc, pd - cac, \text{class } 3 \rangle$ (7.3841)
- $\langle a, b, c, d \mid cb, da - cac, db, dc, caa, pa - ba, pb - cac, pc, pd, \text{class } 3 \rangle$ (7.3842)

- $\langle a, b, c, d \mid cb, da - cac, db, dc, caa, pa - ba, pb, pc, pd, \text{class } 3 \rangle$ (7.3843)
- $\langle a, b, c, d \mid cb, da - cac, db, dc, caa, pa - ba, pb, pc - cac, pd, \text{class } 3 \rangle$ (7.3844)
- $\langle a, b, c, d \mid cb, da, db - cac, dc, caa, pa - ba, pb, pc, pd - xcac, \text{class } 3 \rangle$ (7.3845)
- $\langle a, b, c, d \mid cb, da, db - cac, dc, caa, pa - ba, pb - cac, pc, pd, \text{class } 3 \rangle$ (7.3846)
- $\langle a, b, c, d \mid cb, da, db - cac, dc, caa, pa - ba, pb, pc - cac, pd, \text{class } 3 \rangle$ (7.3847)
- $\langle a, b, c, d \mid cb, da, db - cac, dc, caa, pa - ba - cac, pb, pc, pd + cac, \text{class } 3 \rangle$ (7.3848)
- $\langle a, b, c, d \mid cb, da, db - cac, dc, caa, pa - ba - \omega cac, pb, pc, pd + cac, \text{class } 3 \rangle$ (7.3849)
- $\langle a, b, c, d \mid cb, da, db - caa, dc, cac, pa - ba, pb, pc, pd - xcaa, \text{class } 3 \rangle$ (7.3850)
- $\langle a, b, c, d \mid cb, da, db - caa, dc, cac, pa - ba, pb, pc - caa, pd - xcaa, \text{class } 3 \rangle$ (7.3851)
- $\langle a, b, c, d \mid cb, da, db - caa, dc, cac, pa - ba, pb, pc - \omega caa, pd - xcaa, \text{class } 3 \rangle$ (7.3852)
- $\langle a, b, c, d \mid cb, da, db - caa, dc, cac, pa - ba, pb - caa, pc, pd, \text{class } 3 \rangle$ (7.3853)
- $\langle a, b, c, d \mid cb, da, db - caa, dc, cac, pa - ba, pb - caa, pc - caa, pd, \text{class } 3 \rangle$ (7.3854)
- $\langle a, b, c, d \mid cb, da, db - caa, dc, cac, pa - ba, pb - caa, pc - \omega caa, pd, \text{class } 3 \rangle$ (7.3855)
- $\langle a, b, c, d \mid cb, da, db, dc - caa, cac, pa - ba, pb, pc, pd, \text{class } 3 \rangle$ (7.3856)
- $\langle a, b, c, d \mid cb, da, db, dc - caa, cac, pa - ba, pb - caa, pc, pd, \text{class } 3 \rangle$ (7.3857)
- $\langle a, b, c, d \mid cb, da, db, dc - caa, cac, pa - ba, pb, pc, pd - caa, \text{class } 3 \rangle$ (7.3858)
- $\langle a, b, c, d \mid cb, da, db, dc - caa, cac, pa - ba, pb - caa, pc, pd - caa, \text{class } 3 \rangle$ (7.3859)
- $\langle a, b, c, d \mid cb, da, db, dc - caa, cac, pa - ba, pb - \omega caa, pc, pd - caa, \text{class } 3 \rangle$ (7.3860)
- $\langle a, b, c, d \mid cb, da, db, dc - caa, cac, pa - ba, pb, pc - caa, pd, \text{class } 3 \rangle$ (7.3861)
- $\langle a, b, c, d \mid cb, da, db, dc - caa, cac, pa - ba, pb, pc - \omega caa, pd, \text{class } 3 \rangle$ (7.3862)
- $\langle a, b, c, d \mid cb - caa, da, db, dc, cac, pa - ba, pb, pc, pd - caa, \text{class } 3 \rangle$ (7.3863)
- $\langle a, b, c, d \mid cb - caa, da, db, dc, cac, pa - ba, pb, pc - xcaa, pd, \text{class } 3 \rangle$ (7.3864)

$$\langle a, b, c, d \mid cb - caa, da, db, dc, cac, pa - ba, pb - caa, pc, pd, \text{class } 3 \rangle \quad (7.3865)$$

$$\langle a, b, c, d \mid cb - \omega caa, da, db, dc, cac, pa - ba, pb, pc, pd - caa, \text{class } 3 \rangle \quad (7.3866)$$

$$\langle a, b, c, d \mid cb - \omega caa, da, db, dc, cac, pa - ba, pb, pc - xcaa, pd, \text{class } 3 \rangle \quad (7.3867)$$

$$\langle a, b, c, d \mid cb - \omega caa, da, db, dc, cac, pa - ba, pb - caa, pc, pd, \text{class } 3 \rangle \quad (7.3868)$$

$$\langle a, b, c, d \mid cb, da, db, dc, cac, pa - ba, pb, pc, pd - caa, \text{class } 3 \rangle \quad (7.3869)$$

$$\langle a, b, c, d \mid cb, da, db, dc, cac, pa - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.3870)$$

$$\langle a, b, c, d \mid cb, da, db, dc, cac, pa - ba, pb - caa, pc, pd, \text{class } 3 \rangle \quad (7.3871)$$

$$\langle a, b, c, d \mid cb, da, db, dc, cac, pa - ba, pb, pc - caa, pd, \text{class } 3 \rangle \quad (7.3872)$$

$$\langle a, b, c, d \mid cb, da, db, dc, cac, pa - ba, pb, pc - \omega caa, pd, \text{class } 3 \rangle \quad (7.3873)$$

8.210 Descendants of 6.21

$13p + 27$ algebras

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa, pb - ba, pc, pd - cac, \text{class } 3 \rangle \quad (7.3874)$$

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3875)$$

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa - cac, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3876)$$

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa - \omega cac, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3877)$$

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa - xcac, pb - ba, pc - cac, pd, \text{class } 3 \rangle \quad (7.3878)$$

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa, pb - ba - cac, pc, pd, \text{class } 3 \rangle \quad (7.3879)$$

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa - cac, pb - ba - cac, pc, pd, \text{class } 3 \rangle \quad (7.3880)$$

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa - \omega cac, pb - ba - cac, pc, pd, \text{class } 3 \rangle \quad (7.3881)$$

$$\langle a, b, c, d \mid caa, cb, da, db, dc, pa - xcac, pb - ba - cac, pc - cac, pd, \text{class } 3 \rangle \quad (7.3882)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db, dc, pa, pb - ba, pc, pd - xcac, \text{class } 3 \rangle \quad (7.3883)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db, dc, pa, pb - ba - cac, pc, pd - cac, \text{class } 3 \rangle \quad (7.3884)$$

- $\langle a, b, c, d \mid caa, cb, da - cac, db, dc, pa - cac, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3885)
- $\langle a, b, c, d \mid caa, cb, da - cac, db, dc, pa - \omega cac, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3886)
- $\langle a, b, c, d \mid caa, cb, da - cac, db, dc, pa - xcac, pb - ba, pc - cac, pd, \text{class } 3 \rangle$ (7.3887)
- $\langle a, b, c, d \mid caa, cb, da, db - cac, dc, pa, pb - ba, pc, pd - cac, \text{class } 3 \rangle$ (7.3888)
- $\langle a, b, c, d \mid caa, cb, da, db - cac, dc, pa, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3889)
- $\langle a, b, c, d \mid caa, cb, da, db - cac, dc, pa - cac, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3890)
- $\langle a, b, c, d \mid caa, cb, da, db - cac, dc, pa - \omega cac, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3891)
- $\langle a, b, c, d \mid caa, cb, da, db - cac, dc, pa - xcac, pb - ba, pc - cac, pd, \text{class } 3 \rangle$ (7.3892)
- $\langle a, b, c, d \mid caa, cb, da - cac, db - cac, dc, pa, pb - ba, pc, pd - xcac, \text{class } 3 \rangle$ (7.3893)
- $\langle a, b, c, d \mid caa, cb, da - cac, db - cac, dc, pa, pb - ba - cac, pc, pd - cac, \text{class } 3 \rangle$ (7.3894)
- $\langle a, b, c, d \mid caa, cb, da - cac, db - cac, dc, pa, pb - ba - \omega cac, pc, pd - cac, \text{class } 3 \rangle$ (7.3895)
- $\langle a, b, c, d \mid caa, cb, da - cac, db - cac, dc, pa - cac, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3896)
- $\langle a, b, c, d \mid caa, cb, da - cac, db - cac, dc, pa - \omega cac, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3897)
- $\langle a, b, c, d \mid caa, cb, da - cac, db - cac, dc, pa - xcac, pb - ba, pc - cac, pd, \text{class } 3 \rangle$ (7.3898)
- $\langle a, b, c, d \mid cac, cb, da, db, dc, pa, pb - ba, pc, pd - caa, \text{class } 3 \rangle$ (7.3899)
- $\langle a, b, c, d \mid cac, cb, da, db, dc, pa - caa, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3900)
- $\langle a, b, c, d \mid cac, cb, da, db, dc, pa, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle$ (7.3901)
- $\langle a, b, c, d \mid cac, cb, da, db, dc - caa, pa, pb - ba, pc, pd - caa, \text{class } 3 \rangle$ (7.3902)
- $\langle a, b, c, d \mid cac, cb, da, db, dc - caa, pa - caa, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3903)
- $\langle a, b, c, d \mid cac, cb, da, db, dc - caa, pa, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle$ (7.3904)
- $\langle a, b, c, d \mid cac, cb, da, db - caa, dc, pa, pb - ba, pc - xcaa, pd - caa, \text{class } 3 \rangle$ (7.3905)
- $\langle a, b, c, d \mid cac, cb, da, db - caa, dc, pa - caa, pb - ba, pc, pd, \text{class } 3 \rangle$ (7.3906)

$$\langle a, b, c, d \mid cac, cb, da, db - caa, dc, pa, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle \quad (7.3907)$$

$$\langle a, b, c, d \mid cac, cb, da, db - caa, dc - caa, pa, pb - ba, pc, pd - caa, \text{class } 3 \rangle \quad (7.3908)$$

$$\langle a, b, c, d \mid cac, cb, da, db - caa, dc - caa, pa - caa, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3909)$$

$$\langle a, b, c, d \mid cac, cb, da, db - caa, dc - caa, pa, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle \quad (7.3910)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db, dc, pa, pb - ba, pc, pd - caa, \text{class } 3 \rangle \quad (7.3911)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db, dc, pa - caa, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.3912)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db, dc, pa, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle \quad (7.3913)$$

8.211 Descendants of 6.23

$3p + 41 + 8 \gcd(p - 1, 3) + 2 \gcd(p - 1, 4)$ algebras

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb - ca, pc, pd - bab, \text{class } 3 \rangle \quad (7.3914)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb - ca, pc - bab, pd, \text{class } 3 \rangle \quad (7.3915)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb - ca, pc, pd, \text{class } 3 \rangle \quad (7.3916)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - bab, pb - ca, pc, pd, \text{class } 3 \rangle \quad (7.3917)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - \omega bab, pb - ca, pc, pd, \text{class } 3 \rangle \quad (7.3918)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb - ca - bab, pc, pd, \text{class } 3 \rangle \quad (7.3919)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - bab, pb - ca - bab, pc, pd, \text{class } 3 \rangle \quad (7.3920)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - \omega bab, pb - ca - bab, pc, pd, \text{class } 3 \rangle \quad (7.3921)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa, pb - ca, pc, pd - bab, \text{class } 3 \rangle \quad (7.3922)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa, pb - ca, pc, pd, \text{class } 3 \rangle \quad (7.3923)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - bab, pb - ca, pc, pd, \text{class } 3 \rangle \quad (7.3924)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - \omega bab, pb - ca, pc, pd, \text{class } 3 \rangle \quad (7.3925)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa, pb - ca, pc - bab, pd, \text{class } 3 \rangle \quad (7.3926)$$

- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - bab, pb - ca, pc - bab, pd, \text{class } 3 \rangle$ (7.3927)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - \omega bab, pb - ca, pc - bab, pd, \text{class } 3 \rangle$ (7.3928)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - \omega^2 bab, pb - ca, pc - bab, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.3929)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - \omega^3 bab, pb - ca, pc - bab, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.3930)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb - ca, pc, pd - bab, \text{class } 3 \rangle$ (7.3931)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb - ca, pc - bab, pd, \text{class } 3 \rangle$ (7.3932)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3933)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - bab, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3934)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - \omega bab, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3935)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb - ca, pc, pd - baa, \text{class } 3 \rangle$ (7.3936)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3937)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa - baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3938)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb - ca, pc - baa, pd, \text{class } 3 \rangle$ (7.3939)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb - ca, pc - \omega baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3940)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb - ca, pc - \omega^2 baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3941)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - ca, pc, pd - baa, \text{class } 3 \rangle$ (7.3942)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - ca, pc - baa, pd - baa, \text{class } 3 \rangle$ (7.3943)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - ca, pc - \omega baa, pd - baa, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3944)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - ca, pc - \omega^2 baa, pd - baa, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3945)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3946)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa - baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3947)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - ca, pc - baa, pd, \text{class } 3 \rangle$ (7.3948)

- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - ca, pc - \omega baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3949)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - ca, pc - \omega^2 baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3950)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - ca, pc, pd - baa, \text{class } 3 \rangle$ (7.3951)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3952)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa - baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3953)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - ca, pc - baa, pd, \text{class } 3 \rangle$ (7.3954)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - ca, pc - \omega baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3955)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - ca, pc - \omega^2 baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3956)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb - ca, pc, pd - baa, \text{class } 3 \rangle$ (7.3957)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3958)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa - baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3959)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa - \omega baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3960)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa - \omega^2 baa, pb - ca, pc, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.3961)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa - \omega^3 baa, pb - ca, pc, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.3962)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb - ca, pc - baa, pd, \text{class } 3 \rangle$ (7.3963)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb - ca, pc - \omega baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3964)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb - ca, pc - \omega^2 baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3965)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa, pb - ca, pc, pd - baa, \text{class } 3 \rangle$ (7.3966)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3967)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa - baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3968)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa - \omega baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3969)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa - xbaa, pb - ca - baa, pc, pd, \text{class } 3 \rangle$ (7.3970)

- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa - xbaa, pb - ca - \omega baa, pc, pd, \text{class } 3 \rangle$ (7.3971)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa, pb - ca, pc - baa, pd, \text{class } 3 \rangle$ (7.3972)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa, pb - ca, pc - \omega baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3973)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc, pa, pb - ca, pc - \omega^2 baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3974)
- $\langle a, b, c, d \mid bab - baa, cb, da - baa, db, dc, pa, pb - ca, pc, pd - baa, \text{class } 3 \rangle$ (7.3975)
- $\langle a, b, c, d \mid bab - baa, cb, da - baa, db, dc, pa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3976)
- $\langle a, b, c, d \mid bab - baa, cb, da - baa, db, dc, pa - baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3977)
- $\langle a, b, c, d \mid bab - baa, cb, da - baa, db, dc, pa - \omega baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3978)
- $\langle a, b, c, d \mid bab - baa, cb, da - baa, db, dc, pa, pb - ca, pc - baa, pd, \text{class } 3 \rangle$ (7.3979)
- $\langle a, b, c, d \mid bab - baa, cb, da - baa, db, dc, pa, pb - ca, pc - \omega baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3980)
- $\langle a, b, c, d \mid bab - baa, cb, da - baa, db, dc, pa, pb - ca, pc - \omega^2 baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3981)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc - baa, pa, pb - ca, pc, pd - xbaa, \text{class } 3 \rangle$ (7.3982)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc - baa, pa - baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3983)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc - baa, pa - \omega baa, pb - ca, pc, pd, \text{class } 3 \rangle$ (7.3984)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc - baa, pa, pb - ca, pc - baa, pd, \text{class } 3 \rangle$ (7.3985)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc - baa, pa, pb - ca, pc - \omega baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3986)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc - baa, pa, pb - ca, pc - \omega^2 baa, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.3987)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc - baa, pa, pb - ca - baa, pc, pd - baa, \text{class } 3 \rangle$ (7.3988)
- $\langle a, b, c, d \mid bab - baa, cb, da, db, dc - baa, pa, pb - ca - \omega baa, pc, pd - baa, \text{class } 3 \rangle$ (7.3989)

8.212 Descendants of 6.24

5 algebras

$$\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb - ca, pc, pd - bab, \text{class } 3 \rangle \quad (7.3990)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb - ca, pc, pd, \text{class } 3 \rangle \quad (7.3991)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb - ca, pc - bab, pd, \text{class } 3 \rangle \quad (7.3992)$$

$$\langle a, b, c, d \mid cb, da, db, dc, pa - ba, pb - ca, pc - \omega bab, pd, \text{class } 3 \rangle \quad (7.3993)$$

$$\langle a, b, c, d \mid cb, da, db, dc - bab, pa - ba, pb - ca, pc, pd, \text{class } 3 \rangle \quad (7.3994)$$

8.213 Descendants of 6.29

$10p + 69 + \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb, pc - bab, pd - ca, \text{class } 3 \rangle \quad (7.3995)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb, pc - bab, pd - ca - bab, \text{class } 3 \rangle \quad (7.3996)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb, pc, pd - ca, \text{class } 3 \rangle \quad (7.3997)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - bab, pb, pc, pd - ca, \text{class } 3 \rangle \quad (7.3998)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - \omega bab, pb, pc, pd - ca, \text{class } 3 \rangle \quad (7.3999)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb - bab, pc, pd - ca, \text{class } 3 \rangle \quad (7.4000)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - bab, pb - bab, pc, pd - ca, \text{class } 3 \rangle \quad (7.4001)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - \omega bab, pb - bab, pc, pd - ca, \text{class } 3 \rangle \quad (7.4002)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb, pc, pd - ca - bab, \text{class } 3 \rangle \quad (7.4003)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - bab, pb, pc, pd - ca - bab, \text{class } 3 \rangle \quad (7.4004)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - \omega bab, pb, pc, pd - ca - bab, \text{class } 3 \rangle \quad (7.4005)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa, pb - bab, pc, pd - ca - bab, \text{class } 3 \rangle \quad (7.4006)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - bab, pb - bab, pc, pd - ca - bab, \text{class } 3 \rangle \quad (7.4007)$$

$$\langle a, b, c, d \mid baa, cb, da, db, dc, pa - \omega bab, pb - bab, pc, pd - ca - bab, \text{class } 3 \rangle \quad (7.4008)$$

- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb, pc - xbab, pd - ca - bab, \text{class } 3 \rangle$ ($x \neq 0$) (7.4009)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb, pc - bab, pd - ca, \text{class } 3 \rangle$ (7.4010)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb, pc - \omega bab, pd - ca, \text{class } 3 \rangle$ (7.4011)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb, pc, pd - ca - bab, \text{class } 3 \rangle$ (7.4012)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - bab, pb, pc, pd - ca - bab, \text{class } 3 \rangle$ (7.4013)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - \omega bab, pb, pc, pd - ca - bab, \text{class } 3 \rangle$ (7.4014)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - xbab, pb - bab, pc, pd - ca - bab, \text{class } 3 \rangle$ (7.4015)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4016)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - bab, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4017)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - \omega bab, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4018)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4019)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - bab, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4020)
- $\langle a, b, c, d \mid baa, cb, da - bab, db, dc, pa - \omega bab, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4021)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - xbab, pb, pc - bab, pd - ca - bab, \text{class } 3 \rangle$ (7.4022)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa, pb, pc - bab, pd - ca, \text{class } 3 \rangle$ (7.4023)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - bab, pb, pc - bab, pd - ca, \text{class } 3 \rangle$ (7.4024)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - \omega bab, pb, pc - bab, pd - ca, \text{class } 3 \rangle$ (7.4025)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - xbab, pb, pc, pd - ca - bab, \text{class } 3 \rangle$ (7.4026)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - xbab, pb - bab, pc, pd - ca - bab, \text{class } 3 \rangle$ (7.4027)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4028)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - bab, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4029)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - \omega bab, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4030)

- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4031)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - bab, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4032)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - bab, pa - \omega bab, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4033)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - xbab, pb, pc - bab, pd - ca - bab, \text{class } 3 \rangle$ (7.4034)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa, pb, pc - bab, pd - ca, \text{class } 3 \rangle$ (7.4035)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - bab, pb, pc - bab, pd - ca, \text{class } 3 \rangle$ (7.4036)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - \omega bab, pb, pc - bab, pd - ca, \text{class } 3 \rangle$ (7.4037)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - xbab, pb, pc, pd - ca - bab, \text{class } 3 \rangle$ (7.4038)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - xbab, pb - bab, pc, pd - ca - bab, \text{class } 3 \rangle$ (7.4039)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4040)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - bab, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4041)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - \omega bab, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4042)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4043)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - bab, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4044)
- $\langle a, b, c, d \mid baa, cb, da, db, dc - \omega bab, pa - \omega bab, pb - bab, pc, pd - ca, \text{class } 3 \rangle$ (7.4045)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb, pc - baa, pd - ca, \text{class } 3 \rangle$ (7.4046)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb - baa, pc, pd - ca, \text{class } 3 \rangle$ (7.4047)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb - \omega baa, pc, pd - ca, \text{class } 3 \rangle$ (7.4048)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4049)
- $\langle a, b, c, d \mid bab, cb, da, db, dc, pa - baa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4050)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4051)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa - baa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4052)

- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa - \omega baa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4053)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - baa, pc, pd - ca, \text{class } 3 \rangle$ (7.4054)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - \omega baa, pc, pd - ca, \text{class } 3 \rangle$ (7.4055)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb, pc - baa, pd - ca, \text{class } 3 \rangle$ (7.4056)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - baa, pc - baa, pd - ca, \text{class } 3 \rangle$ (7.4057)
- $\langle a, b, c, d \mid bab, cb, da, db, dc - baa, pa, pb - \omega baa, pc - baa, pd - ca, \text{class } 3 \rangle$ (7.4058)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4059)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa - baa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4060)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa - \omega baa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4061)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa - \omega^2 baa, pb, pc, pd - ca, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.4062)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa - \omega^3 baa, pb, pc, pd - ca, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.4063)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa, pb, pc - baa, pd - ca, \text{class } 3 \rangle$ (7.4064)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa, pb, pc - \omega baa, pd - ca, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.4065)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa, pb, pc - \omega^2 baa, pd - ca, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.4066)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa, pb - baa, pc - xbaa, pd - ca, \text{class } 3 \rangle$ ($x \sim -x$) (7.4067)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc - baa, pa, pb - \omega baa, pc - xbaa, pd - ca, \text{class } 3 \rangle$ ($x \sim -x$) (7.4068)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb, pc - baa, pd - ca, \text{class } 3 \rangle$ (7.4069)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb - baa, pc, pd - ca, \text{class } 3 \rangle$ (7.4070)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb - \omega baa, pc, pd - ca, \text{class } 3 \rangle$ (7.4071)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4072)
- $\langle a, b, c, d \mid bab, cb - baa, da, db, dc, pa - baa, pb, pc, pd - ca, \text{class } 3 \rangle$ (7.4073)
- $\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb, pc - baa, pd - ca, \text{class } 3 \rangle$ (7.4074)

$$\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - baa, pc, pd - ca, \text{class } 3 \rangle \quad (7.4075)$$

$$\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb - \omega baa, pc, pd - ca, \text{class } 3 \rangle \quad (7.4076)$$

$$\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa, pb, pc, pd - ca, \text{class } 3 \rangle \quad (7.4077)$$

$$\langle a, b, c, d \mid bab, cb, da, db - baa, dc, pa - baa, pb, pc, pd - ca, \text{class } 3 \rangle \quad (7.4078)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - baa, dc, pa, pb, pc - baa, pd - ca, \text{class } 3 \rangle \quad (7.4079)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - baa, dc, pa, pb - xbaa, pc, pd - ca, \text{class } 3 \rangle \quad (7.4080)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - baa, dc, pa - baa, pb, pc, pd - ca, \text{class } 3 \rangle \quad (7.4081)$$

8.214 Descendants of 6.33

39 + gcd($p - 1, 3$) algebras

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.4082)$$

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd, pa, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.4083)$$

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd, pa - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.4084)$$

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.4085)$$

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd, pa, pb, pc - bab, pd, \text{class } 3 \rangle \quad (7.4086)$$

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd, pa, pb - bab, pc - bab, pd, \text{class } 3 \rangle \quad (7.4087)$$

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd, pa - bab, pb, pc - bab, pd, \text{class } 3 \rangle \quad (7.4088)$$

$$\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd, pa - \omega bab, pb, pc - bab, pd, \text{class } 3 \rangle \quad (7.4089)$$

$$\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.4090)$$

$$\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.4091)$$

$$\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.4092)$$

$$\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.4093)$$

$$\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa, pb, pc - bab, pd, \text{class } 3 \rangle \quad (7.4094)$$

- $\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa, pb - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.4095)
- $\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa - bab, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.4096)
- $\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa - \omega bab, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.4097)
- $\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.4098)
- $\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa, pb - bab, pc, pd - bab, \text{class } 3 \rangle$ (7.4099)
- $\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa - bab, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.4100)
- $\langle a, b, c, d \mid baa, ca, cb, da - bab, db, dcc, dcd, pa - \omega bab, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.4101)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.4102)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.4103)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.4104)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa, pb, pc - \omega bab, pd, \text{class } 3 \rangle$ (7.4105)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa, pb - bab, pc, pd - bab, \text{class } 3 \rangle$ (7.4106)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa, pb - bab, pc - bab, pd, \text{class } 3 \rangle$ (7.4107)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa, pb - bab, pc - \omega bab, pd, \text{class } 3 \rangle$ (7.4108)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa - bab, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.4109)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa - bab, pb, pc - \omega bab, pd, \text{class } 3 \rangle$ (7.4110)
- $\langle a, b, c, d \mid baa, ca, cb, da, db, dcc, dcd - bab, pa - \omega bab, pb, pc - \omega bab, pd, \text{class } 3 \rangle$ (7.4111)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.4112)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb, pc, pd - bab, \text{class } 3 \rangle$ (7.4113)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb, pc - bab, pd, \text{class } 3 \rangle$ (7.4114)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb, pc - \omega bab, pd, \text{class } 3 \rangle$ (7.4115)
- $\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb - bab, pc, pd - bab, \text{class } 3 \rangle$ (7.4116)

$$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb - bab, pc - bab, pd, \text{class } 3 \rangle \quad (7.4117)$$

$$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb - bab, pc - \omega bab, pd, \text{class } 3 \rangle \quad (7.4118)$$

$$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa - bab, pb, pc - bab, pd, \text{class } 3 \rangle \quad (7.4119)$$

$$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa - bab, pb, pc - \omega bab, pd, \text{class } 3 \rangle \quad (7.4120)$$

$$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa - \omega bab, pb, pc - \omega bab, pd, \text{class } 3 \rangle \quad (7.4121)$$

$$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb - bab, pc, pd - \omega bab, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.4122)$$

$$\langle a, b, c, d \mid baa, ca - bab, cb, da, db, dcc, dcd - bab, pa, pb - bab, pc, pd - \omega^2 bab, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.4123)$$

8.215 Descendants of 6.34

$5p + 38 + 2 \gcd(p - 1, 4)$ algebras

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.4124)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - cac, pb, pc, pd, \text{class } 3 \rangle \quad (7.4125)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - \omega cac, pb, pc, pd, \text{class } 3 \rangle \quad (7.4126)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba, pb, pc - cac, pd, \text{class } 3 \rangle \quad (7.4127)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - cac, pb, pc - cac, pd, \text{class } 3 \rangle \quad (7.4128)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - \omega cac, pb, pc - cac, pd, \text{class } 3 \rangle \quad (7.4129)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba, pb - cac, pc, pd, \text{class } 3 \rangle \quad (7.4130)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - cac, pb - cac, pc, pd, \text{class } 3 \rangle \quad (7.4131)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - \omega cac, pb - cac, pc, pd, \text{class } 3 \rangle \quad (7.4132)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba, pb - cac, pc - cac, pd, \text{class } 3 \rangle \quad (7.4133)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - cac, pb - cac, pc - cac, pd, \text{class } 3 \rangle \quad (7.4134)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - \omega cac, pb - cac, pc - cac, pd, \text{class } 3 \rangle \quad (7.4135)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.4136)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - cac, pb, pc, pd, \text{ class } 3 \rangle \quad (7.4137)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - \omega cac, pb, pc, pd, \text{ class } 3 \rangle \quad (7.4138)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba, pb, pc, pd - cac, \text{ class } 3 \rangle \quad (7.4139)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba + cac, pb, pc, pd - cac, \text{ class } 3 \rangle \quad (7.4140)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba + (1 - \omega)cac, pb, pc, pd - cac, \text{ class } 3 \rangle \quad (7.4141)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba, pb, pc - cac, pd, \text{ class } 3 \rangle \quad (7.4142)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - cac, pb, pc - cac, pd, \text{ class } 3 \rangle \quad (7.4143)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - \omega cac, pb, pc - cac, pd, \text{ class } 3 \rangle \quad (7.4144)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb, pc - cac, pd - cac, \text{ class } 3 \rangle \quad (7.4145)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - cac, pc, pd, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.4146)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - cac, pc - cac, pd, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.4147)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - \omega cac, pc, pd, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.4148)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - \omega cac, pc - cac, pd, \text{ class } 3 \rangle (p = 3 \bmod 4) \quad (7.4149)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - cac, pc, pd, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4150)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - cac, pc - cac, pd, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4151)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - \omega cac, pc, pd, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4152)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - \omega cac, pc - cac, pd, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4153)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - \omega^2 cac, pc, pd, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4154)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - \omega^2 cac, pc - cac, pd, \text{ class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4155)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - \omega^3 cac, pc, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \pmod{4}) \quad (7.4156)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - \omega^3 cac, pc - cac, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \pmod{4}) \quad (7.4157)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.4158)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb, pc - caa, pd, \text{class } 3 \rangle \quad (7.4159)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb, pc - \omega caa, pd, \text{class } 3 \rangle \quad (7.4160)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - caa, pc, pd, \text{class } 3 \rangle \quad (7.4161)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - caa, pc - caa, pd, \text{class } 3 \rangle \quad (7.4162)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - caa, pc - \omega caa, pd, \text{class } 3 \rangle \quad (7.4163)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba - caa, pb, pc, pd, \text{class } 3 \rangle \quad (7.4164)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba - caa, pb - caa, pc, pd, \text{class } 3 \rangle \quad (7.4165)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb, pc, pd, \text{class } 3 \rangle \quad (7.4166)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba - caa, pb, pc, pd, \text{class } 3 \rangle \quad (7.4167)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb - caa, pc, pd, \text{class } 3 \rangle \quad (7.4168)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba - caa, pb - caa, pc, pd, \text{class } 3 \rangle \quad (7.4169)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb, pc - caa, pd, \text{class } 3 \rangle \quad (7.4170)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb, pc - \omega caa, pd, \text{class } 3 \rangle \quad (7.4171)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb - caa, pc - caa, pd, \text{class } 3 \rangle \quad (7.4172)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb - caa, pc - \omega caa, pd, \text{class } 3 \rangle \quad (7.4173)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb, pc, pd - caa, \text{class } 3 \rangle \quad (7.4174)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba + caa, pb, pc, pd - caa, \text{class } 3 \rangle \quad (7.4175)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba + (1 - \omega)caa, pb, pc, pd - caa, \text{class } 3 \rangle \quad (7.4176)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb, pc - caa, pd - caa, \text{class } 3 \rangle \quad (7.4177)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb, pc - \omega caa, pd - caa, \text{class } 3 \rangle \quad (7.4178)$$

8.216 Descendants of 6.35

10p + 26 algebras

- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba, pc, pd, \text{ class } 3 \rangle \quad (7.4179)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - caa, pb - ba, pc, pd, \text{ class } 3 \rangle \quad (7.4180)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba - caa, pc, pd, \text{ class } 3 \rangle \quad (7.4181)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - caa, pb - ba - caa, pc, pd, \text{ class } 3 \rangle \quad (7.4182)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba, pc - caa, pd, \text{ class } 3 \rangle \quad (7.4183)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba, pc - \omega caa, pd, \text{ class } 3 \rangle \quad (7.4184)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba - caa, pc - caa, pd, \text{ class } 3 \rangle \quad (7.4185)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba - caa, pc - \omega caa, pd, \text{ class } 3 \rangle \quad (7.4186)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba, pc, pd - caa, \text{ class } 3 \rangle \quad (7.4187)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa + caa, pb - ba, pc, pd - caa, \text{ class } 3 \rangle \quad (7.4188)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba, pc - caa, pd - caa, \text{ class } 3 \rangle \quad (7.4189)$$
- $$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa, pb - ba, pc - \omega caa, pd - caa, \text{ class } 3 \rangle \quad (7.4190)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba, pc, pd, \text{ class } 3 \rangle \quad (7.4191)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - caa, pb - ba, pc, pd, \text{ class } 3 \rangle \quad (7.4192)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba - caa, pc, pd, \text{ class } 3 \rangle \quad (7.4193)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - caa, pb - ba - caa, pc, pd, \text{ class } 3 \rangle \quad (7.4194)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba, pc - caa, pd, \text{ class } 3 \rangle \quad (7.4195)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba, pc - \omega caa, pd, \text{ class } 3 \rangle \quad (7.4196)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba - caa, pc - caa, pd, \text{ class } 3 \rangle \quad (7.4197)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba - caa, pc - \omega caa, pd, \text{ class } 3 \rangle \quad (7.4198)$$
- $$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba, pc, pd - caa, \text{ class } 3 \rangle \quad (7.4199)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa + caa, pb - ba, pc, pd - caa, \text{class } 3 \rangle \quad (7.4200)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba, pc - caa, pd - caa, \text{class } 3 \rangle \quad (7.4201)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa, pb - ba, pc - \omega caa, pd - caa, \text{class } 3 \rangle \quad (7.4202)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle \quad (7.4203)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa, pb - ba, pc - xcaa, pd - caa, \text{class } 3 \rangle \quad (7.4204)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa, pb - ba - caa, pc - xcaa, pd, \text{class } 3 \rangle \quad (7.4205)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa, pb - ba - caa, pc - xcaa, pd - caa, \text{class } 3 \rangle \quad (7.4206)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - caa, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4207)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - caa, pb - ba - caa, pc, pd, \text{class } 3 \rangle \quad (7.4208)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - xcaa, pb - ba, pc, pd - caa, \text{class } 3 \rangle (x \neq 0, -2, x \sim -x - 2) \quad (7.4209)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - xcaa, pb - ba - caa, pc, pd - caa, \text{class } 3 \rangle (x \neq 0, -2, x \sim -x - 2) \quad (7.4210)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle \quad (7.4211)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa, pb - ba, pc - xcaa, pd - caa, \text{class } 3 \rangle \quad (7.4212)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa, pb - ba - caa, pc - xcaa, pd, \text{class } 3 \rangle \quad (7.4213)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa, pb - ba - caa, pc - xcaa, pd - caa, \text{class } 3 \rangle \quad (7.4214)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - caa, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4215)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - caa, pb - ba - caa, pc, pd, \text{class } 3 \rangle \quad (7.4216)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - xcaa, pb - ba, pc, pd - caa, \text{class } 3 \rangle (x \neq 0, -2, x \sim -x - 2) \quad (7.4217)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - xcaa, pb - ba - caa, pc, pd - caa, \text{class } 3 \rangle (x \neq 0, -2, x \sim -x - 2) \quad (7.4218)$$

8.217 Descendants of 6.36

$p^3 + 9p^2 + 20p + 18 + \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4219)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - cac, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4220)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - \omega cac, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4221)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - xcac, pb - ba - cac, pc, pd, \text{class } 3 \rangle \quad (7.4222)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - xcac, pb - ba - \omega cac, pc, pd, \text{class } 3 \rangle \quad (7.4223)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba, pb - ba, pc, pd - cac, \text{class } 3 \rangle \quad (7.4224)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba + cac, pb - ba, pc, pd - cac, \text{class } 3 \rangle \quad (7.4225)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba + (1 - \omega)cac, pb - ba, pc, pd - cac, \text{class } 3 \rangle \quad (7.4226)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba, pb - ba, pc - cac, pd, \text{class } 3 \rangle \quad (7.4227)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - cac, pb - ba, pc - cac, pd, \text{class } 3 \rangle \quad (7.4228)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - \omega cac, pb - ba, pc - cac, pd, \text{class } 3 \rangle \quad (7.4229)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - xcac, pb - ba - cac, pc - cac, pd, \text{class } 3 \rangle \quad (7.4230)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - xcac, pb - ba - \omega cac, pc - cac, pd, \text{class } 3 \rangle \quad (7.4231)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba, dc, pa - ba - xcac, pb - ba, pc - cac, pd - cac, \text{class } 3 \rangle \quad (7.4232)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - xcac, pb - ba, pc - ycac, pd - cac, \text{class } 3 \rangle ((x, y) \sim (x, -y), p = 3 \bmod 4) \quad (7.4233)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - xcac, pb - ba, pc - ycac, pd - \omega cac, \text{class } 3 \rangle ((x, y) \sim (x, -y), p = 3 \bmod 4) \quad (7.4234)$$

In the following presentation $x \neq 0$, $(x, y) \sim (x, -y) \sim (-x, iy)$ if $i^2 = -1$:

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - xcac, pb - ba, pc - ycac, pd - cac, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.4235)$$

In the next presentation we have $x \neq 1 - \omega$, $(x, y) \sim (x, -y) \sim (-x + 2(1 - \omega), iy)$ if $i^2 = -1$:

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - xcac, pb - ba, pc - ycac, pd - \omega cac, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.4236)$$

In the next presentation we have $x \neq 1 - \omega^2$, $(x, y) \sim (x, -y) \sim (-x + 2(1 - \omega^2), iy)$ if $i^2 = -1$:

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - xcac, pb - ba, pc - ycac, pd - \omega^2 cac, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.4237)$$

In the next presentation we have $x \neq 1 - \omega^3$, $(x, y) \sim (x, -y) \sim (-x + 2(1 - \omega^3), iy)$ if $i^2 = -1$:

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - xcac, pb - ba, pc - ycac, pd - \omega^3 cac, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.4238)$$

In the next four presentations we have $x \sim ax$ if $a^4 = 1$, $p = 1 \bmod 4$:

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba, pb - ba, pc - xcac, pd - cac, \text{class } 3 \rangle \quad (7.4239)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - (1 - \omega)cac, pb - ba, pc - xcac, pd - \omega cac, \text{class } 3 \rangle \quad (7.4240)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - (1 - \omega^2)cac, pb - ba, pc - xcac, pd - \omega^2 cac, \text{class } 3 \rangle \quad (7.4241)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - (1 - \omega^3)cac, pb - ba, pc - xcac, pd - \omega^3 cac, \text{class } 3 \rangle \quad (7.4242)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - cac, pb - ba - cac, pc, pd, \text{class } 3 \rangle \quad (7.4243)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - cac, pb - ba - cac, pc - cac, pd, \text{class } 3 \rangle \quad (7.4244)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - cac, pb - ba - cac, pc - \omega cac, pd, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4245)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - cac, pb - ba - cac, pc - \omega^2 cac, pd, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4246)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - cac, pb - ba - 2cac, pc - xcac, pd, \text{class } 3 \rangle (x \sim -x) \quad (7.4247)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - cac, pb - ba - (1 + \omega)cac, pc - xcac, pd, \text{class } 3 \rangle (x \sim -x) \quad (7.4248)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - 2cac, pb - ba - xcac, pc - ycac, pd, \text{class } 3 \rangle ((x, y) \sim (x, -y)) \quad (7.4249)$$

$$\langle a, b, c, d \mid caa, cb, da, db - ba - cac, dc, pa - ba - (1 + \omega)cac, pb - ba - xcac, pc - ycac, pd, \text{class } 3 \rangle ((x, y) \sim (x, -y)) \quad (7.4250)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - ba - ycac, pc - zcac, pd - cac, \text{class } 3 \rangle ((x, y, z) \sim (x, y, -z)) \quad (7.4251)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - ba - ycac, pc, pd, \text{class } 3 \rangle \quad (7.4252)$$

$$\langle a, b, c, d \mid caa, cb, da - cac, db - ba, dc, pa - ba - xcac, pb - ba - ycac, pc - cac, pd, \text{class } 3 \rangle \quad (7.4253)$$

$$\langle a, b, c, d \mid caa, cb, da - \omega cac, db - ba, dc, pa - ba - xcac, pb - ba - ycac, pc - zcac, pd - cac, \text{class } 3 \rangle ((x, y, z) \sim (x, y, -z)) \quad (7.4254)$$

$$\langle a, b, c, d \mid caa, cb, da - \omega cac, db - ba, dc, pa - ba - xcac, pb - ba - ycac, pc, pd, \text{class } 3 \rangle \quad (7.4255)$$

$$\langle a, b, c, d \mid caa, cb, da - \omega cac, db - ba, dc, pa - ba - xcac, pb - ba - ycac, pc - cac, pd, \text{class } 3 \rangle \quad (7.4256)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4257)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba - caa, pc, pd, \text{class } 3 \rangle \quad (7.4258)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba - caa, pb - ba - xcaa, pc, pd, \text{class } 3 \rangle \quad (7.4259)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba, pc, pd - caa, \text{class } 3 \rangle \quad (7.4260)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba + caa, pb - ba, pc, pd - caa, \text{class } 3 \rangle \quad (7.4261)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba, pc - caa, pd, \text{class } 3 \rangle \quad (7.4262)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba - caa, pc - caa, pd, \text{class } 3 \rangle \quad (7.4263)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba, pc - \omega caa, pd, \text{class } 3 \rangle \quad (7.4264)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba - caa, pc - \omega caa, pd, \text{class } 3 \rangle \quad (7.4265)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba, pc - caa, pd - caa, \text{class } 3 \rangle \quad (7.4266)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc, pa - ba, pb - ba, pc - \omega caa, pd - caa, \text{class } 3 \rangle \quad (7.4267)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4268)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb - ba - caa, pc, pd, \text{class } 3 \rangle \quad (7.4269)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba - caa, pb - ba - xcaa, pc, pd, \text{class } 3 \rangle \quad (7.4270)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle (x \neq 0) \quad (7.4271)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb - ba - caa, pc - xcaa, pd, \text{class } 3 \rangle (x \neq 0) \quad (7.4272)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba - xcaa, pb - ba, pc, pd - caa, \text{class } 3 \rangle \quad (7.4273)$$

$$\langle a, b, c, d \mid cac, cb - caa, da, db - ba, dc, pa - ba, pb - ba, pc - xcaa, pd - caa, \text{class } 3 \rangle (x \neq 0) \quad (7.4274)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4275)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - ba, pb - ba - caa, pc, pd, \text{class } 3 \rangle \quad (7.4276)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - ba - caa, pb - ba - xcaa, pc, pd, \text{class } 3 \rangle \quad (7.4277)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - ba, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle (x \neq 0) \quad (7.4278)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - ba, pb - ba - caa, pc - xcaa, pd, \text{class } 3 \rangle (x \neq 0) \quad (7.4279)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - ba - ycaa, pb - ba - xcaa, pc, pd - caa, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (-x, -y - 2)) \quad (7.4280)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - ba - xcaa, pb - ba, pc, pd - caa, \text{class } 3 \rangle (x \sim -x - 2) \quad (7.4281)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - caa, pa - ba, pb - ba - ycaa, pc - xcaa, pd - caa, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (x, -y)) \quad (7.4282)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - ba, pb - ba, pc, pd, \text{class } 3 \rangle \quad (7.4283)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - ba, pb - ba - caa, pc, pd, \text{class } 3 \rangle \quad (7.4284)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - ba - caa, pb - ba - xcaa, pc, pd, \text{class } 3 \rangle \quad (7.4285)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - ba, pb - ba, pc - xcaa, pd, \text{class } 3 \rangle (x \neq 0) \quad (7.4286)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - ba, pb - ba - caa, pc - xcaa, pd, \text{class } 3 \rangle (x \neq 0) \quad (7.4287)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - ba - ycaa, pb - ba - xcaa, pc, pd - caa, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (-x, -y - 2)) \quad (7.4288)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - ba - xcaa, pb - ba, pc, pd - caa, \text{class } 3 \rangle (x \sim -x - 2) \quad (7.4289)$$

$$\langle a, b, c, d \mid cac, cb, da, db - ba, dc - \omega caa, pa - ba, pb - ba - ycaa, pc - xcaa, pd - caa, \text{class } 3 \rangle (x \neq 0, (x, y) \sim (x, -y)) \quad (7.4290)$$

8.218 Descendants of 6.48

$27 + 3 \gcd(p - 1, 3) + 2 \gcd(p - 1, 4)$ algebras

- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc, pa, pb, pc - baa, pd, \text{ class } 3 \rangle \quad (7.4291)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc, pa, pb, pc, pd - baa, \text{ class } 3 \rangle \quad (7.4292)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc, pa, pb, pc, pd, \text{ class } 3 \rangle \quad (7.4293)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc, pa - baa, pb, pc, pd, \text{ class } 3 \rangle \quad (7.4294)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc, pa, pb - baa, pc, pd, \text{ class } 3 \rangle \quad (7.4295)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3 \rangle \quad (7.4296)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb, pc - baa, pd, \text{ class } 3 \rangle \quad (7.4297)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb, pc, pd - baa, \text{ class } 3 \rangle \quad (7.4298)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb, pc, pd, \text{ class } 3 \rangle \quad (7.4299)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc, pa - baa, pb, pc, pd, \text{ class } 3 \rangle \quad (7.4300)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb - baa, pc, pd, \text{ class } 3 \rangle \quad (7.4301)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3 \rangle \quad (7.4302)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc - baa, pd, \text{ class } 3 \rangle \quad (7.4303)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc - \omega baa, pd, \text{ class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.4304)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc - \omega^2 baa, pd, \text{ class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.4305)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa - baa, pb, pc, pd, \text{ class } 3 \rangle \quad (7.4306)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc, pd, \text{ class } 3 \rangle \quad (7.4307)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc, pd - baa, \text{ class } 3 \rangle \quad (7.4308)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - baa, pc, pd, \text{ class } 3 \rangle \quad (7.4309)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - baa, pc, pd - baa, \text{ class } 3 \rangle \quad (7.4310)$$
- $$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - \omega baa, pc, pd, \text{ class } 3 \rangle \quad (7.4311)$$

- $\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - \omega baa, pc, pd - baa, \text{class } 3 \rangle$ (7.4312)
- $\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - \omega^2 baa, pc, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.4313)
- $\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - \omega^2 baa, pc, pd - baa, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.4314)
- $\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - \omega^3 baa, pc, pd, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.4315)
- $\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - \omega^3 baa, pc, pd - baa, \text{class } 3 \rangle$ ($p = 1 \pmod{4}$) (7.4316)
- $\langle a, b, c, d \mid baa, bab, bac, cb, da, db - ca, dc, pa, pb, pc - bad, pd, \text{class } 3 \rangle$ (7.4317)
- $\langle a, b, c, d \mid baa, bab, bac, cb, da, db - ca, dc, pa, pb, pc - \omega bad, pd, \text{class } 3 \rangle$ (7.4318)
- $\langle a, b, c, d \mid baa, bab, bac, cb, da, db - ca, dc, pa - bad, pb, pc, pd, \text{class } 3 \rangle$ (7.4319)
- $\langle a, b, c, d \mid baa, bab, bac, cb, da, db - ca, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.4320)
- $\langle a, b, c, d \mid baa, bab, bac, cb, da, db - ca, dc, pa, pb, pc, pd - bad, \text{class } 3 \rangle$ (7.4321)
- $\langle a, b, c, d \mid baa, bab, bac, cb, da, db - ca, dc, pa, pb - bad, pc, pd, \text{class } 3 \rangle$ (7.4322)
- $\langle a, b, c, d \mid baa, bab, bac, cb, da, db - ca, dc, pa, pb - bad, pc, pd - bad, \text{class } 3 \rangle$ (7.4323)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb, pc - bad, pd, \text{class } 3 \rangle$ (7.4324)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb, pc - \omega bad, pd, \text{class } 3 \rangle$ (7.4325)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa - bad, pb, pc, pd, \text{class } 3 \rangle$ (7.4326)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb, pc, pd, \text{class } 3 \rangle$ (7.4327)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb - bad, pc, pd, \text{class } 3 \rangle$ (7.4328)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb, pc, pd - bad, \text{class } 3 \rangle$ (7.4329)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb - bad, pc, pd - bad, \text{class } 3 \rangle$ (7.4330)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb, pc, pd - \omega bad, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.4331)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb - bad, pc, pd - \omega bad, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.4332)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb, pc, pd - \omega^2 bad, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.4333)
- $\langle a, b, c, d \mid baa, bab, bac, cb - bad, da, db - ca, dc, pa, pb - bad, pc, pd - \omega^2 bad, \text{class } 3 \rangle$ ($p = 1 \pmod{3}$) (7.4334)

8.219 Descendants of 6.51

$8p + 15 + (2p + 5) \gcd(p - 1, 3) + (p + 2) \gcd(p - 1, 4) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb, pc - bab, pd, \text{class } 3 \rangle \quad (7.4335)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb, pc, pd - bab, \text{class } 3 \rangle \quad (7.4336)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb, pc - bab, pd - bab, \text{class } 3 \rangle \quad (7.4337)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb, pc, pd - \omega bab, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.4338)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb, pc - bab, pd - \omega bab, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.4339)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb, pc, pd - \omega^2 bab, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.4340)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb, pc - bab, pd - \omega^2 bab, \text{class } 3 \rangle \quad (p = 1 \pmod{3}) \quad (7.4341)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb, pc, pd, \text{class } 3 \rangle \quad (7.4342)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.4343)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.4344)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.4345)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca - bab, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.4346)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - ca - \omega bab, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.4347)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - xbab, pd - bab, \text{class } 3 \rangle \quad (x \sim ax \text{ if } a^3 = 1) \quad (7.4348)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - xbab, pd - \omega bab, \text{class } 3 \rangle \quad (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.4349)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - xbab, pd - \omega^2 bab, \text{class } 3 \rangle \quad (x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.4350)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - bab, pd, \text{class } 3 \rangle \quad (7.4351)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - \omega bab, pd, \text{class } 3 \rangle \quad (p = 1 \pmod{5}) \quad (7.4352)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - \omega^2 bab, pd, \text{class } 3 \rangle \quad (p = 1 \pmod{5}) \quad (7.4353)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - \omega^3 bab, pd, \text{class } 3 \rangle \quad (p = 1 \pmod{5}) \quad (7.4354)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - \omega^4 bab, pd, \text{class } 3 \rangle (p = 1 \bmod 5) \quad (7.4355)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca - xbab, pb - bab, pc, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4356)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca - xbab, pb - \omega bab, pc, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4357)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca - xbab, pb - \omega^2 bab, pc, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4358)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca - xbab, pb - \omega^3 bab, pc, pd, \text{class } 3 \rangle (x \sim -x, p = 1 \bmod 4) \quad (7.4359)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca - xbab, pb - bab, pc, pd, \text{class } 3 \rangle (p = 3 \bmod 4) \quad (7.4360)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca - xbab, pb - \omega bab, pc, pd, \text{class } 3 \rangle (p = 3 \bmod 4) \quad (7.4361)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc, pd, \text{class } 3 \rangle \quad (7.4362)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca - bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.4363)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - ca - \omega bab, pb, pc, pd, \text{class } 3 \rangle \quad (7.4364)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - ca, pb, pc, pd - xbab, \text{class } 3 \rangle \quad (7.4365)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - ca, pb, pc - bab, pd - xbab, \text{class } 3 \rangle \quad (7.4366)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - ca, pb - bab, pc, pd, \text{class } 3 \rangle \quad (7.4367)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - ca - xbab, pb, pc, pd + bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 =) \quad (7.4368)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - ca - xbab, pb, pc - bab, pd + bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.4369)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega bab, pa - ca, pb, pc, pd - xbab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4370)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega bab, pa - ca, pb, pc - bab, pd - xbab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4371)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega bab, pa - ca, pb - bab, pc, pd, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4372)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega bab, pa - ca - xbab, pb, pc, pd + \omega bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.4373)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega bab, pa - ca - xbab, pb, pc - bab, pd + \omega bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.4374)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega^2 bab, pa - ca, pb, pc, pd - xbab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4375)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega^2 bab, pa - ca, pb, pc - bab, pd - xbab, \text{class } 3 \rangle (\text{all } x, p = 1 \bmod 3) \quad (7.4376)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega^2 bab, pa - ca, pb - bab, pc, pd, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4377)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega^2 bab, pa - ca - xbab, pb, pc, pd + \omega^2 bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.4378)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - \omega^2 bab, pa - ca - xbab, pb, pc - bab, pd + \omega^2 bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.4379)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa - ca, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.4380)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa - ca, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.4381)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa - ca, pb, pc, pd, \text{class } 3 \rangle \quad (7.4382)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa - ca - baa, pb, pc, pd, \text{class } 3 \rangle \quad (7.4383)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa - ca, pb - baa, pc, pd, \text{class } 3 \rangle \quad (7.4384)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa - ca, pb - \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.4385)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa - ca, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.4386)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa - ca, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.4387)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa - ca, pb, pc, pd - \omega baa, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.4388)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa - ca, pb - xbaa, pc, pd, \text{class } 3 \rangle \quad (7.4389)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa - ca - baa, pb + baa, pc, pd, \text{class } 3 \rangle \quad (7.4390)$$

$$\langle a, b, c, d \mid bab, cb - \omega baa, da, db - ca, dc, pa - ca, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.4391)$$

$$\langle a, b, c, d \mid bab, cb - \omega baa, da, db - ca, dc, pa - ca, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.4392)$$

$$\langle a, b, c, d \mid bab, cb - \omega baa, da, db - ca, dc, pa - ca, pb, pc, pd - \omega baa, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.4393)$$

$$\langle a, b, c, d \mid bab, cb - \omega baa, da, db - ca, dc, pa - ca, pb - xbaa, pc, pd, \text{class } 3 \rangle \quad (7.4394)$$

$$\langle a, b, c, d \mid bab, cb - \omega baa, da, db - ca, dc, pa - ca - baa, pb + \omega baa, pc, pd, \text{class } 3 \rangle \quad (7.4395)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - ca, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.4396)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - ca, pb, pc - \omega baa, pd, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.4397)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - ca, pb, pc - \omega^2 baa, pd, \text{class } 3 \rangle \quad (p = 1 \bmod 3) \quad (7.4398)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - ca, pb, pc, pd - xbaa, \text{class } 3 \rangle \quad (7.4399)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - ca, pb - baa, pc, pd - xbaa, \text{class } 3 \rangle \quad (7.4400)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - ca, pb - \omega baa, pc, pd - xbaa, \text{class } 3 \rangle \quad (7.4401)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - ca, pb - \omega^2 baa, pc, pd - xbaa, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.4402)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - ca, pb - \omega^3 baa, pc, pd - xbaa, \text{class } 3 \rangle \quad (p = 1 \bmod 4) \quad (7.4403)$$

8.220 Descendants of 6.52

$4p^2 + 15p + 15 + (p + 1) \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa, pb, pc - ca, pd - bab, \text{class } 3 \rangle \quad (7.4404)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa, pb - bab, pc - ca, pd, \text{class } 3 \rangle \quad (7.4405)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa, pb - bab, pc - ca - bab, pd, \text{class } 3 \rangle \quad (7.4406)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa, pb, pc - ca, pd, \text{class } 3 \rangle \quad (7.4407)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - bab, pb, pc - ca, pd, \text{class } 3 \rangle \quad (7.4408)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - \omega bab, pb, pc - ca, pd, \text{class } 3 \rangle \quad (7.4409)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa, pb, pc - ca - bab, pd, \text{class } 3 \rangle \quad (7.4410)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - bab, pb, pc - ca - bab, pd, \text{class } 3 \rangle \quad (7.4411)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc, pa - \omega bab, pb, pc - ca - bab, pd, \text{class } 3 \rangle \quad (7.4412)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa, pb, pc - ca, pd - bab, \text{class } 3 \rangle \quad (7.4413)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa, pb, pc - ca, pd - \omega bab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4414)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa, pb, pc - ca, pd - \omega^2 bab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4415)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - bab, pb, pc - ca, pd, \text{class } 3 \rangle \quad (7.4416)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - \omega bab, pb, pc - ca, pd, \text{class } 3 \rangle \quad (7.4417)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - \omega^2 bab, pb, pc - ca, pd, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.4418)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa - \omega^3 bab, pb, pc - ca, pd, \text{class } 3 \rangle (p = 1 \bmod 4) \quad (7.4419)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa, pb, pc - ca, pd, \text{class } 3 \rangle \quad (7.4420)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa, pb - bab, pc - ca, pd, \text{class } 3 \rangle \quad (7.4421)$$

$$\langle a, b, c, d \mid baa, cb, da, db - ca, dc - bab, pa, pb - \omega bab, pc - ca, pd, \text{class } 3 \rangle \quad (7.4422)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa, pb, pc - ca, pd - xbab, \text{class } 3 \rangle \quad (7.4423)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa, pb, pc - ca - bab, pd - bab, \text{class } 3 \rangle \quad (7.4424)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - xbab, pb - bab, pc - ca, pd, \text{class } 3 \rangle \quad (7.4425)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - bab, pb, pc - ca, pd, \text{class } 3 \rangle \quad (7.4426)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc, pa - \omega bab, pb, pc - ca, pd, \text{class } 3 \rangle \quad (7.4427)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - bab, pa, pb, pc - ca, pd - xbab, \text{class } 3 \rangle \quad (7.4428)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - bab, pa, pb, pc - ca - xbab, pd - bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.4429)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - bab, pa - xbab, pb, pc - ca, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.4430)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - bab, pa, pb - xbab, pc - ca, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1) \quad (7.4431)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - \omega bab, pa, pb, pc - ca, pd - xbab, \text{class } 3 \rangle (p = 1 \bmod 3) \quad (7.4432)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - \omega bab, pa, pb, pc - ca - xbab, pd - bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \bmod 3) \quad (7.4433)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - \omega bab, pa - xbab, pb, pc - ca, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.4434)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - \omega bab, pa, pb - xbab, pc - ca, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.4435)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - \omega^2 bab, pa, pb, pc - ca, pd - xbab, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.4436)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - \omega^2 bab, pa, pb, pc - ca - xbab, pd - bab, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.4437)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - \omega^2 bab, pa - xbab, pb, pc - ca, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.4438)$$

$$\langle a, b, c, d \mid baa, cb, da - bab, db - ca, dc - \omega^2 bab, pa, pb - xbab, pc - ca, pd, \text{class } 3 \rangle (x \neq 0, x \sim ax \text{ if } a^3 = 1, p = 1 \pmod{3}) \quad (7.4439)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa, pb, pc - ca, pd - baa, \text{class } 3 \rangle \quad (7.4440)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa, pb, pc - ca - baa, pd - baa, \text{class } 3 \rangle \quad (7.4441)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa, pb - xbaa, pc - ca, pd, \text{class } 3 \rangle \quad (7.4442)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa, pb - xbaa, pc - ca - baa, pd, \text{class } 3 \rangle \quad (7.4443)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa - baa, pb - xbaa, pc - ca, pd, \text{class } 3 \rangle \quad (7.4444)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc, pa - baa, pb - xbaa, pc - ca - baa, pd, \text{class } 3 \rangle \quad (7.4445)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa, pb, pc - ca - xbaa, pd - baa, \text{class } 3 \rangle (x \sim -x) \quad (7.4446)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa, pb, pc - ca - xbaa, pd - \omega baa, \text{class } 3 \rangle (x \sim -x) \quad (7.4447)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa - xbaa, pb - ybaa, pc - ca - baa, pd, \text{class } 3 \rangle \quad (7.4448)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa, pb - xbaa, pc - ca, pd, \text{class } 3 \rangle \quad (7.4449)$$

$$\langle a, b, c, d \mid bab, cb - baa, da, db - ca, dc, pa - baa, pb - xbaa, pc - ca, pd, \text{class } 3 \rangle \quad (7.4450)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa, pb - ybaa, pc - ca - xbaa, pd - baa, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.4451)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa, pb - ybaa, pc - ca - xbaa, pd, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.4452)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - baa, pa - baa, pb - ybaa, pc - ca - xbaa, pd, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.4453)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - \omega baa, pa, pb - ybaa, pc - ca - xbaa, pd - baa, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.4454)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - \omega baa, pa, pb - ybaa, pc - ca - xbaa, pd, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.4455)$$

$$\langle a, b, c, d \mid bab, cb, da, db - ca, dc - \omega baa, pa - baa, pb - ybaa, pc - ca - xbaa, pd, \text{class } 3 \rangle ((x, y) \sim (-x, y)) \quad (7.4456)$$

8.221 Descendants of 6.60

7 + gcd($p - 1, 3$) algebras

$$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - \omega ba, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.4457)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - \omega ba, pa, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.4458)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - \omega ba, pa, pb - baa, pc - xbaa, pd, \text{class } 3 \rangle (\text{unique } x \text{ with } x^2 - \omega \text{ not a square}) \quad (7.4459)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb, da, db - ca, dc - \omega ba, pa, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.4460)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc - \omega ba, pa, pb, pc, pd, \text{class } 3 \rangle \quad (7.4461)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc - \omega ba, pa, pb, pc - baa, pd, \text{class } 3 \rangle \quad (7.4462)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc - \omega ba, pa, pb - baa, pc - xbaa, pd, \text{class } 3 \rangle (\text{unique } x \text{ with } x^2 - \omega \text{ not a square}) \quad (7.4463)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc - \omega ba, pa, pb, pc, pd - baa, \text{class } 3 \rangle \quad (7.4464)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc - \omega ba, pa, pb, pc, pd - \omega baa, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.4465)$$

$$\langle a, b, c, d \mid bab, bac, bad, cb - baa, da, db - ca, dc - \omega ba, pa, pb, pc, pd - \omega^2 baa, \text{class } 3 \rangle (p = 1 \pmod{3}) \quad (7.4466)$$

8.222 Descendants of 6.63

4 algebras

$$\langle a, b, c, d \mid ba, ca, cb, da, db, dc, pb, pc, pd, \text{class } 4 \rangle \quad (7.4467)$$

$$\langle a, b, c, d \mid ba - p^3 a, ca, cb, da, db, dc, pb, pc, pd, \text{class } 4 \rangle \quad (7.4468)$$

$$\langle a, b, c, d \mid ba, ca, cb - p^3 a, da, db, dc, pb, pc, pd, \text{class } 4 \rangle \quad (7.4469)$$

$$\langle a, b, c, d \mid ba, ca, cb - p^3 a, da - p^3 a, db, dc, pb, pc, pd, \text{class } 4 \rangle \quad (7.4470)$$

8.223 Descendants of 6.67

$18 + 8 \gcd(p - 1, 3) + 3 \gcd(p - 1, 4)$ algebras

- $$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4471)$$
- $$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb - baaa, pc, pd, \text{ class } 4 \rangle \quad (7.4472)$$
- $$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb - \omega baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4473)$$
- $$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4474)$$
- $$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4475)$$
- $$\langle a, b, c, d \mid bab, ca, cb, da, db, dc, pa - baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4476)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa, pb, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4477)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa, pb - baaa, pc, pd, \text{ class } 4 \rangle \quad (7.4478)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa, pb - \omega baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4479)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4480)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4481)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa - baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4482)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa - \omega baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4483)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa - \omega^2 baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.4484)$$
- $$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc, pa - \omega^3 baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.4485)$$
- $$\langle a, b, c, d \mid bab, ca, cb - baaa, da, db, dc, pa, pb, pc, pd - baaa, \text{ class } 4 \rangle \quad (7.4486)$$
- $$\langle a, b, c, d \mid bab, ca, cb - baaa, da, db, dc, pa, pb, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4487)$$
- $$\langle a, b, c, d \mid bab, ca, cb - baaa, da, db, dc, pa, pb - baaa, pc, pd, \text{ class } 4 \rangle \quad (7.4488)$$
- $$\langle a, b, c, d \mid bab, ca, cb - baaa, da, db, dc, pa, pb - \omega baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4489)$$
- $$\langle a, b, c, d \mid bab, ca, cb - baaa, da, db, dc, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4490)$$
- $$\langle a, b, c, d \mid bab, ca, cb - baaa, da, db, dc, pa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4491)$$

$$\langle a, b, c, d \mid bab, ca, cb - baaa, da, db, dc, pa - baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4492)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa, pb, pc, pd - baaa, \text{ class } 4 \rangle \quad (7.4493)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa, pb, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4494)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa, pb, pc - \omega baaa, pd, \text{ class } 4 \rangle \quad (7.4495)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa, pb - baaa, pc, pd, \text{ class } 4 \rangle \quad (7.4496)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa, pb - \omega baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4497)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4498)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4499)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa - baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4500)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa - \omega baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4501)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa - \omega^2 baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.4502)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb - baaa, da, db, dc, pa - \omega^3 baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 4) \quad (7.4503)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4504)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa, pb, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4505)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa, pb - baaa, pc, pd, \text{ class } 4 \rangle \quad (7.4506)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa, pb - baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4507)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa, pb - \omega baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4508)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa, pb - \omega baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4509)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4510)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (p = 1 \bmod 3) \quad (7.4511)$$

$$\langle a, b, c, d \mid bab, ca, cb, da, db, dc - baaa, pa - baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4512)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4513)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa, pb, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4514)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - baaa, pc, pd, \text{ class } 4 \rangle \quad (7.4515)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4516)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - \omega baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.4517)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - \omega baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.4518)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.4519)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.4520)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa - baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4521)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa - \omega baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4522)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa - \omega^2 baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.4523)$$

$$\langle a, b, c, d \mid bab - baaa, ca, cb, da, db, dc - baaa, pa - \omega^3 baaa, pb, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{4}) \quad (7.4524)$$

8.224 Descendants of 6.72

$17 + (p + 7) \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb, pc, pd - baaa, \text{ class } 4 \rangle \quad (7.4525)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb, pc, pd, \text{ class } 4 \rangle \quad (7.4526)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4527)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb - baaa, pc, pd, \text{ class } 4 \rangle \quad (7.4528)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb - baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (7.4529)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb - \omega baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.4530)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb - \omega baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.4531)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.4532)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa, pb - \omega^2 baaa, pc - baaa, pd, \text{ class } 4 \rangle \quad (p = 1 \pmod{3}) \quad (7.4533)$$

- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa - baaa, pb, pc, pd, \text{class } 4 \rangle$ (7.4534)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc, pa - baaa, pb, pc - baaa, pd, \text{class } 4 \rangle$ (7.4535)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb, pc, pd - baaa, \text{class } 4 \rangle$ (7.4536)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb, pc, pd, \text{class } 4 \rangle$ (7.4537)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb, pc - baaa, pd, \text{class } 4 \rangle$ (7.4538)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb - baaa, pc, pd, \text{class } 4 \rangle$ (7.4539)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb - baaa, pc - baaa, pd, \text{class } 4 \rangle$ (7.4540)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb - \omega baaa, pc, pd, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.4541)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb - \omega baaa, pc - baaa, pd, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.4542)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb - \omega^2 baaa, pc, pd, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.4543)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa, pb - \omega^2 baaa, pc - baaa, pd, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.4544)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc, pa - baaa, pb, pc, pd, \text{class } 4 \rangle$ (7.4545)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb, pc, pd - baaa, \text{class } 4 \rangle$ (7.4546)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb, pc, pd - \omega baaa, \text{class } 4 \rangle$ (7.4547)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb, pc, pd, \text{class } 4 \rangle$ (7.4548)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb, pc - baaa, pd, \text{class } 4 \rangle$ (7.4549)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb - baaa, pc, pd, \text{class } 4 \rangle$ (7.4550)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb - baaa, pc - baaa, pd, \text{class } 4 \rangle$ (7.4551)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb - \omega baaa, pc, pd, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.4552)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb - \omega baaa, pc - baaa, pd, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.4553)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc, pd, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.4554)
- $\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc - baaa, pd, \text{class } 4 \rangle$ ($p = 1 \pmod{3}$) (7.4555)

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db, dc - baaa, pa - baaa, pb, pc, pd, \text{class } 4 \rangle \quad (7.4556)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb, pc, pd - baaa, \text{class } 4 \rangle \quad (7.4557)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb, pc, pd - \omega baaa, \text{class } 4 \rangle \quad (7.4558)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb - xbaaa, pc - baaa, pd, \text{class } 4 \rangle \quad (7.4559)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb - xbaaa, pc - \omega baaa, pd, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.4560)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb - xbaaa, pc - \omega^2 baaa, pd, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.4561)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb - baaa, pc, pd, \text{class } 4 \rangle \quad (7.4562)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb - \omega baaa, pc, pd, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.4563)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb - \omega^2 baaa, pc, pd, \text{class } 4 \rangle (p = 1 \bmod 3) \quad (7.4564)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa, pb, pc, pd, \text{class } 4 \rangle \quad (7.4565)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa - baaa, pb, pc, pd, \text{class } 4 \rangle \quad (7.4566)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa - \omega baaa, pb, pc, pd, \text{class } 4 \rangle \quad (7.4567)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa - \omega^2 baaa, pb, pc, pd, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.4568)$$

$$\langle a, b, c, d \mid ca, cb - baa, bab, da, db - baaa, dc - baaa, pa - \omega^3 baaa, pb, pc, pd, \text{class } 4 \rangle (p = 1 \bmod 4) \quad (7.4569)$$

8.225 Descendants of 6.9

7 algebras

$$\langle a, b, c, d \mid ba, ca, cb, da, db, dc, p^2b, pc, pd, \text{class } 3 \rangle \quad (7.4570)$$

$$\langle a, b, c, d \mid ba - p^2a, ca, cb, da, db, dc, p^2b, pc, pd, \text{class } 3 \rangle \quad (7.4571)$$

$$\langle a, b, c, d \mid ba, ca - p^2a, cb, da, db, dc, p^2b, pc, pd, \text{class } 3 \rangle \quad (7.4572)$$

$$\langle a, b, c, d \mid ba, ca, cb - p^2a, da, db, dc, p^2b, pc, pd, \text{class } 3 \rangle \quad (7.4573)$$

$$\langle a, b, c, d \mid ba, ca, cb - p^2a, da - p^2a, db, dc, p^2b, pc, pd, \text{class } 3 \rangle \quad (7.4574)$$

$$\langle a, b, c, d \mid ba, ca, cb, da, db, dc - p^2a, p^2b, pc, pd, \text{class } 3 \rangle \quad (7.4575)$$

$$\langle a, b, c, d \mid ba - p^2a, ca, cb, da, db, dc - p^2a, p^2b, pc, pd, \text{class } 3 \rangle \quad (7.4576)$$

8.226 Descendants of 5.1

178 algebras if $p = 3$, and $p^2 + 15p + 125$ otherwise

- $\langle a, b, c, d, e \mid ba, ca, da, ea, cb, db, eb, dc, ec, ed, pc, pd, pe, \text{ class } 2 \rangle$ (7.4577)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4578)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pd, pe, \text{ class } 2 \rangle$ (7.4579)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pd - ba, pe, \text{ class } 2 \rangle$ (7.4580)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pa - ba, pb, pd, pe, \text{ class } 2 \rangle$ (7.4581)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc - ba, pd, pe, \text{ class } 2 \rangle$ (7.4582)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc, ec, ed, pb - ba, pc, pd, pe, \text{ class } 2 \rangle$ (7.4583)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4584)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pa, pb, pc, pd, \text{ class } 2 \rangle$ (7.4585)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pa - ba, pb, pc, pd, \text{ class } 2 \rangle$ (7.4586)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pb, pc, pd, pe - ba, \text{ class } 2 \rangle$ (7.4587)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pb - ba, pc, pd, pe, \text{ class } 2 \rangle$ (7.4588)
- $\langle a, b, c, d, e \mid ca, da, ea, cb, db, eb, dc - ba, ec, ed, pb, pc - ba, pd, pe, \text{ class } 2 \rangle$ (7.4589)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4590)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ba, eb, dc, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4591)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4592)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc - \omega ba, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4593)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4594)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4595)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa - ba, pb, pc, pd, pe, \text{ class } 2 \rangle$ (7.4596)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb - ba, pc, pd, pe, \text{ class } 2 \rangle$ (7.4597)

- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb - ca, pc, pd, pe, \text{class } 2 \rangle$ (7.4598)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb, pc, pd - ca, pe, \text{class } 2 \rangle$ (7.4599)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba, pb, pc, pd, pe, \text{class } 2 \rangle$ (7.4600)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa, pb - ba, pc, pd, pe, \text{class } 2 \rangle$ (7.4601)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba, pb - ba, pc, pd, pe, \text{class } 2 \rangle$ (7.4602)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba - ca, pb, pc, pd, pe, \text{class } 2 \rangle$ (7.4603)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba - ca, pb - ba - ca, pc, pd, pe, \text{class } 2 \rangle$ (7.4604)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe - ba, \text{class } 2 \rangle$ (7.4605)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe - ba - ca, \text{class } 2 \rangle$ (7.4606)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa - ba, pb, pc, pd, pe, \text{class } 2 \rangle$ (7.4607)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb, pc - ba, pd, pe, \text{class } 2 \rangle$ (7.4608)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa - ca, pb, pc, pd, pe, \text{class } 2 \rangle$ (7.4609)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb, pc - ca, pd, pe, \text{class } 2 \rangle$ (7.4610)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe - ba, \text{class } 2 \rangle$ (7.4611)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe - ca, \text{class } 2 \rangle$ (7.4612)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc - \omega ba, ea, eb, ec, ed, pa - ba, pb, pc, pd, pe, \text{class } 2 \rangle$ (7.4613)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc - \omega ba, ea, eb, ec, ed, pa, pb, pc, pd, pe - ba, \text{class } 2 \rangle$ (7.4614)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ba, pc, pd, pe, \text{class } 2 \rangle$ (7.4615)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc, pd, pe, \text{class } 2 \rangle$ (7.4616)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc - ca, pd, pe, \text{class } 2 \rangle$ (7.4617)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc, pd - ba, pe, \text{class } 2 \rangle$ (7.4618)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd - ba, pe, \text{class } 2 \rangle$ (7.4619)

- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc, pd - ca, pe, \text{class } 2 \rangle$ (7.4620)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ca, pd - ca, pe, \text{class } 2 \rangle$ (7.4621)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd, pe, \text{class } 2 \rangle$ (7.4622)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ca, pd, pe, \text{class } 2 \rangle$ (7.4623)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ba, pb, pc, pd, pe, \text{class } 2 \rangle$ (7.4624)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ca, pb, pc, pd, pe, \text{class } 2 \rangle$ (7.4625)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc, pd, pe - ba, \text{class } 2 \rangle$ (7.4626)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd, pe - ba, \text{class } 2 \rangle$ (7.4627)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc - \omega ba, pd, pe - ba, \text{class } 2 \rangle$ (7.4628)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ba, pc, pd, pe, \text{class } 2 \rangle$ (7.4629)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd, pe, \text{class } 2 \rangle$ (7.4630)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ba, pb, pc, pd, pe, \text{class } 2 \rangle$ (7.4631)
- $\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc, pd - ba, pe, \text{class } 2 \rangle$ (7.4632)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed, pa, pb, pc, pd - ba, pe - ca, \text{class } 2 \rangle$ (7.4633)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa - ca, pb - ba, pc, pd, pe, \text{class } 2 \rangle$ (7.4634)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa - ba, pb - ca, pc, pd, pe, \text{class } 2 \rangle$ (7.4635)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb - ba, pc - xca, pd, pe, \text{class } 2 \rangle$ ($x \neq 0, x \sim x^{-1}$) (7.4636)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb - ba - ca, pc - ca, pd, pe, \text{class } 2 \rangle$ (7.4637)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb - \omega ca, pc - ba, pd, pe, \text{class } 2 \rangle$ (7.4638)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb - xca, pc - ba - ca, pd, pe, \text{class } 2 \rangle$ ($1 + 4x$ not a square) (7.4639)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa - ba, pb, pc, pd - ca, pe, \text{class } 2 \rangle$ (7.4640)

- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb - ba, pc, pd - ca, pe, \text{class } 2 \rangle$ (7.4641)
- $\langle a, b, c, d, e \mid cb, da, db, dc, ea, eb, ec, ed, pa, pb, pc - ba, pd - ca, pe, \text{class } 2 \rangle$ (7.4642)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba, pb - ba, pc - ca, pd - ca, pe, \text{class } 2 \rangle$ (7.4643)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba, pb - ba, pc, pd - ca, pe, \text{class } 2 \rangle$ (7.4644)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba + ca, pb - ba, pc, pd - ca, pe, \text{class } 2 \rangle$ (7.4645)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa + ca, pb - ba, pc, pd - ca, pe, \text{class } 2 \rangle$ (7.4646)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa, pb - ba, pc, pd - ca, pe, \text{class } 2 \rangle$ (7.4647)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba - ca, pb - ba - xca, pc, pd, pe, \text{class } 2 \rangle$ ($x \neq 0, 1$) (7.4648)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba - ca, pb - ba, pc, pd, pe, \text{class } 2 \rangle$ (7.4649)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba - ca, pb - ca, pc, pd, pe, \text{class } 2 \rangle$ (7.4650)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ba, pb - ca, pc, pd, pe, \text{class } 2 \rangle$ (7.4651)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ca, pb - ba, pc, pd, pe, \text{class } 2 \rangle$ (7.4652)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ca, pb, pc, pd, pe - ba, \text{class } 2 \rangle$ (7.4653)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa + ca, pb, pc, pd - ca, pe - ba, \text{class } 2 \rangle$ (7.4654)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa, pb, pc, pd - ca, pe - ba, \text{class } 2 \rangle$ (7.4655)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa - ca, pb, pc, pd, pe - ba - ca, \text{class } 2 \rangle$ (7.4656)
- $\langle a, b, c, d, e \mid cb, da, db - ba, dc, ea, eb, ec, ed, pa, pb, pc, pd - ca, pe - ba - ca, \text{class } 2 \rangle$ (7.4657)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa - ba, pb - ca, pc, pd, pe, \text{class } 2 \rangle$ (7.4658)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa - ca, pb, pc - ba, pd, pe, \text{class } 2 \rangle$ (7.4659)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb - ca, pc - ba, pd, pe, \text{class } 2 \rangle$ (7.4660)
- $\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb - \omega ca, pc - ba, pd, pe, \text{class } 2 \rangle$ (7.4661)

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa - ba, pb, pc - ca, pd, pe, \text{ class } 2 \rangle \quad (7.4662)$$

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb - xba, pc - ca, pd, pe, \text{ class } 2 \rangle \quad (x \neq 0) \quad (7.4663)$$

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb, pc - ba, pd - ca, pe, \text{ class } 2 \rangle \quad (7.4664)$$

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa - ca, pb, pc, pd, pe - ba, \text{ class } 2 \rangle \quad (7.4665)$$

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb, pc - ca, pd, pe - ba, \text{ class } 2 \rangle \quad (7.4666)$$

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa - ba, pb, pc, pd, pe - ca, \text{ class } 2 \rangle \quad (7.4667)$$

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc, ea, eb, ec, ed, pa, pb, pc - ba, pd, pe - ca, \text{ class } 2 \rangle \quad (7.4668)$$

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc - \omega ba, ea, eb, ec, ed, pa, pb, pc - ba, pd - ca, pe, \text{ class } 2 \rangle \quad (7.4669)$$

$$\langle a, b, c, d, e \mid cb, da, db - ca, dc - \omega ba, ea, eb, ec, ed, pa, pb - ba, pc - xba - yca, pd, pe, \text{ class } 2 \rangle \quad (7.4670)$$

In presentation 7.4670 we have two parameters x, y with $y \neq 0$. Parameter pairs (x, y) and (z, t) give isomorphic algebras if and only if

$$\begin{pmatrix} 1 & 0 \\ z & t \end{pmatrix} = \begin{pmatrix} \mu & \nu \\ \omega\nu & \mu \end{pmatrix} \begin{pmatrix} 1 & 0 \\ x & y \end{pmatrix} \begin{pmatrix} \mu + \nu x & \nu y \\ \omega\nu y & \mu + \nu x \end{pmatrix}^{-1}$$

for some non-singular matrix $\begin{pmatrix} \mu & \nu \\ \omega\nu & \mu \end{pmatrix}$.

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ca, pb, pc, pd, pe - ba, \text{ class } 2 \rangle \quad (7.4671)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc, pd - ba, pe - ca, \text{ class } 2 \rangle \quad (7.4672)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd - ba, pe - ca, \text{ class } 2 \rangle \quad (7.4673)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ca, pd - ba, pe - ca, \text{ class } 2 \rangle \quad (7.4674)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ca, pb, pc, pd - ba, pe, \text{ class } 2 \rangle \quad (7.4675)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ba, pb, pc, pd - ca, pe, \text{ class } 2 \rangle \quad (7.4676)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ca, pb, pc - ba, pd - ba, pe, \text{ class } 2 \rangle \quad (7.4677)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd - ca, pe, \text{ class } 2 \rangle \quad (7.4678)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ba, pb, pc - ca, pd - ca, pe, \text{ class } 2 \rangle \quad (7.4679)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ca, pd - ba, pe, \text{ class } 2 \rangle \quad (7.4680)$$

- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb, pc - ca, pd - ba - ca, pe, \text{ class } 2 \rangle$ (7.4681)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ca, pb, pc - ba, pd, pe, \text{ class } 2 \rangle$ (7.4682)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ba, pb, pc - ca, pd, pe, \text{ class } 2 \rangle$ (7.4683)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ba, pc - xca, pd, pe, \text{ class } 2 \rangle (x \neq 0)$ (7.4684)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ba, pc - xca, pd - ca, pe, \text{ class } 2 \rangle (x \neq 0)$ (7.4685)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ba, pc, pd - ca, pe, \text{ class } 2 \rangle$ (7.4686)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ca, pb - ba, pc, pd, pe, \text{ class } 2 \rangle$ (7.4687)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ca, pb - ba, pc, pd - ca, pe, \text{ class } 2 \rangle$ (7.4688)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ba, pc - ba - ca, pd, pe, \text{ class } 2 \rangle$ (7.4689)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ba, pc - ba - ca, pd - ca, pe, \text{ class } 2 \rangle$ (7.4690)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ba, pb - ca, pc, pd, pe, \text{ class } 2 \rangle$ (7.4691)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc, pd - ba, pe, \text{ class } 2 \rangle$ (7.4692)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ba, pb - ca, pc, pd - ba, pe, \text{ class } 2 \rangle$ (7.4693)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ba, pb - ca, pc - ca, pd, pe, \text{ class } 2 \rangle$ (7.4694)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc - ca, pd - ba, pe, \text{ class } 2 \rangle$ (7.4695)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa - ba, pb - ca, pc - ca, pd - ba, pe, \text{ class } 2 \rangle$ (7.4696)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc - ba, pd, pe, \text{ class } 2 \rangle$ (7.4697)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc - ba, pd - ba, pe, \text{ class } 2 \rangle$ (7.4698)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc - \omega ba, pd, pe, \text{ class } 2 \rangle$ (7.4699)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc - \omega ba, pd - ba, pe, \text{ class } 2 \rangle$ (7.4700)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc - xba - ca, pd, pe, \text{ class } 2 \rangle (x \neq 0)$ (7.4701)
- $\langle a, b, c, d, e \mid da, ea, cb, db, eb, dc, ec, ed - ba, pa, pb - ca, pc - xba - ca, pd - ba, pe, \text{ class } 2 \rangle (x \neq 0)$ (7.4702)

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ba, pb, pc, pd - xca, pe, \text{class } 2 \rangle (x \neq 0, x \sim x^{-1}) \quad (7.4703)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ba - ca, pb, pc, pd - ca, pe, \text{class } 2 \rangle \quad (7.4704)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - \omega ca, pb, pc, pd - ba, pe, \text{class } 2 \rangle \quad (7.4705)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - xca, pb, pc, pd - ba - ca, pe, \text{class } 2 \rangle (1 + 4x \text{ not a square}) \quad (7.4706)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ca, pb, pc, pd, pe - ba, \text{class } 2 \rangle \quad (7.4707)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc, pd - ca, pe - ba, \text{class } 2 \rangle \quad (7.4708)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ca, pb, pc, pd - ca, pe - ba, \text{class } 2 \rangle \quad (7.4709)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ca, pb, pc - ba, pd, pe - ba, \text{class } 2 \rangle \quad (7.4710)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - xca, pb, pc - ba, pd - ca, pe - ba, \text{class } 2 \rangle (x \sim -x) \quad (7.4711)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ca, pb, pc - \omega ba, pd, pe - ba, \text{class } 2 \rangle \quad (7.4712)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - xca, pb, pc - \omega ba, pd - ca, pe - ba, \text{class } 2 \rangle (x \sim -x) \quad (7.4713)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ba, pc, pd - ca, pe, \text{class } 2 \rangle \quad (7.4714)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ca, pb - ba, pc, pd, pe, \text{class } 2 \rangle \quad (7.4715)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ca, pb - ba, pc, pd - ca, pe, \text{class } 2 \rangle \quad (7.4716)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd - ca, pe, \text{class } 2 \rangle \quad (7.4717)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa - ca, pb, pc - ba, pd, pe, \text{class } 2 \rangle \quad (7.4718)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc - xba - ca, pd, pe - ba - yca, \text{class } 2 \rangle (xy \neq 1, (x, y) \sim (y, x)) \quad (7.4719)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc - xba, pd, pe - ba - ca, \text{class } 2 \rangle (x \neq 0) \quad (7.4720)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd, pe - ca, \text{class } 2 \rangle \quad (7.4721)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb, pc - ba, pd, pe - \omega ca, \text{class } 2 \rangle (p = 1 \bmod 3) \quad (7.4722)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc - tba - xca, pd, pe - yba - zca, \text{class } 2 \rangle \quad (7.4723)$$

We consider the orbits of matrices

$$\begin{pmatrix} u & v \\ t & x \\ y & z \end{pmatrix}$$

where $(tz - xy)^2 - (ux - vt)(uz - vy)$ is not a square, under the action of non-singular matrices $\begin{pmatrix} \alpha & \varepsilon \\ \gamma & \eta \end{pmatrix}$ given by

$$\begin{pmatrix} u & v \\ t & x \\ y & z \end{pmatrix} \rightarrow (\alpha\eta - \gamma\varepsilon)^{-2} \begin{pmatrix} (\alpha\eta + \gamma\varepsilon) & 2\gamma\eta & -2\alpha\varepsilon \\ \varepsilon\eta & \eta^2 & -\varepsilon^2 \\ -\alpha\gamma & -\gamma^2 & \alpha^2 \end{pmatrix} \begin{pmatrix} u & v \\ t & x \\ y & z \end{pmatrix} \begin{pmatrix} \eta & -\gamma \\ -\varepsilon & \alpha \end{pmatrix}.$$

Each orbit contains a matrix with $u = 0$ and $v = 1$, and we pick one matrix of this form to obtain a representative set of parameters t, x, y, z for presentation 7.4723. The number of orbits is 4 when $p = 3$, $(p^2 - 1)/2$ when $p = 1 \bmod 3$, and $(p^2 + 1)/2$ when $p = 2 \bmod 3$.

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc, pd, pe - xba, \text{class } 2 \rangle (x \neq 0) \quad (7.4724)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc, pd - ca, pe - xba, \text{class } 2 \rangle (x \neq 0) \quad (7.4725)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc, pd, pe + ba - ca, \text{class } 2 \rangle \quad (7.4726)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc, pd - ca, pe + ba - ca, \text{class } 2 \rangle \quad (7.4727)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc - ba, pd, pe, \text{class } 2 \rangle \quad (7.4728)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc - ba, pd - ba, pe, \text{class } 2 \rangle \quad (7.4729)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc - \omega ba, pd, pe, \text{class } 2 \rangle \quad (7.4730)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc - \omega ba, pd - ba, pe, \text{class } 2 \rangle \quad (7.4731)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc - xba - ca, pd, pe, \text{class } 2 \rangle (x \neq 0) \quad (7.4732)$$

$$\langle a, b, c, d, e \mid da, ea, cb, db - ca, eb, dc, ec, ed - ba, pa, pb - ca, pc - xba - ca, pd - ba, pe, \text{class } 2 \rangle (x \neq 0) \quad (7.4733)$$

8.227 Descendants of 6.2

5 algebras

$$\langle a, b, c, d, e \mid ba, ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4734)$$

$$\langle a, b, c, d, e \mid ba - p^2a, ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4735)$$

$$\langle a, b, c, d, e \mid ba, ca, da, ea, cb - p^2a, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4736)$$

$$\langle a, b, c, d, e \mid ba, ca, da - p^2a, ea, cb - p^2a, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4737)$$

$$\langle a, b, c, d, e \mid ba, ca, da, ea, cb - p^2a, db, eb, dc, ec, ed - p^2a, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4738)$$

8.228 Descendants of 6.3

25 algebras

$$\langle a, b, c, d, e \mid ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4739)$$

$$\langle a, b, c, d, e \mid ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa - baa, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4740)$$

$$\langle a, b, c, d, e \mid ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa, pb - baa, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4741)$$

$$\langle a, b, c, d, e \mid ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa, pb - \omega baa, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4742)$$

$$\langle a, b, c, d, e \mid ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc - baa, pd, pe, \text{ class } 3 \rangle \quad (7.4743)$$

$$\langle a, b, c, d, e \mid ca, cb - baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4744)$$

$$\langle a, b, c, d, e \mid ca, cb - baa, bab, da, db, dc, ea, eb, ec, ed, pa - baa, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4745)$$

$$\langle a, b, c, d, e \mid ca, cb - baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb - baa, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4746)$$

$$\langle a, b, c, d, e \mid ca, cb - baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb - \omega baa, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4747)$$

$$\langle a, b, c, d, e \mid ca, cb - baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc - baa, pd, pe, \text{ class } 3 \rangle \quad (7.4748)$$

$$\langle a, b, c, d, e \mid ca, cb - baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc, pd - baa, pe, \text{ class } 3 \rangle \quad (7.4749)$$

$$\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb, ec, ed, pa, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4750)$$

$$\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb, ec, ed, pa - baa, pb, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4751)$$

$$\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb, ec, ed, pa, pb - baa, pc, pd, pe, \text{ class } 3 \rangle \quad (7.4752)$$

- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb, ec, ed, pa, pb - \omega baa, pc, pd, pe, \text{ class } 3 \rangle$ (7.4753)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb, ec, ed, pa, pb, pc - baa, pd, pe, \text{ class } 3 \rangle$ (7.4754)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb, ec, ed, pa, pb - baa, pc - baa, pd, pe, \text{ class } 3 \rangle$ (7.4755)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb, ec, ed, pa, pb - \omega baa, pc - baa, pd, pe, \text{ class } 3 \rangle$ (7.4756)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb, ec, ed, pa, pb, pc, pd, pe - baa, \text{ class } 3 \rangle$ (7.4757)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb - baa, ec, ed, pa, pb, pc, pd, pe - baa, \text{ class } 3 \rangle$ (7.4758)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb - baa, ec, ed, pa, pb, pc - baa, pd, pe, \text{ class } 3 \rangle$ (7.4759)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb - baa, ec, ed, pa, pb - baa, pc, pd, pe, \text{ class } 3 \rangle$ (7.4760)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb - baa, ec, ed, pa, pb - \omega baa, pc, pd, pe, \text{ class } 3 \rangle$ (7.4761)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb - baa, ec, ed, pa, pb, pc, pd, pe, \text{ class } 3 \rangle$ (7.4762)
- $\langle a, b, c, d, e \mid bab, ca, cb, da, db, dc - baa, ea, eb - baa, ec, ed, pa - baa, pb, pc, pd, pe, \text{ class } 3 \rangle$ (7.4763)

8.229 Descendants of 6.1

9 algebras

- $\langle a, b, c, d, e, f \mid ba, ca, da, ea, fa, cb, db, eb, fb, dc, ec, fc, ed, fd, fe, pb, pc, pd, pe, pf, \text{ class } 2 \rangle$ (7.4764)
- $\langle a, b, c, d, e, f \mid ca, da, ea, fa, cb, db, eb, fb, dc, ec, fc, ed, fd, fe, pa, pb, pc, pd, pe, pf, \text{ class } 2 \rangle$ (7.4765)
- $\langle a, b, c, d, e, f \mid ca, da, ea, fa, cb, db, eb, fb, dc, ec, fc, ed, fd, fe, pa - ba, pb, pc, pd, pe, pf, \text{ class } 2 \rangle$ (7.4766)
- $\langle a, b, c, d, e, f \mid ca, da, ea, fa, cb, db, eb, fb, dc, ec, fc, ed, fd, fe, pa, pb, pc - ba, pd, pe, pf, \text{ class } 2 \rangle$ (7.4767)
- $\langle a, b, c, d, e, f \mid ca, da, ea, fa, cb, db, eb, fb, dc - ba, ec, fc, ed, fd, fe, pa, pb, pc, pd, pe, pf, \text{ class } 2 \rangle$ (7.4768)
- $\langle a, b, c, d, e, f \mid ca, da, ea, fa, cb, db, eb, fb, dc - ba, ec, fc, ed, fd, fe, pa - ba, pb, pc, pd, pe, pf, \text{ class } 2 \rangle$ (7.4769)
- $\langle a, b, c, d, e, f \mid ca, da, ea, fa, cb, db, eb, fb, dc - ba, ec, fc, ed, fd, fe, pa, pb, pc, pd, pe - ba, pf, \text{ class } 2 \rangle$ (7.4770)
- $\langle a, b, c, d, e, f \mid ca, da, ea, fa, cb, db, eb, fb, dc - ba, ec, fc, ed, fd, fe - ba, pa, pb, pc, pd, pe, pf, \text{ class } 2 \rangle$ (7.4771)
- $\langle a, b, c, d, e, f \mid ca, da, ea, fa, cb, db, eb, fb, dc - ba, ec, fc, ed, fd, fe - ba, pa - ba, pb, pc, pd, pe, pf, \text{ class } 2 \rangle$ (7.4772)

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