

# Algebra 6.114

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Algebra 6.114 has presentation

$$\langle a, b, c \mid pa - ba, pb - cb, pc - kba - ca, \text{class } 2 \rangle \quad (k = 0, 1, \dots, p - 1).$$

Over all  $p$  values of  $k$ , algebra 6.114 has  $4p - 4$  descendants of order  $p^7$  and  $p$ -class 3. The cases  $k = -1$  and  $k = 3$  are straightforward, but things are more complicated when  $k \neq -1, 3$ . In these cases we have a parametrized family of algebras

$$\langle a, b, c \mid bac - zbab, pa - ba, pb - cb, pc - kba - ca, \text{class } 3 \rangle,$$

where (for a given  $k \neq -1, 3$ )  $z$  and  $z'$  define isomorphic algebras if the ratios  $1 : z$  and  $1 : z'$  are in the same orbit of ratios  $\alpha : \beta$  under the action

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \rightarrow A \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

where  $A$  equals

$$\begin{pmatrix} k - 1 & 1 \\ -1 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} k^2 - 2k & k - 1 \\ 1 - k & -1 \end{pmatrix} \text{ or } \begin{pmatrix} (1 + \gamma k)(\gamma k - 2\gamma + 1) & \gamma(\gamma k + 2 - \gamma) \\ -\gamma(\gamma k + 2 - \gamma) & -(-1 + \gamma)(\gamma + 1) \end{pmatrix}$$

with  $\gamma \neq -1$  and  $\gamma$  not a root of  $\gamma^2 + (k - 1)\gamma + 1 = 0$ . (Note that the ratio  $1 : 0$  is in the same orbit as the ratio  $0 : 1$ .)

A MAGMA program to compute a set of representative pairs  $(k, z)$  is given in Notes6.114.m.